Duke University
Department of Physics

Physics 391 — Fall Term 2010

WUN2K FOR LECTURE 1

These are notes summarizing the main concepts you need to understand and be able to apply.

- Reported measurements must be accompanied by "uncertainties" or "errors" in order to interpret them meaningfully. Uncertainties can be categorized as:
  - "Statistical" or "random" uncertainties: this type of error approaches zero as the number of measurements goes to infinity.
  - "Systematic" uncertainties: this type of error may not approach zero as the number of measurements approaches infinity; it can be a true "error" due to some kind of problem with the apparatus, an unknown perturbation or background, a poor calibration, *etc.*

  In many cases one treats statistical and systematic errors in the same way when interpreting and combining uncertainties. However systematic errors can be much more difficult to estimate than statistical ones (if one *knew* the error, one could correct for it).

- For N measurements of some quantity x, $\{x_i\}$, the *sample*, one assumes that an underlying *parent* distribution $P(x)$ describes the probability of making a measurement x.
  - The *sample mean* is $\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$, and the parent distribution "true" mean μ is $\mu = \lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} x_i$.
  - The *sample variance* is $s^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2$; this approaches the "true" variance σ^2 in the limit $N \to \infty$. It is a measure of the spread of the parent distribution.

  We use $\bar{x}$ and s^2 as estimates of the "true" μ and σ^2, respectively.

- Propagation of errors: for some quantity $x = f(u, v, ...)$, the variance of x, σ_x^2, is given (to first order) by

$$\sigma_x^2 = \sigma_u^2 \left(\frac{\partial x}{\partial u}\right)^2 + \sigma_v^2 \left(\frac{\partial x}{\partial v}\right)^2 + ... + 2\sigma_{uv}^2 \left(\frac{\partial x}{\partial u}\right)\left(\frac{\partial x}{\partial v}\right) + ...$$

where $\sigma_u^2 = \lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N}(u_i - \bar{u})^2$ is the variance of u, *etc.*, and $\sigma_{uv}^2 = \lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N}[(u_i - \bar{u})(v_i - \bar{v})]$ is the *covariance* of u and v. For uncorrelated measurements, the covariance terms vanish as $N \to \infty$.