Duke University
Department of Physics

Physics 391 Fall Term 2010

WUN2K FOR LECTURE 2

These are notes summarizing the main concepts you need to understand and be able to apply.

- For a situation in which one assumes some underlying "truth" description, but for which specific results can vary, the probability of a particular result is the ratio of the number of times that result occurs to the total number of occurrences. A probability is a number in the range 0 to 1, and the distribution of probabilities must be normalized, *i.e.* $\sum_{\text{possible results i}} P(i) = 1$ for discrete results, and $\int_{-\infty}^{\infty} P(x)dx$ for continuous results.
- Rules of probability:
  - The probability of A OR B occurring, $P(A+B)$, must satisfy $P(A+B) \leq P(A) + P(B)$, where the equality holds if A and B are independent.
  - The probability of A AND B occurring, $P(AB)$, is given by $P(AB) = P(A|B)P(B) = P(B|A)P(A)$, where the notation $P(A|B)$ means "the probability of A, given B". This relation is also known as "Bayes' Law".
- The *binomial distribution* is $B(x;n,p) = \begin{pmatrix} n \\ x \end{pmatrix} p^x q^{n-x}$: this is the probability of x "successes" in n trials, where the probability of success is p, and $q = 1-p$. (Here $\begin{pmatrix} n \\ x \end{pmatrix} = \frac{n!}{x!(n-x)!}$.) The mean of a binomial is $\mu = np$ and the variance is $\sigma^2 = np(1-p)$.
- The *Poisson distribution* is the limit of the binomial for the case $\mu << n$ and $p << 1$: $\mathcal{P}(x;\mu) = \frac{\mu^x}{x!}e^{-\mu}$. Its mean is μ and its variance is also

$\sigma^2 = \mu$; it's described by the single parameter μ. The Poisson is often useful for counting statistics.

- The *Gaussian distribution* is the limit of the binomial for $np >> 1$, and also the limit of the Poisson for $\mu >> 1$: it's $G(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$, with mean μ and variance σ^2. Some useful properties:

  $G(\mu \pm \sigma; \mu, \sigma) = G(\mu; \mu, \sigma)/e$.

  The "full width half max (FWHM)", for which $G(\mu \pm \frac{\Gamma}{2}; \mu, \sigma) = \frac{G(\mu;\mu,\sigma)}{2}$ is $\Gamma = 2.354\sigma$.

  The area between $\mu - \sigma$ and $\mu + \sigma$ is 68.3% of the total area; between $\mu - 2\sigma$ and $\mu + 2\sigma$ is 95.5%; and between $\mu - 3\sigma$ and $\mu + 3\sigma$ is 99.7%. Beware of "non-Gaussian tails" when going out many sigmas, though!