Review Guide and Problems For Physics 53

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Overview

The following is a brief collection of possible study strategies for the Physics 53 final exam. These are strictly optional; feel free to use any study strategy you like. Bear in mind that you should *balance* your preparation for the Physics 53 final with your preparations for your other finals. Limit your physics study (for better or worse) to what you can afford, probably no more than 2-4 hours a day (the latter only if broken up into a couple of sessions). This should be plenty of time, provided you've worked diligently on the material for the entire semester.

When confronted with the need to prepare for the Physics 53 final, it is easy to panic. Don't panic, just relax. Sure, we covered a lot of material. On the other hand, you could easily fit all the equations you have to "memorize" onto a couple of sheets of paper without writing particularly small, especially if you don't just memorize them but rather **learn** them. Many of the relations you might be tempted to memorize can easily be derived in a few steps from a relatively small set of starting equations, and understanding how to go about this is a major part of understanding or learning the material.

I personally don't remember off the top of my head any of the details of any particular solution to (e.g.) an inclined plane problem or gravity problem or wave problem. What I remember instead are Newton's Law(s) and how to solve constant acceleration problems in general, Newton's Law of Gravitation and its potential energy equivalent (and the various kinematical relations like the definition of kinetic energy, angular momentum and what have you) and the general form(s) of solutions to the wave equation. For a specific problem, I then take these relatively simple equations, relations, and rules and fit them to the physics of the problem, and then systematically (algebraically) solve for the unknown requested. If I had to (or tried to) remember all the equations in the book, I'd go nuts too.

That is why I emphasize (again) that the "right" way to study for the test is **not** by memorizing formulae; it is by solving and re-solving your webassign, homework, quiz, and exam problems until you can do them all perfectly. The webassign and homework problems will generally teach you what you need to memorize in terms of starting points even as they help you gut-level learn what those starting points mean in terms of pushes and pulls and masses, which (after all) is mostly what this semester has been all about. Most of the problems we solve come down to figuring out what pushes on what, how things respond to the pushes, and using your intuition to guide the algebra from a few simple starting points.

Consider rereading the online notes and the text (very quickly, of course)– frequently it contains insight and understanding is more important than memorization, especially for answering the "concept" multiple choice or T/F questions. If there are any points that still confuse you about the right answer to a concept-type question, go to the review session(s) and ask questions until you are Enlightened.

After a while, though, one can get to the point where just going over problems one has already seen ceases to be of benefit. One needs new problems "like" the previously assigned problems to work with to see if one has learned what is needed to solve them. A large collection of problems follows these study suggestions. Feel free to use them working alone or in groups to test your knowledge and understanding of the core material covered in this course. It is impossible to guarantee that this collection contains *all* of the possible kinds of problems you might see on the final, but *most* of the possible kinds are probably represented by one or more examples in this collection.

With that said, here are some specific study hints and suggestions.

Study Hint 1:

To start with, don't *worry* about the course or your grade, at least if you've been working hard and really trying to learn all semester. In my experience it is rare for a student who has worked hard, done their homework, attended lecture and recitation, and done their best to learn the material to fail the course. It is not impossible, but it is very rare. So **relax**.

Plan out your study schedule for the next four days – don't just wing it and study until you drop. I'd recommend studying physics at least three hours per day, and at most five, in one or two sessions. The two times physics study is likely to be the most effective is mid-morning when you are fully awake and able to really focus and in the evening well after dinner but before you are really tired (say, from 8 to 10 or 9 to 11, depending on your sleep schedule). Mid to late afternoon may be ok depending on your personal rhythms. Avoid studying right after lunch or dinner – a big meal robs the brain of needed blood while it digests and it is difficult to focus.

Organize (if you haven't done so already) physics **study groups**. Studying in a group (like the recitation groups) is almost certainly going to be less frustrating and more time-efficient than studying alone. At this point, there should be few problems indeed that nobody in a group of four students from our class cannot get between them, but any given student could well be stumped by any given problem.

Stay calm and get **plenty of sleep**. Sleep actually aids in the formation of long term memory and understanding (not to mention keeping you mentally healthy in other ways. At this point you SHOULDN'T be spending as much time LEARNING the material as you are REVIEWING the material and reminding yourself of how this problem or that problem is solved. If you are one of the few students in the class who really did blow off working honestly on most of the material and is counting on "learning" it now in a few 20 hour days of study, you have my profound sympathies as I'm deeply skeptical of your the chances of your strategy succeeding.

The last piece of "state advice" I have is to consider drinking a **caffienated beverage** immediately before the exam (provided that you are not one of those people that overrespond to caffiene). A moderate dose of caffiene (a single cup of coffee or tea or a coke) is known to measurably increase your IQ by about 5-10 points, probably by bringing you to a state of full alertness and postponing mental fatigue. Too much isn't useful, of course – I don't mean to suggest that you drink so much coffee that you come in "wired".

You will do your best on the test *by far* if you can enter the exam room on test day **relaxed** (with no concern for the results), **rested** (a good night's sleep and NOT after studying physics all day Monday), **alert** (after a *light* meal accompanied by a nice cup of tea or coffee), and possibly in possession of a nice *dark chocolate bar* (or even two) to fortify your brain an hour or so in – a bit more caffiene-analogue in the chocolate and some sugar to feed those grey cells. No drinks in the exam hall, of course, please – to easy to have a disaster.

Study Hint 2:

A *good* student would be sure to come in prepared at least to answer any of the previously assigned problems (from the homework, webassign, lab, recitation, quizzes, hour exams) perfectly. A *good* student would take *special* care to learn why their incorrect answer to problem or a multiple choice or true/false question was incorrect and how and why they should have solved for or identified the correct answer. A *good* student would take any of their previously assigned problems they still don't understand and cannot readily solve to the review session(s) or study group sessions and work out their remaining issues.

The point is that although the actual problems themselves may be different from your homework or quizzes or previous exams on the final (although some might *not* be different) but the physics concepts being tested are for ever the same.

So, be a *good student!*

If you can answer *all* of your homework and exam problems **without looking back in the text and understanding the methodology or reasoning used**, and **know the various laws, rules, and theorems by heart** you can almost certainly get in the high 90's on the final. It therefore makes sense to study (by doing and redoing) your homework, quiz and hour exam problems until you can answer them **quickly and intelligently** without looking.

Pay particular attention to learning **how to start a problem**, i.e. – identifying the relevant physics and starting equations – so that algebraic skill has a *chance* of carrying you to the correct answer. If you don't start a problem right (and don't *understand* the problem and what it is asking you to do) you don't have much chance of solving it.

More pragmatically, the graders typically give substantial partial credit for just having the relevant equations (preferrably not mixed in with a ton of irrelevant equations in a "shotgun" attempt) on paper even if you don't know what to do with them once you get them there. For example, when discussing the work done by the **adiabatic expansion** of an ideal gas it is nice to see $PV^\gamma =$ constant somewhere on the page, even if you don't figure out that you need to integrate e.g. $W = \int P_0 V_0^\gamma dV/V^\gamma$. Blank pages get no credit at all.

Here is an instructive moral: I have in the past in more than one semester presented a particularly lovely problem in lecture and then assigned it for homework. Then I've given that problem as the quiz for that chapter. I've put the problem *again* on the hour exam for that chapter. Then I've gone and put the problem *yet again* on the final, and *still* had students miss it on the final.

These students are *not good students* of course. Almost by definition, although one can always encounter a poor unfortunate soul who isn't a bad student but simply cannot learn physics. I and my colleagues are *perfectly happy to fail bad students*, however much we'll try *not* to fail a student who tries very hard but simply cannot succeed.

Study Hint 3:

There are a few things that you *must* enter the exam having *learned*. You know what they are (or should) but here is a list anyway:

- SI units and dimensional analysis
- Vector algebra – adding, subtracting, dot product, cross product.
- Kinematic relations: constant acceleration results and circular motion
- Dynamic relations: Newton's Laws
- Forces in Nature (and forces used in the course): Gravity (near earth's surface and planetary), weak nuclear, electromagnetic, strong nuclear, friction, drag forces, springs, bouyant forces, the various moduli (Young's, shear, bulk).
- "Effective" forces in accelerating frames.
- Work-Energy Theorem and Friends
- Potential Energy and Conservation of Energy (one of the Friends)
- Systems of particles and momentum. Conservation of Momentum.
- Angular variables and relations.
- Angular Physics: Torque, angular momentum, angular acceleration, angular energy.
- Conditions for static equilibrium.
- Kepler's Laws
- Archimedes' Principle, Bernoulli Equation, Pascal Principle, Toricelli's Law
- Oscillation, Resonance and Damping
- Waves: on a string, sound waves, doppler shift, standing waves, interference
- Laws of thermodynamics (0-3, but especially 1 and 2)
- Equipartition (degrees of freedom), specific heat(s), heat capacity(s)
- Engines and refrigerators
- Thermal expansion and contraction
- Latent heats

"Learned" in this context means "able to solve problems in or with". It also means "understands well enough to be able to conceptually reason out puzzles with". Some of these items (e.g. Laws) are just equations or 1-2 sentence phrases. They can be memorized (and certainly shouldn't be forgotten) but they really should be learned, which involves a bit more. Other items are topics where these laws are applied.

Study Hint 4:

The **labs**, **unassigned homework** problems, problems found **on the web**, problems found in other **introductory physics texts**, problems found **below**, problems found in the records of **other professors' quizzes and exams for related courses** are also good sources for **practice exam problems**, if you manage to get to where you can do all the *assigned* homework and quizzes and exams perfectly and are still hungry for more problems.

You will find that a lot of these sources contain problems that are surprisingly alike. The same general elementary physics problems are **recycled** or **reinvented** from year to year, book to book, professor to professor. It simply is not possible to come up with a Bernoulli's equation problem (to pick just one of a myriad of examples) that does not look much like all the rest, once you strip the "story" down to its essential elements. The story "frame" can be dressed up differently (beer vs water vs liquid sodium can flow from a keg or a pump or in the cooling system of a nuclear reactor) but it is still basically fluid flowing under pressure uphill and/or downhill through pipes of varying sizes. Similarly, how many different kinds of inclined plane problem or pulley problem can one really encounter? With or without friction, rolling or sliding, with or without a massive pulley – even a plane with or without a pair of masses and a pulley – but these are all easy variations to handle, with just a bit of practice.

It is a good idea to do *a few* of almost any kind of problem or work your way through a set of problems that explore different aspects of a given topic or concept, but **once you get the idea, go on to the next concept**. Don't waste a huge amount of time studying constant acceleration motion when you already know it and don't expect more than one or two questions on the exam on it.

While studying for the final (as in all things in life) remain aware of the **cost/benefit** of your efforts. Spend your time where it is likely to have concrete benefit. We don't deliberately make up exams to punish you – we tend to draw *most* of the questions from the things that are the *most important* in a totally straightforward way, and include one or two on the "esoteric" topics or more difficult problems to see who is *really* good and perhaps deserves an "A" thereby.

In a medical career, a common aphorism is "if you hear hoofbeats, think horses, not zebras". This is intended to remind a physician that most of the time, patients presenting with symptoms that belong both to (say) the common cold and the ebola virus are likely to have the former (at least in this country – in Africa one might well think zebras). It's harmless to be aware of the latter possibility, but focus your energies on the likely outcome.

In physics, a similar analysis would have you focus the bulk of your study energies on the bread and butter problems of the course – Newton's Law (second, of course) in any of a dozen contexts, energy conservation, torque and rotation – and less on the single-topic chapters. After all, you'll need Newton's second law and/or energy conservation or momentum conservation on more than one problem – I guarantee it. You *might* have a problem on Archimedes' Principle, on thermal expansion, on shear stress and shear strain (and collectively you're nearly certain to have a problem on one of them) but with only eight problems there will be topics with no coverage or coverage only in a short answer MC or T/F question.

Obviously, I don't *recommend* going into the exam knowing anything less than everything, but when it comes to recommending study focus, clearly you're better off ensuring that you do well on Newton's Second Law than on Archimedes' Principle. In fact, you very likely would be unable to do a lot of Archimedes' Principle problems without also knowing Newton's Second Law and how to use it! Basics first, then add on the specific, topical laws and applications (which, if you think of it, is precisely how the entire course was developed).

Conclusion

Good luck, and it has been a pleasure teaching you this semester. I hope that you got something out of the course and that it challenged you to start seeing the world as a rational, reasonably deterministic place where you can, if you work at it, figure out "how things work" in a very general sense. If "magic" is something that happens outside of physics, I sincerely hope that the course taught you to not believe in magic. Any *real* magic must live within the bounds of *some* physics (just like most of the industrial and technological magic that we currently take for granted but that would get you burned at the stake 600 years ago), even if it is a larger one than the one we currently understand. For those of you continuing in a scientific, medical or engineering discipline, I hope that you also learned those details that will help you in your future careers. Most of all, I hope that you learned a little bit more than you did before about how to *think*, or critically analyze almost any real problem you might encounter.

The following collection of problems is drawn from my years teaching physics 51 in a different format than currently used. These problems were assigned on quizzes and hour exams and final exams, and have been selected on the basis of being "compatible" with the requirements and expectations of this course. I've tried a bit to eliminate the totally irrelevant and redundant and supply at least some of the figures missing from the first posting of this review guide.

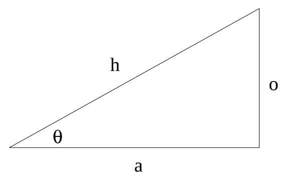
Hopefully you will find it useful.

Sample Problems

Basic Skills

Everybody should know this sort of thing (and more, but certainly this).

1. Trigonometry



$$\sin(\theta) =$$
$$\cos(\theta) =$$
$$\tan(\theta) =$$

2. $$\int \sin(\theta) d\theta =$$

3. Solve for x:

$$5x + 5y = 10$$
$$5x - 2z = 4$$
$$2z - y = 0$$

4. $$\frac{d}{dt}(at^5 + be^{-ct^2} + sin(dt)) =$$

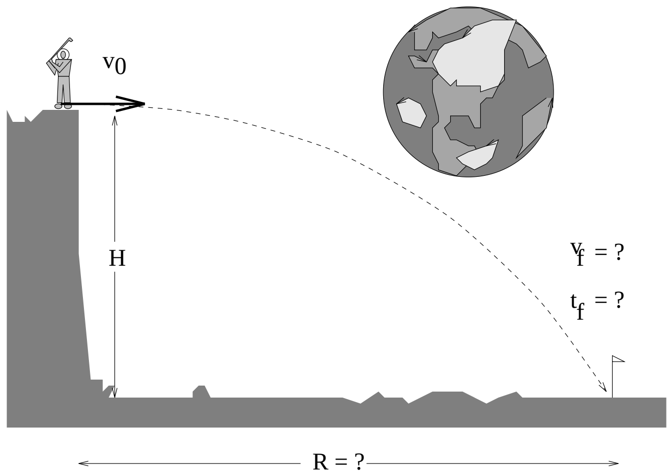
5. Solve for x:
$$ax^2 + bx + c = 0$$

6. Cross product (give magnitude and direction)
$$\vec{A} \times \vec{B} =$$

7. Dot product
$$\vec{A} \cdot \vec{B} =$$

8. Binomial expansion (I'm just giving you this because I think it is immensely useful for understanding things like tides, which are the *difference* between two almost equal reciprocal powers).
$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$
(Note that this converges for $|x| < 1$ only, which dictates the algebra used in its typical application. Well worth learning for next semester, if nothing else.)

Problems of Years Past

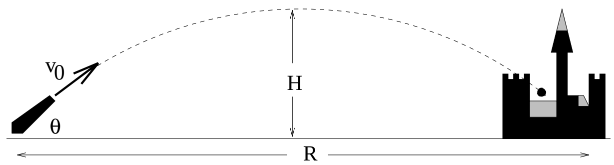


1. An astronaut on the moon hits a golf ball horizontally from the top of a cliff 750 m high. The initial speed of the ball is $v_0 = 120$ m/sec. Assume that the acceleration due to gravity on the moon is $a_m = g/6 = 5/3 = 1.67$ m/sec^2.

   a) How long does it take the ball to reach the ground (neglect the curvature of the moon)?

   b) How far from the base of the cliff does the ball strike?

   c) How fast is the ball going when it hits the ground?



2. A cannoneer is trying to hit a city on a plain. The city is 1 kilometer horizontally away from the cannon mouth. The cannon fires projectiles only at $\sqrt{2} \times 10^2$ m/sec. Assume $g = 10$ m/sec^2.

   a) Find the angles at which the cannoneer can set the cannon in order to destroy the city.

   b) How high does the cannonball go in each case?

   c) How long do the people in the city have to wait between seeing the flash of the cannon and the arrival of the cannonball? Obviously we should ignore the speed of light in this one...

3. A car is travelling at 50 meters/second directly towards a brick wall. When the car is 250 meters away from the wall, its brakes are applied and the car slows down with a (negative) acceleration of -5 m/sec^2.

   Does the car smash into the wall? If so, at what speed.

4. Patrick (my son) walks out of the house and sees the schoolbus about to pull away. He runs at it from behind at 4 meters/second. When he is twenty meters behind the still-open schoolbus door, the bus starts to pull away at an acceleration of 0.5 m/sec^2.

   Does Patrick catch the bus before it pulls away, avoiding the disaster of having DAD drive him to school? If so, how far has the bus gone when he reaches it?

5. A string of mass density μ is stretched to a tension T and fixed at $x = 0$ and $x = L$. The transverse string displacement is measured in the y direction. All answers should be given in terms of these quantities or new quantities you define in terms of these quantities.

   a) Write down the wave equation (the differential equation of motion) for waves on a string. You do not have to derive it.

   b) Find $k_n, \omega_n, f_n, \lambda_n$ for the first four modes supported by the string. Sketch them in on the figure above, labelling nodes and antinodes.

   c) Write down the equation for standing waves on this string, with mode index n, assuming that each mode is maximally displaced at $t = 0$.

   d) Find the total energy in **one** of these modes, assuming that it has an amplitude A.

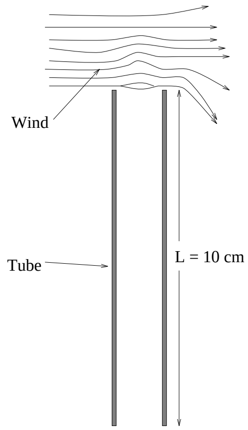
6. A string of mass density μ is stretched to a tension T and is fixed at $x = 0$ but *free* at $x = L$. The transverse string displacement is measured in the y direction. All answers should be given in terms of these quantities or new quantities you define in terms of these quantities.

   a) Write down the wave equation (the differential equation of motion) for waves on a string. You do not have to derive it.

   b) Find $k_n, \omega_n, f_n, \lambda_n$ for the first four modes supported by the string. Sketch them in on the figure above, labelling nodes and antinodes.

   c) Write down the equation for standing waves on this string, with mode index n, assuming that each mode is maximally displaced at $t = 0$.

   d) Find the total energy in **one** of these modes, assuming that it has an amplitude A.

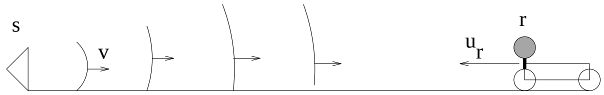


7. Wind or human breath running over the top of a pan-pipe (a tube open at both ends) can drive resonances and make several harmonic tones. Suppose sound waves are generated in this manner in a narrow tube of length $L = 10$ cm open at both ends. Express your answers to the questions below in terms of L and v (the speed of sound in air) before substituting any values to get a numerical answer. You do **not** have to derive the answers to a-c, just show me that you know the answers.

   a) Write down the wave equation (the differential equation of motion) for longitudinal *displacement* (sound) waves in air.

   b) Write down the *solution* to this equation that describes resonant standing waves in a tube **open at both ends**, assuming that each mode is maximally displaced at $t = 0$.

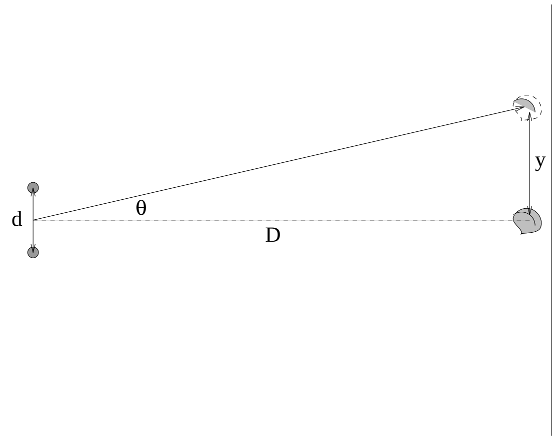
   c) Find $k_n, \omega_n, f_n, \lambda_n$ for the first four modes (resonant frequencies) supported by the tube. Sketch the displacement amplitudes in on the figure above the same way it was done in the book and lecture, labelling nodes and antinodes.



8. A microphone mounted on a cart is moved directly toward a harmonic source at a speed of 34 m/sec. The harmonic source is emitting sound waves at a frequency of 1360 Hz.

   a) **Derive** an expression for the frequency of the waves picked up by the moving microphone.

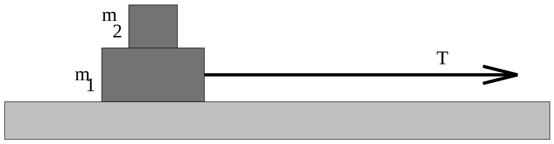
   b) What is that frequency?



9. Sarah puts two small speakers $d = 2$ meters apart on one wall of her spacious dorm room. Unfortunately, she fails to note that her speakers are connected **out of phase**. She plays a stereo rendition of "the harmonic frequency 340 Hz" and notices that she can hardly hear it with her chair in the middle a distance $D = 10$ meters away (told you it was a spacious dorm room).

   a) What is the smallest distance y she can move her chair up along the wall to be able to hear the sound clearly? (Show explicitly why this is true, don't just write down a formula).

   b) What else could she do to enable herself to listen with her chair in the middle (where it is)? Explain algebraically.

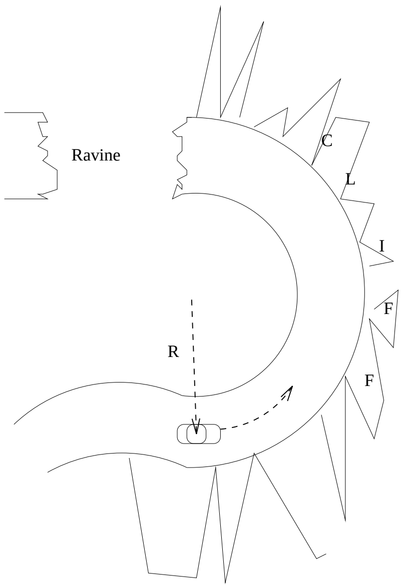


10. A person pulls some blocks resting on a table with friction by means of a rope as shown. The coefficients of friction between the bottom block and the table are $\mu_s(bt) = 0.5$ and $\mu_k(bt) = 0.3$. The coefficients of friction between the blocks themselves are $\mu_s(bb) = 0.2$ and $\mu_k(bb) = 0.1$. The mass of the blocks is $m_1 = 6$ kg and $m_2 = 3$ kg. The blocks are initially at rest.

    Find:

    a) The minimum tension in the rope such that the blocks start to move.

    b) Suppoe that the tension in the rope is 54 N (bigger than the minimum tension needed to start the blocks moving). Find the acceleration of **both** blocks. (Do they move together or separately?)

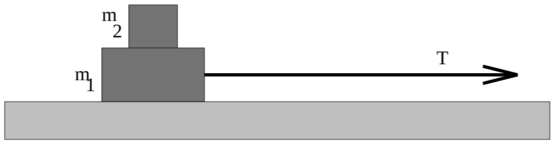


11. A car is rounding the unbanked Dead Man's Curve at a speed of 40 m/sec when the driver observes (at the initial position shown) that the bridge is out and hits the brakes. $R = 500$ m for this curve, and the tires have a coefficient of static friction $\mu_s = 0.5$.

    a) Can the car go around the curve travelling a constant speed without skidding off of the cliff and Plunging to its Doom?

    b) If the car were equipped with antilock brakes could their tangential force (exerted as if the car were travelling in a straight line of length πR), could it stop before Plunging to its Doom in the ravine?

    c) Will the car Plunge to its Doom no matter what the driver does? Justify this answer?



12. A 75 kg person skis down a frictionless slope of height $H = 60$ m, then rounds a circular bottom, goes up 5 m, and flies off of a jump as shown at an angle $\theta = 45\circ$.

Find:

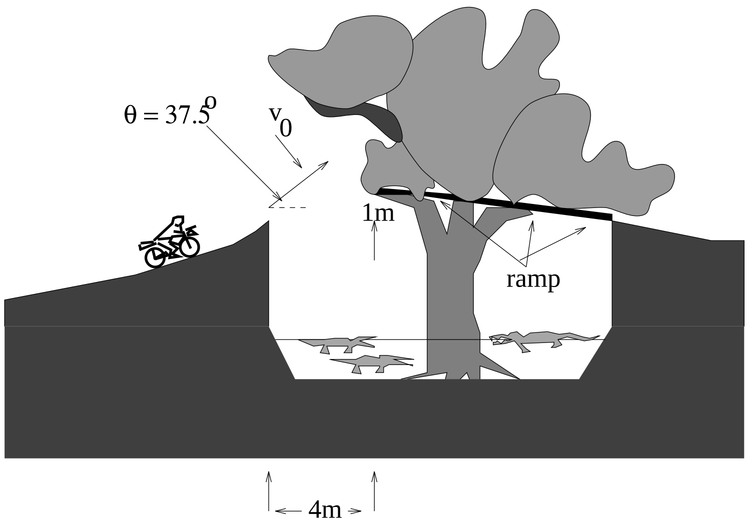
a) The maximum speed of the person at the lowest point (at the bottom). (Note – FYI – that if I told you the radius of curvature of the slope you could easily find the normal force exerted by the slope or the person's legs at this point, right?)

b) The maximum height that the person attains after flying off of the jump. (Note that I could just as easily have asked you the range of the jump or how long it takes to hit (requiring the use of Newton's 2nd Law), but that this answer you can get either from N2L or from energy conservation...)

13. A block of mass $M = 1$ kg is propelled by a spring with spring constant $k = 10$ N/m onto a smooth (frictionless) track. The spring is initially compressed a distance of 0.5m from its equilibrium configuration ($x_i - x_0 = 0.5$ m). At the end of the track there is a rough inclined plane at an angle of 45o with respect to the horizontal and with a coefficient of kinetic friction $\mu_k = 0.5$.

a) How far up the incline will the block slide before coming to rest (find H_f)?

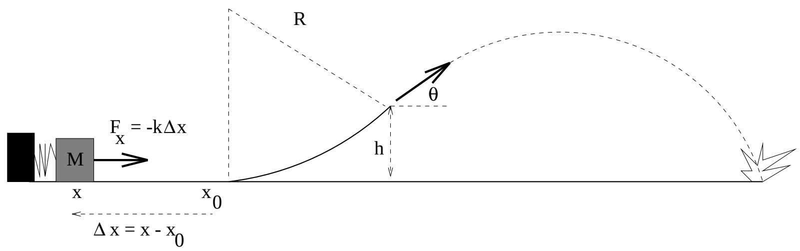
b) The coefficient of static friction is $\mu_s = 0.7$. Will the block remain at rest on the incline? If not, how fast will it be going when it reaches the bottom again?



14. A stunt woman for a movie[1] is trying to jump a motorcycle 4 meters across a ditch and land on a special ramp 1 meter higher than her takeoff point that is built into the branch of a convenient tree. The angle of the takeoff ramp is fixed at 37.5°. The "plot" of the movie requires that the ditch be filled with large, hungry crocodiles and the director is a sucker for realism. If she jumps even a bit too high, she will wreck on an overhanging branch and the crocs will dine well. If she jumps too low, she bounces back and also becomes crocodile-bait.

    With what speed v_0 must she take off to complete the jump successfully (and live to get paid)?

[1] Hopefully, this problem convinces all of you that physics can be useful to a variety of interesting future professions.



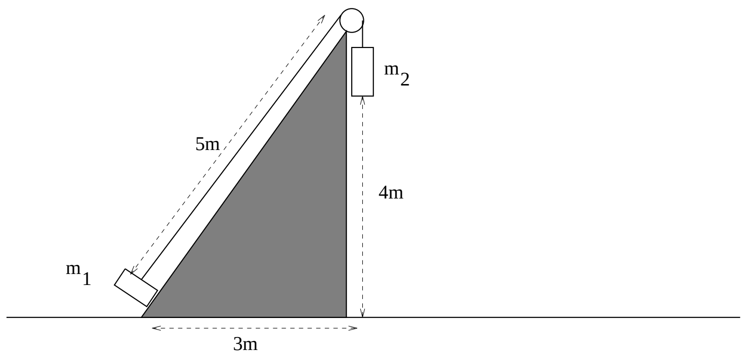
15. A 2 kg mass is launched from a spring with spring constant $k = 10^4$ N/m compressed a distance $\Delta x = 10$ cm as shown. It slides up a frictionless ramp to the top and flies off the ramp at a height $h = 1$ m above the ground at an angle of $\theta = 60°$.

    Find:

    a) The final speed of the mass when it hits the ground.

    b) If the ramp is circular with a radius of curvature $R = 2$ m (as drawn on the figure), find the normal force the ramp exerts on the mass just before it flies off the track.

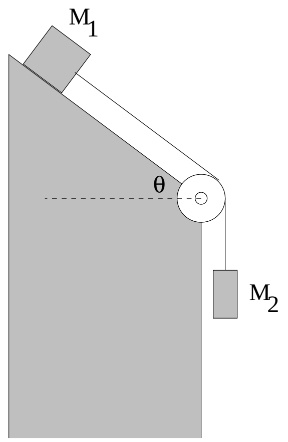
    c) If the ramp had friction, would the normal force be greater or smaller at the top? Explain.



16. Two blocks with mass $m_1 = 5$ kg and $m_2 = 10$ kg are arranged as shown on a "3-4-5" inclined plane and connected with a massless unstretchable string running over a massless, frictionless pulley. The two masses are released initially with m_2 4 m above the ground. The inclined plane has a coefficient of static friction $\mu_2 = 0.5$ and a coefficient of kinetic friction of $\mu_k = 0.25$.

    a) How fast are the masses moving when m_2 hits the ground?

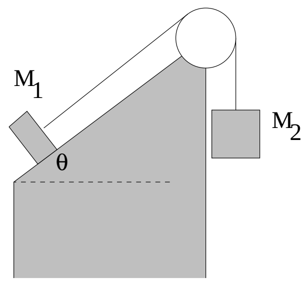
    b) Assuming the string goes limp at that instant (so m_1 keeps moving on the rough incline) does m_1 hit the pulley?



17. Two blocks with mass $M_1 = 10$ kg and $m_2 = 2.5$ kg are arranged as shown with M_1 sitting on an inclined plane and connected with a massless unstretchable string running over a massless, frictionless pulley to M_2, which is hanging over a drop. The two masses are released initially with M_2 4 m above the ground. The inclined plane has a coefficient of static friction $\mu_s = 0.5$ and a coefficient of kinetic friction of $\mu_k = 0.25$.

    a) What is the *largest* that the angle θ can be such that the system remains at rest?

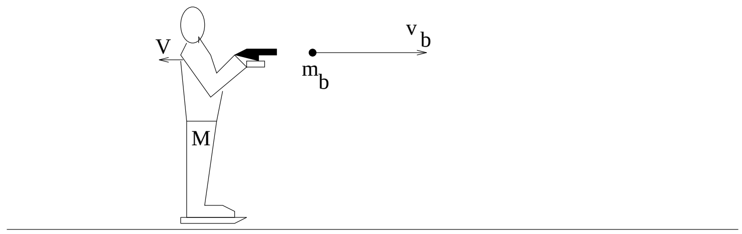
    b) If $\theta = 37.5°$ (so that it forms a 3-4-5 right triangle) and the top mass is "nudged" just enough to overcome static friction (if necessary) how fast is M_2 travelling when it hits the ground?



18. Two blocks with mass $M_1 = 5$ kg and $M_2 = 5$ kg are arranged as shown with M_1 sitting on an inclined plane and connected with a massless unstretchable string running over a massless, frictionless pulley to M_2, which is hanging over a drop. The two masses are released initially with M_2 4 m above the ground. The inclined plane has a coefficient of static friction $\mu_s = 0.5$ and a coefficient of kinetic friction of $\mu_k = 0.25$.

    a) Find an *algebraic expression* involving M_1, M_2, and μ_s from which one can find the *largest* angle θ_{max} such that the system remains at rest. If you can solve this somewhat complicated expression, you will get some extra credit.

    b) If $\theta = 37.5°$ (so that it forms a 3-4-5 right triangle) and the top mass is "nudged" just enough to overcome static friction (if necessary) how fast is M_2 travelling when it hits the ground?



19. An ice skater of mass 50 kg (on frictionless ice) fires a bullet of mass 10 gm that leaves the barrel at 500 m/sec (in the x-direction relative to the ground) as shown.

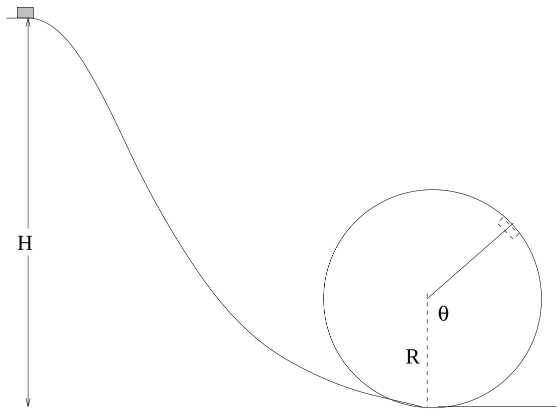
    a) What is the recoil velocity of the skater relative to the ground?

    b) How much work was done by the exploding gunpowder? (Note that this much energy would have to be *dissipated* if the skater *caught* the bullet travelling the other way in an inelastic process).

20. The Duke Communications company wants to put a satellite into a circular geosynchronous orbit over the equator. (Note: this means that the period of the satellite's orbit is one day.) Ignoring perturbations like the moon and the sun, and using $G = 6.67 \times 10^{-11} \frac{N-m^2}{kg^2}$ and $M_E = 5.98 \times 10^{24} kg$, find:

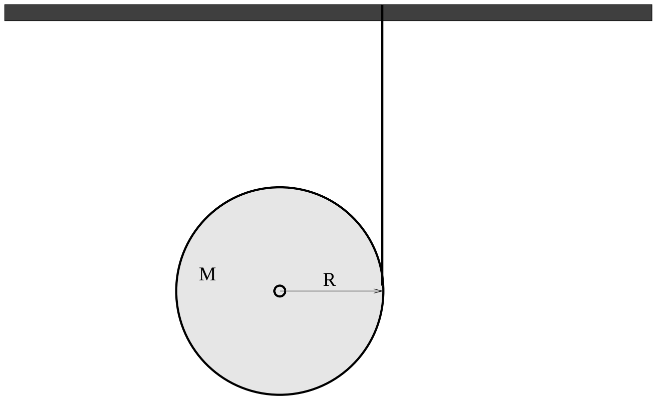
a) The radius of such an orbit.

b) The velocity of the satellite in the orbit.



21. A block of mass M sits at the top of a frictionless loop-the-loop.

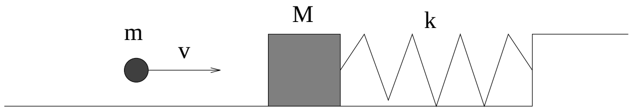
    Find the normal force exerted by the track when the mass is at an angle θ on the loop as shown.



22. A spool of fishing line is tied to a beam and released from rest in the position shown at time $t = 0$. The spool is a disk and has a mass of 50 grams and a radius of 5 cm. The line itself has negligible mass per unit length. Once released, the disk falls as the taut line unrolls.

    a) What is the tension in the line as the disk falls (unrolling the line)?

    b) After the disk has fallen 2m, what is its speed?



23. A bullet of mass m and velocity $\vec{v}_0$ hits a block of mass M that is sitting on a frictionless table, at rest, attached to a spring of mass k as shown. At $t = 0$ the bullet enters the block and "sticks" (as in the ballistic pendulum). At this instant, the block is in its equilibrium position, which you should consider to be $x = 0$.

    a) The initial position of the bullet and block combination is $x = 0$, as noted. Find the initial *velocity* of the block and bullet immediately *after* the collision;

    b) What is the total energy of the bullet/block/spring system immediately after the collision? How much energy was lost to heat?

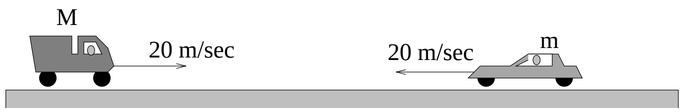
    c) Find the greatest distance x that the spring will compress before the bullet/block come to rest.

24. A pendulum of mass M and length L hangs at rest. At time $t = 0$, the pendulum is struck sharply and given a horizontal speed to the right of v_0 such that its initial kinetic energy is much less than MgL.

a) What is the natural frequency of the pendulum?

b) What are $s(t)$ and $v(t)$ (position and velocity of the mass M on the arc as functions of time)?

c) Show the derivation of the equations of motion used to get the answers to a) and b). Why was it important that the initial kinetic energy be $<< MgL$?

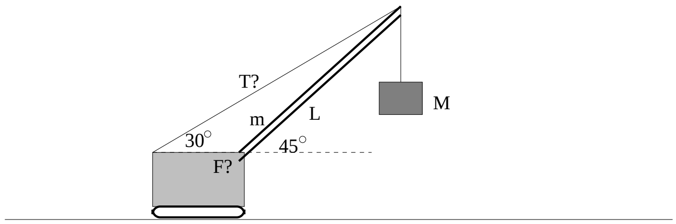


25. A heavy truck with mass $M = 5000$ kg is travelling at 20 m/sec to the right. It collides head on with a passenger car with mass $m = 2000$ kg travelling at 20 m/sec to the left. The two vehicles stick together and are carried down the road with locked brakes. The coefficient of kinetic friction between their tires and the road is $\mu_k = 0.3$. Find:

a) The velocity of the wrecked vehicles immediately after the collision (before friction has had any effect).

b) The energy dissipated during the collision.

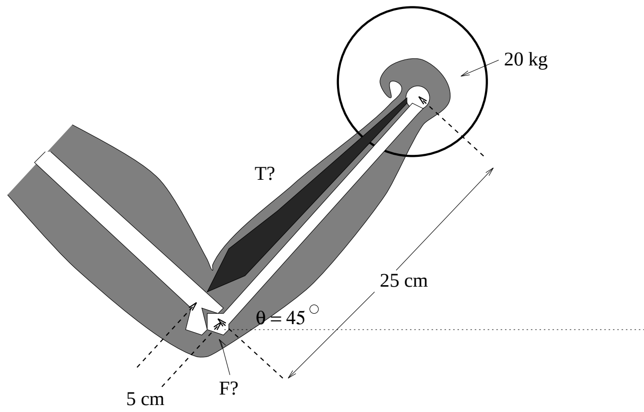
c) How far the locked vehicles skid down the road after the collision before coming to rest.



26. A crane with a boom of mass $m = 500$ kg and length $L = 20$ meters holds a mass $M = 2000$ kg suspended as shown.

a) Find the tension in the wire.

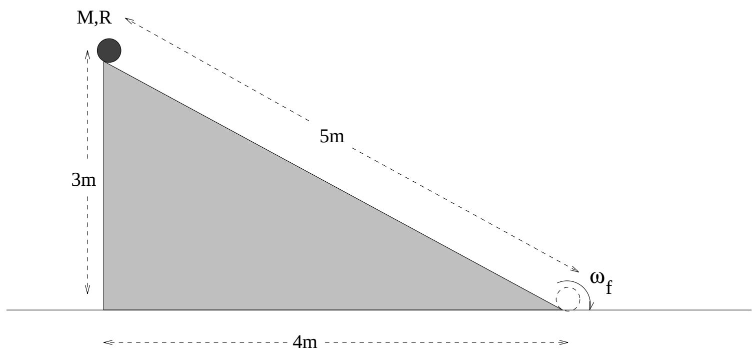
b) Find the force exerted on the boom by the crane body.



27. A human person holds a 20 kg barbell stationary half-way through a 45° curl. The muscle that supports this is connected as shown 5 cm up from the elbow joint. The bone that supports the weight is 25 cm long.

    a) Neglecting the mass of the arm itself, find the tension T in the muscle.

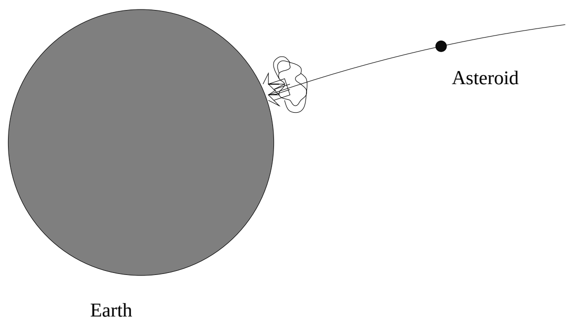
    b) Find the force $\vec{\mathbf{F}}$ exerted on the supporting bone by the elbow joint.



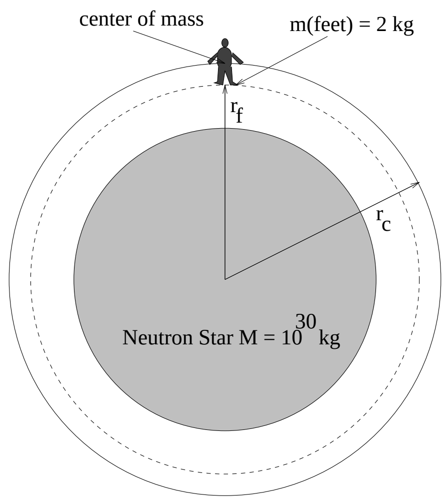
28. A round **ball** of mass $M = 1$ kg and radius $R = 10$ cm sits at the top of a 345 inclined plane as shown. The coefficient of static friction is high enough that the ball will roll without slipping. At time $t = 0$ the ball is released. Note that the moment of inertia of a ball is $\frac{2}{5}MR^2$.

   a) Find the acceleration of the ball down the inclined plane.

   b) Find the angular velocity with which it is spinning about its center of mass when it reaches the bottom of the plane.



29. Estimate the total energy released when a spherical asteroid with a density $\rho = 10$ kg/m^3 and radius $R = 1000$ meters falls onto the surface of the earth from "outer space" (far away). Obviously your answer should be justified by a good physical argument.

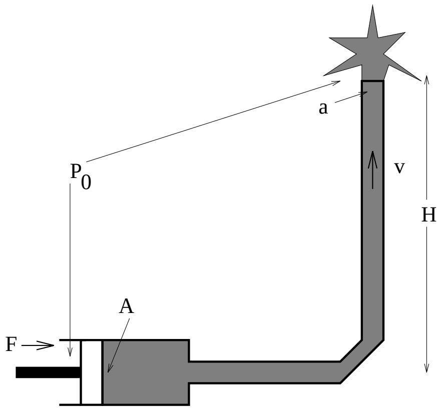


30. Tides can be dangerous. You are a scientist in orbit around a neutron star with a mass $M = 10^{30}$ kg and a radius of 8 km. Your center of mass moves in a perfect circle 10 km around the center of the star. You have just enough angular momentum that your feet always point "down" toward the center of the star and your head points away. Your feet are therefore also in a circular trajectory around the center of the star, but they cannot also be in orbit (free fall).

    Assuming that your feet have a mass of approximately 2 kg and are located approximately 1 meter closer to the star than your center of mass, how much force do your legs have to provide to keep your feet from falling off? Do they fall off?

    (Hint: proceed by finding the centripetal acceleration/force of your center of mass in terms of the gravitational field/force of the star at that location. Repeat this for your feet separately, assuming that they have the same angular frequency of circular motion as your center of mass but are in a (much!) stronger gravitational field. The difference in the force required to keep the feet in a circular orbit (the total centripetal force) and the actual gravitational force must be provided by your legs.)

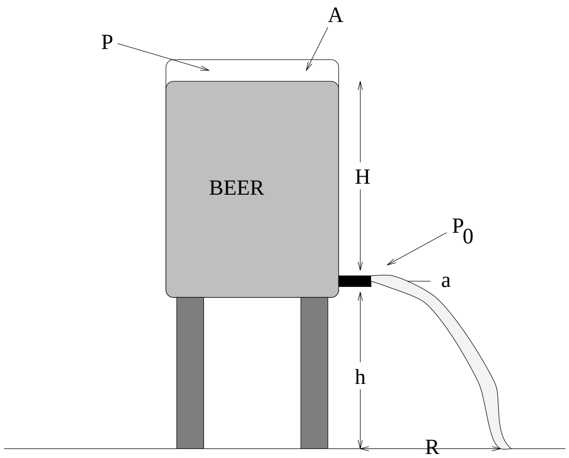
    (Double Hint: The binomial expansion might well be useful here...)



31. A piston is pressed with a force $\vec{\mathbf{F}}$ on a hydraulic cylinder containing water ($\rho = 10^3$ kg/m^3). The cross sectional area of the cylinder is $A = 100$ cm^2. The water therein is forced into a pipe with a cross sectional area of $a = 1$ cm^2 that rises vertically a height $H = 50$ meters. Both the end of the pipe (at the top) and the back of the piston (at the bottom) are open to atmospheric pressure.

    a) What does F have to be to make the water spurt from the pipe with a speed of 10 meters/sec at the top?

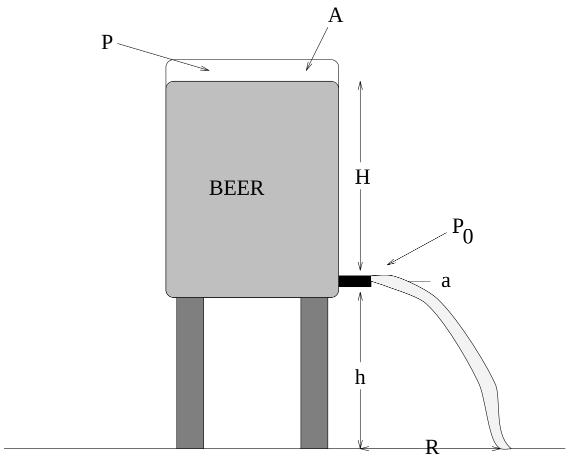
    b) What is the pressure (in atmospheres, where 1 atm $= 10^5$ Pascals) inside the cylinder at the bottom at that time?



32. In the figure above, somebody has pumped a beer keg with a cross sectional area $A = 0.25$ m^2 up to $P = 2$ atmospheres of pressure at the top (1 atm $= 10^5$ Pascals) and pulled the tube off of the tap (which has a cross sectional area of $a = 0.25$ cm^2) at the bottom. The surface of the beer is $H = 50$ cm above the tap at the bottom. Fortunately, the keg is up on a stand $h = 1$ m above the ground, so that you can catch it in a bucket.

   a) Where should you put the bucket to catch the beer? (Find R. Don't forget P_0).

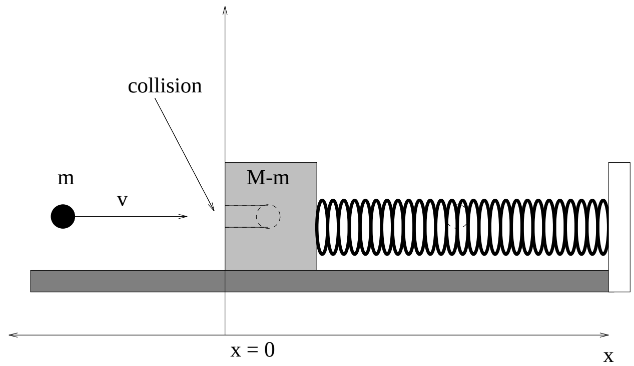
   b) How long (approximately) do you have to find a bucket before the beer is all gone?



33. In the figure above, a gas cylinder (not shown) maintains $P = 1.5$ atmospheres (1 atm $=$ 10^5 Pascals) of pressure at the top of a beer keg with a cross sectional area $A = 400$ cm^2. A drunken student falls agains the keg and knocks the spigot off of the tap at the bottom (which has a cross sectional area of $a = 1$ cm^2). The surface of the beer is $H = 50$ cm above the tap at the bottom when the tap is knocked off. Fortunately, the keg is up on a bar-stand $h = 1.5$ m above the ground, so that you can catch it in a bucket placed on the floor

    a) Where should you put the bucket to catch the beer? (Find R. Don't forget P_0).

    b) How long (approximately) do you have to find a bucket before the beer is all gone?



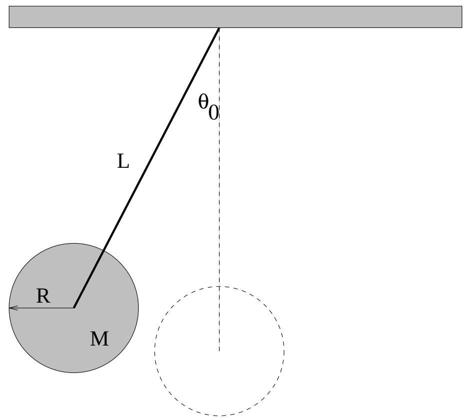
34. A bullet of mass m, travelling at speed v, hits a block of mass $M - m$ resting at the equilibrium position of a connected spring with constant k as shown above. The block is sitting on a frictionless table (i.e. – ignore damping). Assume that the collision occurs at $t = 0$. All answers below should be given in terms of m, M, k, v, although once you have defined a new quantity (e.g. $P = mv$) you can express answers thereafter in terms of it instead.

a) What is the maximum displacement X_0 of the block?

b) What is the angular frequency ω of oscillation of the combined bullet-block system?

c) What is the phase δ of the oscillator?

d) Write down $X(t)$.



35. A physical pendulum is constructed from a thin rod of negligible mass inserted into a uniform ball of mass M and radius R. The rod has length L from the pivot point to the center of the ball. At time $t = 0$ the ball is released from rest when the rod is at an initial **small** angle θ_0 with respect to its vertical equilibrium position.

    Answer all the questions below in terms of M, R, L, g, θ_0. You may make the small angle approximation where appropriate.

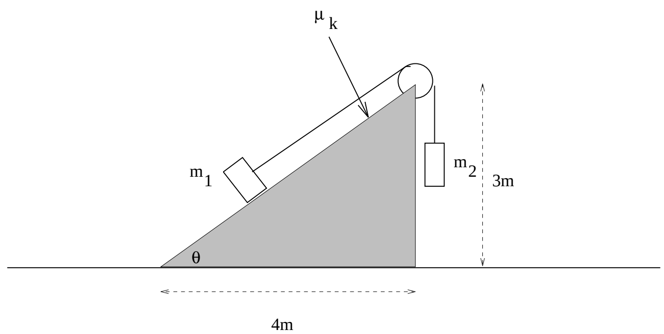
    a) Determine the equation of motion for the system, solving for $\alpha = \frac{d^2\theta}{dt^2}$.

    b) Determine the angular frequency of oscillation ω and write down $\theta(t)$ for the ball.

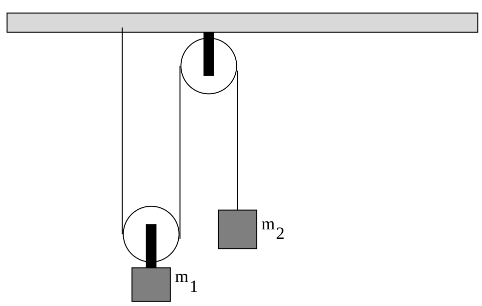
    c) Find the maximum speed v of the ball. Is this larger or smaller than it would have been if the ball had been a point mass M at the end of the rod? Why?

36. For each of the following figures: a) Draw a "free body diagram" for each mass shown, that is, draw all *real* forces acting on it; b) Apply **Newton's Second Law** *algebraically* (with no numbers) to each mass in each figure, but do not proceed further; c) Pick **one** figure and solve for the acceleration(s) of all masses and any unknown forces (such as a normal force or the tension in a string). Don't forget that the acceleration is a **vector** and must be given as a magnitude and a direction (for example, "along the plane to the right" is ok) or in vector components.

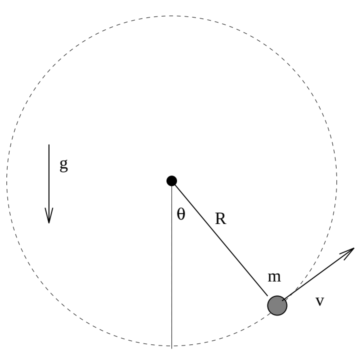
a) $\theta = 36.87°$, $m_1 = 10$ kg, $m_2 = 20$ kg, $\mu_k = 0.1$

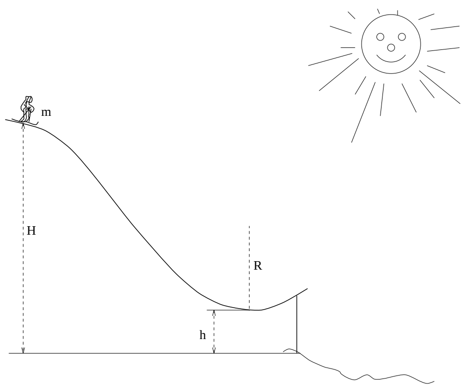


b) $m_1 = m_2 = 8$ kg.



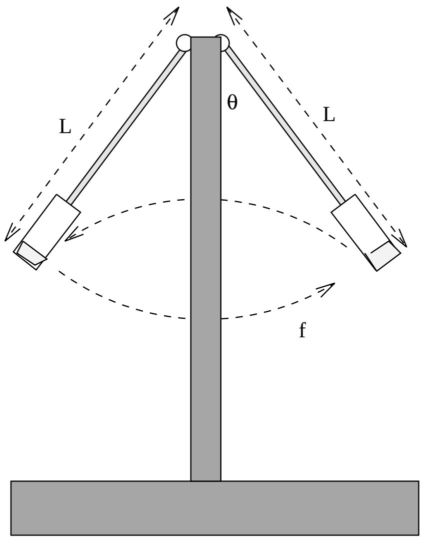
c) $r = 2.0$ m, $\theta = 45\circ$, $m = 5$ kg, $|\vec{v}| = 10$ m/sec.





37. Ambitious Amy (who masses $m = 50$ kg) skis down the (frictionless) slope of height $H = 60$ m to a ski ramp whose radius of curvature $R = 25$ m and whose lowest point is $h = 10$ m above the ground (as shown).

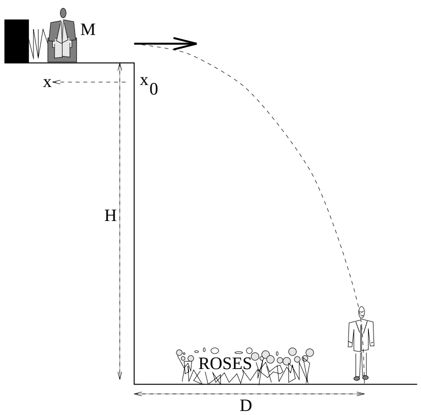
    Amy's legs can withstand up to 3000 N without collapsing (she's quite strong). Can she make the jump without breaking a leg (before she lands, anyway)?



38. A centrifuge with very light arms (neglect their mass) 10 cm long spins down samples of a colloidal mixture (one in each tube) at 3000 rpm. The arms are jointed at the top and swing out as the device spins up so that the bottom of the sample tubes exert a (normal) force on the samples that is always directed back along the arms. Find:

    a) The equilibrium angle θ that the arms make with the stand when the centrifuge is operating at full speed;

    b) The effective centrifugal "pseudogravity" produced by the device, expressed as a multiple of g. This is why centrifuges work so well to separate mixtures.



39. A physics student, irritated by the personal mannerisms of their physics professor, decides to build an infernal device to rid the world of him altogether. The student plans to push a large, massive object (the statue of Washington Duke, actually) from a tall building of height H by means of a powerful spring (with spring constant k) stolen from the suspension of a jacked-up Camero. However, the student (being a careful sociopath) wants to make sure that the mass M will make it over the roses to the path a distance D from the base of the building.

    Unfortunately, the student isn't very good at physics and comes to *you* for help. Since they don't want to tell you *which* building or *which* path or spring or mass they want to use (you might be able to testify against them!) they want you to find a *general formula* for how far to compress the spring.

    a) Help them out. Find $\Delta x = x - x_0$ in terms of H, M, D, k and g (the gravitational constant) that will drop M on RGB assuming *no* friction on the roof. That way I'm still pretty safe.

    b) Explicitly show that your answer to a) has the right units. If you are clueless in a) try to find SOME combination of these letters that has the right units and varies the way you expect the answer to (more height H means *smaller* Δx, for example, so it probably belongs on the bottom).

40. A basketball player goes up for a 3-pointer from 7m out. To shoot over his opponent's outstretched arm, he releases the ball at 60$\circ$ with respect to the horizontal, at a height 1m below the height of the rim. Assuming that his release is on line and undeflected: a) How fast must the ball be thrown to go through the hoop "perfectly"? b) If the game has exactly 1 second left when the ball is released, (by how much time) will the ball go through before or after the buzzer?

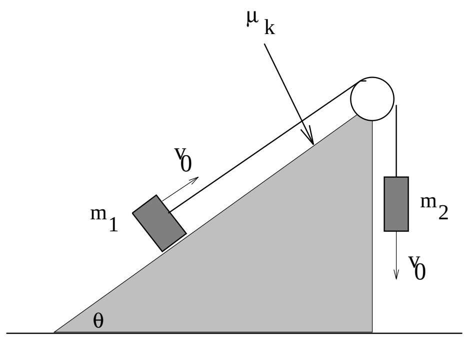
41. A speedway 13 m wide is banked on a curve so that the outside edge is 5 m higher than the inside edge. A racecar goes around the curve at 50 m/sec (a little better than 110 mph). The coefficient of static friction of the racecar's tires is $\mu_s = 0.6$ (good tires). a) Find an **algebraic expression** for the smallest radius the curve could have such that the car can make the turn; b) IF YOU HAVE TIME you can evaluate this expression for extra credit.

42. For **three** out of the **four** following figures:

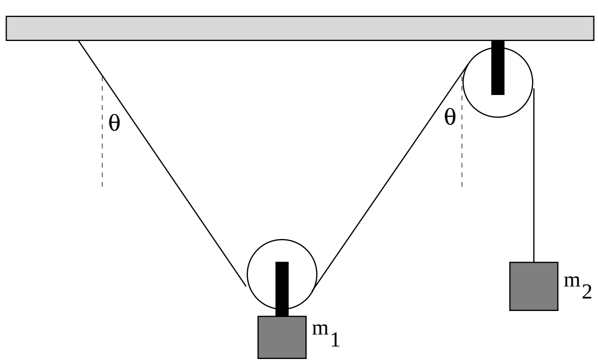
- Draw a "free body diagram" for **each** mass shown, that is, draw in and label all *real* forces acting on it;
- Apply **Newton's Second Law** in appropriate coordinates to **each** mass shown. In two of your three choices, do not proceed further; (5 points each for three.)
- Pick **one** figure and solve for the acceleration(s) of **each** mass shown and evaluate **all** unknown forces (such as a normal force or the tension in a string) in terms of the given quantities. (5 points for the one.)

Don't forget that the acceleration is a **vector** and must be given as a magnitude and a direction (for example, "along the plane to the right" is ok) or in vector components. (5 points each + 5 points for finding unknowns for one of them.)

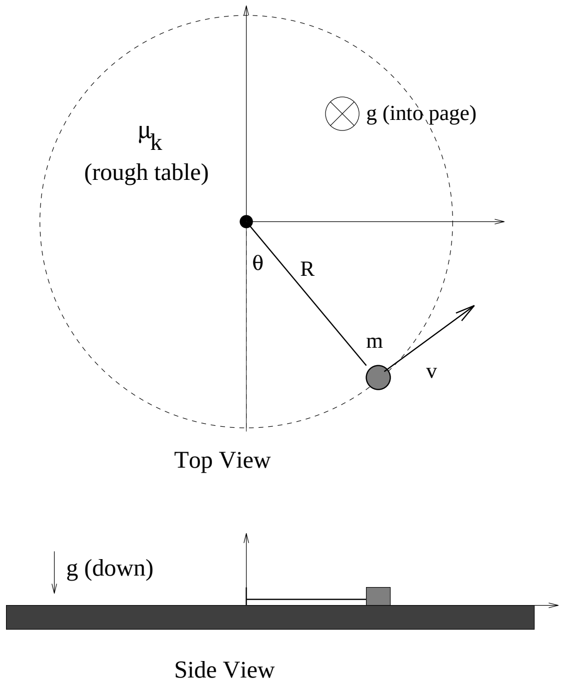
a) Given: θ, m_1, m_2, μ_k and $v_0 \neq 0$ to the right (the objects are already moving, so only kinetic friction applies regardless of their masses).



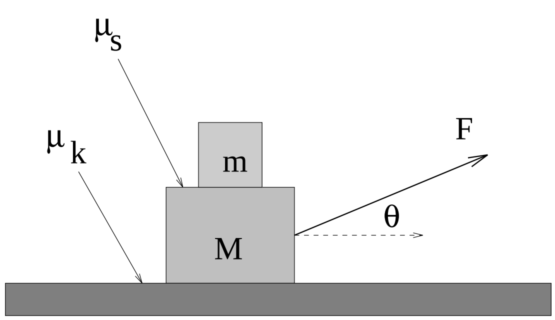
b) Given: θ, m_1, m_2.

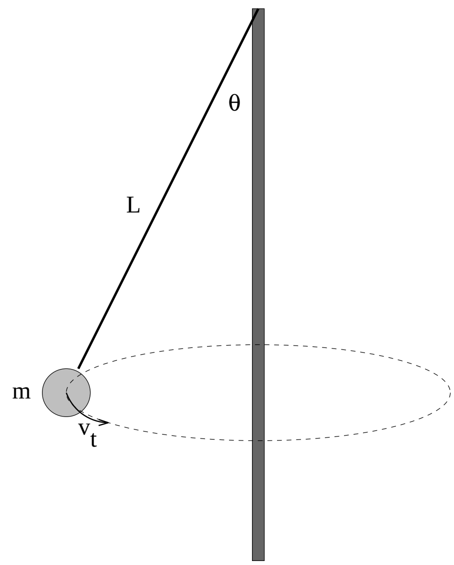


c) This is a block whirled on the end of a string sitting on a rough table. Gravity is **into** the page. Given: R, m, v, μ_k, θ.

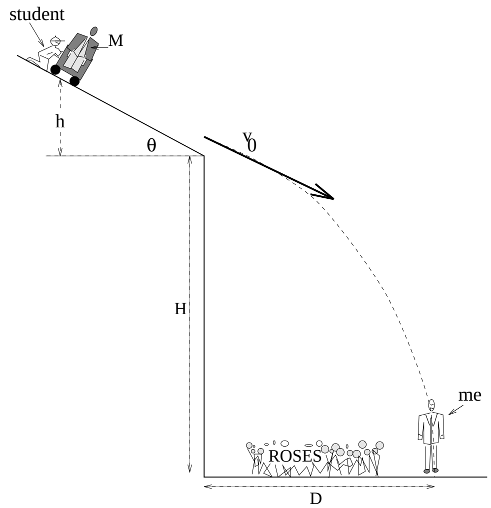


d) A rope at an angle θ with the horizontal is pulled with a force $\mathbf{F}$. It pulls, in turn, two blocks, the bottom with mass M and the top with mass m. The coefficients of friction are μ_s between the top and bottom block (assume that they do not slide for the given force $\mathbf{F}$) and μ_k between the bottom block and the table. Remember to show (and possibly evaluate) *all* forces acting on *both* blocks, including internal forces between the blocks.





43. The tether ball on a rope pictured above is going around in a horizontal circle as drawn. The length of the rope is L, the mass of the ball is m, and the rope makes a fixed angle θ with the pole. Find: a) the tension T in the rope; b) the tangential speed v_t of the ball; c) the angular frequency ω with which the ball goes around the pole. (20 points.)

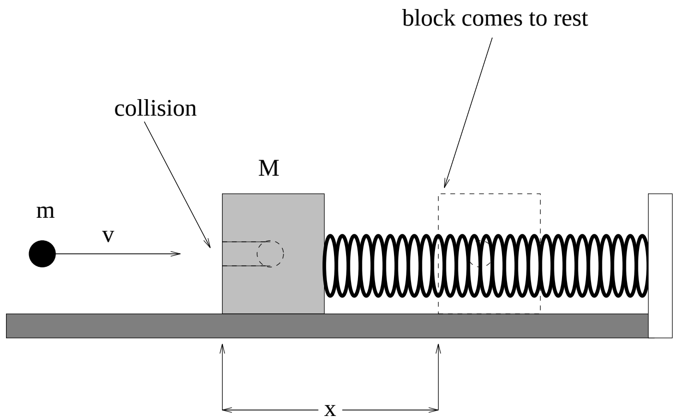


44. A physics student irritated by the personal mannerisms of their physics professor decides to rid the world of him. The student plans to drop a large, massive object (the statue of Washington Duke, actually, recently stolen by pranksters from his fraternity), mounted on nearly frictionless casters, from a tall building of height H with a smooth roof sloped at the angle θ as shown. However, the student (being a thoughtful sociopath) wants to make sure that the mass M will make it *over the roses* to the path a distance D from the base of the building and needs to know how far to let the statue roll down the roof to get the right speed.

    Unfortunately, the student isn't very good at physics and comes to *you* for help. Since they don't want to tell you *which* building or *which* path they want to use (you might be able to testify against them!) they want you to find (in **two steps**, each counting as a separate problem) a *general formula* for the requisite distance.

    Help them out. Start by finding v_0 in terms of H, M, D, θ and g (the gravitational constant) that will drop M on RGB assuming *no friction or drag forces.* (That way I'm still pretty safe). (20 points.)

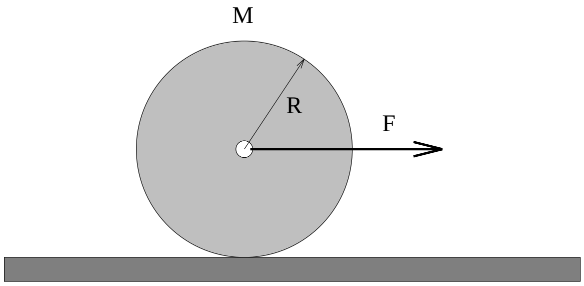
45. Now that you know the speed (or rather, assuming that you know the speed, as the case may be) find h (the vertical distance the statue must roll down, released from rest, to come off with the right speed). Explicitly show that your overall answer (in which v_0 should NOT appear) has the right units. If you were clueless in problem 4) you may leave v_0 in your answer but should still try to find SOME combination of the letters H, M, D, θ and g that has the right units and varies the way you expect the answer to (more height H means *smaller* h, for example, so it probably belongs on the bottom). (20 points.)



46. A ball bearing of mass $m = 50$ grams travelling at 200 m/sec smacks into a block of mass $M = 950$ gms and sticks in a hole drilled therein. The block is initially at rest on a table with coefficient of kinetic friction $\mu_k = 0.4$ and is also connected to a spring with spring constant $k = 400$ N/m at its equilibrium position (see figure).

    a) What is the maximum distance x the spring is compressed by the recoiling ball bearing-block system?

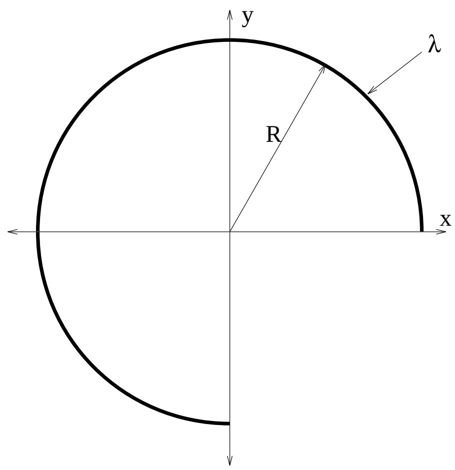
    b) How much does the presence of friction decrease the stopping distance (find and compare the answer when there is no friction)?



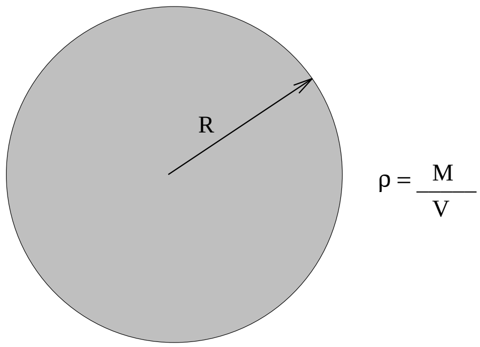
47. A force $\vec{\mathbf{F}} = 48$ N to the right is applied to the frictionless axle of a wheel made of a uniform disk with mass $M = 4$ kg and radius $R = 10$ cm. It rolls without slipping on a rough table. Find:

a) the net acceleration of the wheel.

b) The minimum coefficient of static friction μ_s such that the wheel does not slip for this force.



48. Find the center of mass of the 3/4 hoop of wire above. It has a uniform mass per unit length $\lambda = 10$ grams/meter, and its radius $R = 100$ cm.



49. Planetary rock has an average density around 10^4 kg/m^3. Assuming that you can throw a fastball around 40 m/sec (nearly 90 mph) what is the approximate radius R of the *largest* spherical planet where you can stand on the surface and throw a baseball away (to infinity)?

50. A block of soft wood with a mass $M = 1$ kg sits at rest on a rough table with a coefficient of kinetic friction $\mu_k = 0.5$. A bullet of mass $m = 10$ grams and travelling at 50 m/sec hits the block and becomes embedded. Find:

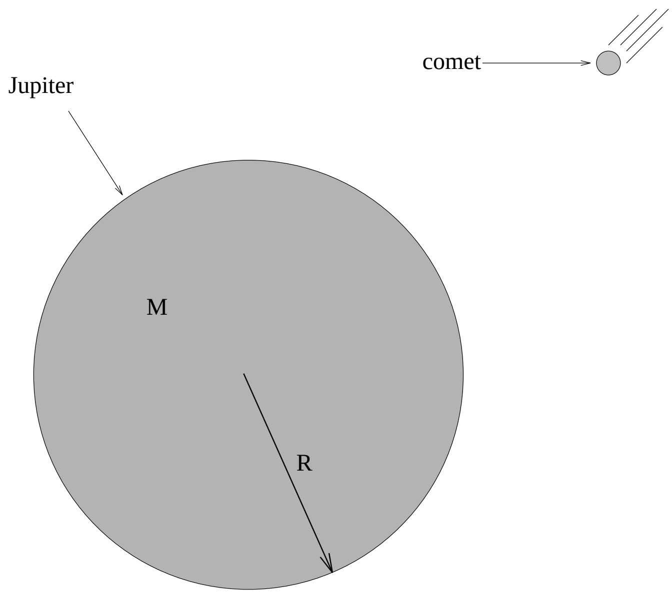
a) The distance the block will slide before coming to rest.

b) Is the kinetic energy of the bullet+block system conserved **during** the collision (that is, **before** the block begins to slide)? If not, what is the kinetic energy gained or lost by the system.

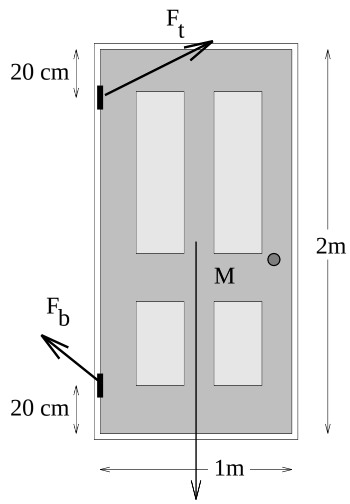
51. Tom is a hefty guy with a good sense of balance who wants to push down a brick wall. The wall, however, is strong enough to withstand a horizontal push of 2000 N and Tom can only exert a push of about his 1000 N weight with his muscles. Fortunately, Tom has a perfectly rigid 4×4 beam (of negligible mass), and there is a solid rock (that can withstand essentially any push) 5 m from the wall to brace it on. Even more fortunately, Tom has taken physics 51! He therefore cuts the beam to lean against the house as shown and proceeds to walk up the beam towards the house.

a) Suppose the coefficient of static friction between the beam and the wall is 0.4. What is the highest Tom can make the point of contact of the beam with the wall (H) and still knock it down if he walks to the end?

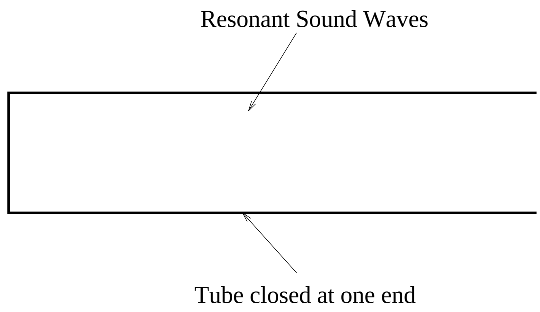
b) Tom is smart, and uses some grease and a smooth braceplate to render the brick wall "frictionless" where the beam hits it. He then cuts the beam so that it rests 1.5 m up on the wall. How far must he walk (horizontally) before the wall breaks?



52. Several years ago several chunks of a comet slammed into the far side of Jupiter. Given that $M_{\text{Jupiter}} = 1.9 \times 10^{27}$ kg and $R_{\text{Jupiter}} = 7 \times 10^4$ km, **estimate** how much energy was released in the (inelastic) collisions of a typical comet fragment with Jupiter. Assume that the comet fragment is a ball of ice with density ≈ 1 gm/cm^3 and of radius 0.5 km. State clearly the basis of your argument, drawing pictures or graphs if necessary.



53. A door of mass $M = 30$ kg that is 2 m high and 1 m wide is hung from two hinges located 20 cm from the top and bottom, respectively. Assuming that the weight of the door is equally distributed between the two hinges, find the total force (magnitude and direction) exerted by each hinge. (Neglect the mass of the doorknob. The force directions drawn for you are **NOT** likely to be correct or even close.)

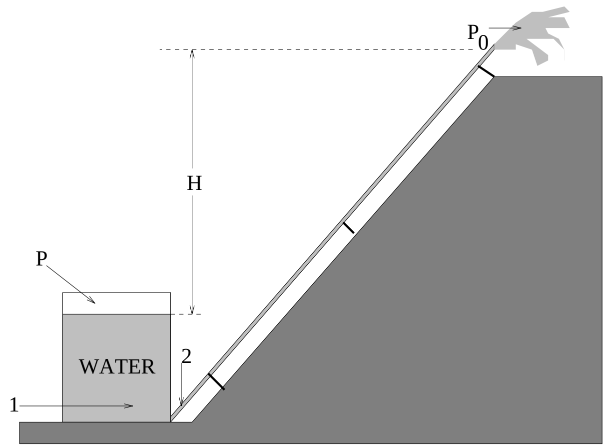


54. An organ pipe is made from a brass tube closed at one end as shown. The pipe is 3.4 meters long. When driven it produces a sound that is a mixture of the first, third and sixth harmonic (mode).

a) What are the frequencies of these modes?

b) Sketch the wave amplitudes for the third harmonic mode (only) in on the figure, indicating the nodes and antinodes. Be sure to indicate whether the nodes or antinodes are for **pressure/density** waves or **displacement** waves!

c) The temperature in the church where the organ plays varies by around 30° C between summer and winter. By how much (**approximately**) does this vary the frequency of the fundamental harmonic? (Indicate **why** your answer is what it is, don't just put down a guess).



55. In the figure above, a pump maintains a pressure of $P = 2.5$ atm. (1 atm $= 10^5$ Pascals) on top of a tank of water with a cross sectional area $A = 10\ \mathrm{m}^2$. An irrigation pipe at the bottom leads up a slope to a farmer's field. The vertical distance between the top surface of water in the tank and the opening of the pipe is $H = 10$ m. The cross-sectional area of the pipe is $a = 4\ \mathrm{cm}^2$. The top pipe is open to air pressure $P_0 = 1$ atm. Recall that the density of water is $\rho = 10^3\ \mathrm{kg/m}^3$.

a) What is the velocity of the water coming from the pipe?

b) Is the pressure at the bottom of the tank greater inside the main vessel (point 1 on figure above) or inside the pipe (point 2)? **Briefly** explain.

56. Jane is fishing in still water off of the old dock. She is using a cylindrical bobber as shown. The bobber has a cross sectional area of A, a length of H, a mean density of $\rho = \rho_w/2$ (recall $\rho_w = 1$ gram/cm^3), and is balanced so that it remains vertical. When it is floating at equilibrium (supporting the weight of the hook and worm dangling underneath) $2H/3$ of its length is submerged in the water. (Hint: What is the combined mass of the bobber, hook and worm from this data?) A fish gives a tug on the worm and pulls the bobber straight down a distance $y_0 < H/3$. At $t = 0$ it lets go.

a) What is the **net** restoring force on the bobber as a function of y? (Hint: The word net means that you don't need to worry about the weight of the worm or bobber explicitly.) Use this force and the calculated mass to write Newton's 2nd Law for the motion of the bobber up and down (in y).

b) Neglecting the damping effects of the water, write an equation for the displacement of the bobber from its equilibrium depth as a function of time, $y(t)$. With what frequency does the bobber bob?

57. A string with mass per unit length $\mu = 2$ g/m is stretched between a 60 Hz oscillator and a clamp 1.5 meters away. The string is under a tension $T = Mg$ produced by a weight suspended from the string as shown (you may use $g = 10$ m/sec^2 if you wish).

a) What must mass M be to drive a resonance with the standing wave pattern shown above?

b) What is the velocity of waves in the string at that time?

c) If the mass M from part a) is left unchanged on the string, what frequency f must the oscillator be changed to to excite the fundamental harmonic?

58. A plan for an **adiabatic fire starter** is shown above. (This is the sort of thing bored physicists design in their spare time.) It consists of an adiabatic cylinder fitted with a frictionless, insulated piston. Tinder placed in the bottom of the cylinder that will ignite if the air around it ever gets above 900 °K. If one whacks the piston shaft hard with a sledgehammer the piston does work on the gas as it compresses it. If one hits hard enough, you pull out the piston, shake out the tinder, blow on it and voila! A hard-earned flame!

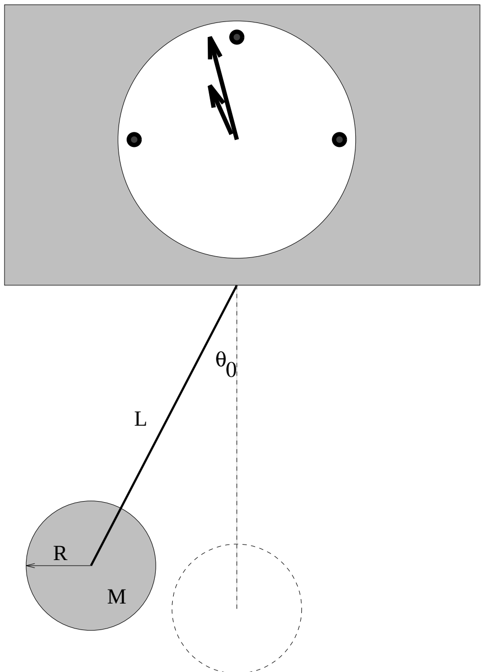
a) Assuming that the air in the cylinder is an ideal diatomic gas and that it starts with $P_i = 1$ atm $\approx 10^5$ N/m^2, $V_i = 0.01$ liter (10 cm^3), and $T_i = 300°$ K, what is its heat capacity at constant volume, C_v? At constant pressure, C_p? How many molecules of air are there in the cylinder?

b) Find the approximate amount of initial (kinetic) energy that must be imparted to the piston by the sledgehammer in order to ignite the tinder. Is it reasonable to expect to get this much energy from a 2.5 kg sledgehammer operated by muscle power?

59. Certain kinds of pumps work by creating a vacuum that “sucks water up” a rigid pipe. Unfortunately, these pumps can only lift water a certain distance before they mysteriously fail.

a) Your on-site construction boss has just called you into her office to either explain why they aren’t getting any water out of the pump on top of the 25 meter high cliff even though they are using a pump that can maintain 10 atm of pressure difference between its intake and its output. Examine the schematic above and show why it cannot possibly deliver water that high. Your explanation should include an explicit calculation of the highest distance such a pump could lift water. Why is the notion that the pump sucks water up misleading? What really moves the water up?

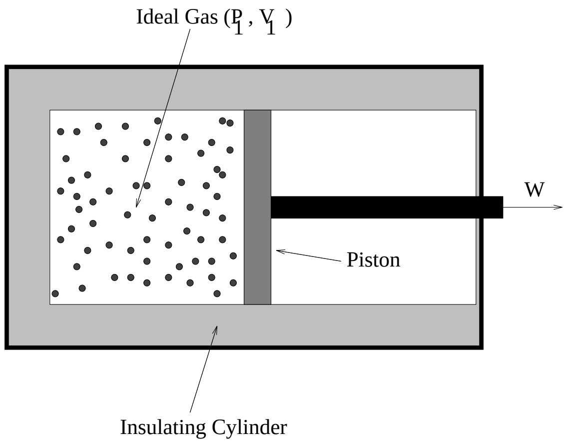
b) If you answered a), you get to keep your job. If you answer b), you might even get a raise (or at least, get full credit on this problem)! Tell your boss where this single pump should be located to move water up to the top and how it should be hooked up.



60. A Grandfather clock's pendulum is constructed from a thin rod of negligible mass inserted into a uniform **disk** of mass $M = 1$ kg and radius $R = 5$ cm. The rod has a length L from the pivot point to the center of the disk that can be adjusted from 0.20 m to 0.30 m in length so that the clock keeps the correct time.

    a) **Algebraically** determine the (differential) equation of motion for the system, making the small angle approximation to put it in the form of a simple harmonic oscillator equation.

    b) The clock keeps correct time when the period of its pendulum is $T = 1$ second. What should L be (to **3** digits) so that this is true. (Use the algebraic form for ω^2 from your answer to part a to solve for L.)



61. N molecules of an ideal monoatomic gas at initial volume V_1 and pressure P_1 expands in a cylinder quasi-statically and adiabatically (so that $dU + PdV = 0$) to volume V_2 and pressure P_2. All answers should be expressed in terms of these quantities, k and $\gamma = C_p/C_v$. Calculate the work done two ways:

a) Integrate $dW = -C_v dT$ (and put the answer in a form with only the variables noted above). I did this in lecture for you.

b) Integrate $dW = PdV$ directly, using the equation of state satisfied by an adiabatically expanding gas. Show that the two answers are equal (I noted the equality in class and assigned this problem for homework).

62. A certain heat engine uses an ideal gas as a medium and follows the cycle a-b: isothermal expansion at $T_1 = 600°$ K; b-c: constant volume cooling to a new temperature $T_2 = 300°$ k; c-d: isothermal compression to the original volume; d-a: constant volume heating back to T_1. This cycle is shown on the PV-diagram above. $V_a = V_d = 1$ liter, $V_b = V_c = 2$ liters and $P_a = 2$ atm.

a) Evaluate Nk for this engine, and find N.

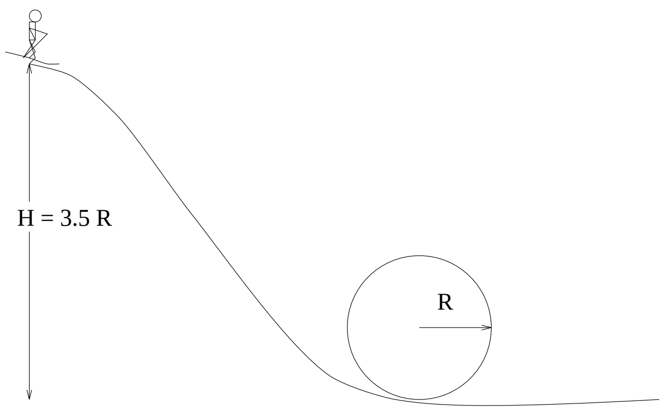
b) How much work is done per cycle of this engine?

c) For **extra credit**, determine how much heat is absorbed (d-a and a-b) and rejected (b-c and c-d) by the engine and evaluate its efficiency. You will need to use the first law extensively. Note that even if you can't do all of this part, showing what you can do might get you some extra credit.

63. A cannoneer is trying to hit a foxhole with a mortar. The foxhole is 384 m horizontally away from the mortar mouth. The mortar fires at the fixed angle of 53° but has a charge that can be adjusted so that its mortar shells come out of the mortar with any desired velocity (within reason). (Note: 345 triangle, $\sin(53) = 0.8$, $\cos(53) = 0.6$)

a) Find the initial velocity required to hit the foxhole.

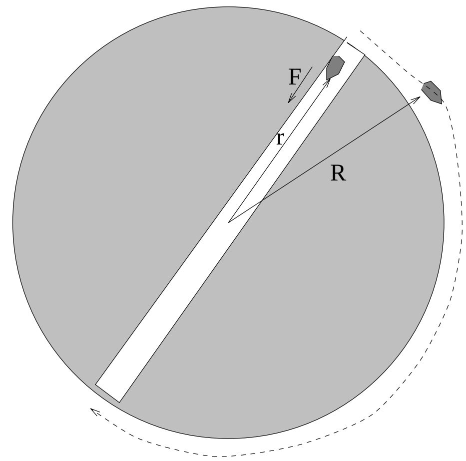
b) How long should the fuse on the mortar be set for (assuming that one wishes it to detonate just as it reaches the ground at the other end.



64. A skier of mass m at an exhibition wants to loop-the-loop on a special (frictionless) ice track of radius R set up as shown. Suppose $H = 3.5R$. All answers should be given in terms of g, m and R. (Note that the picture is not drawn strictly to scale for ease of viewing.)

a) What is her apparent "weight" (the normal force exerted by the track on her skis) when she is upside down at the top of the loop-the-loop?

b) What is her maximum apparent "weight" on the loop-the-loop and where (at what point on the loop-the-loop track) does it occur? Indicate the position on the figure.



65. A straight, smooth (frictionless) transit tunnel is dug through an airless moon of radius R whose mass density ρ_0 is constant. The moon does not rotate, and the tunnel is left in vacuum to eliminate drag forces. All answers should be given in terms of ρ, R, G (and/or M_{moon} once you have evaluated/defined it).

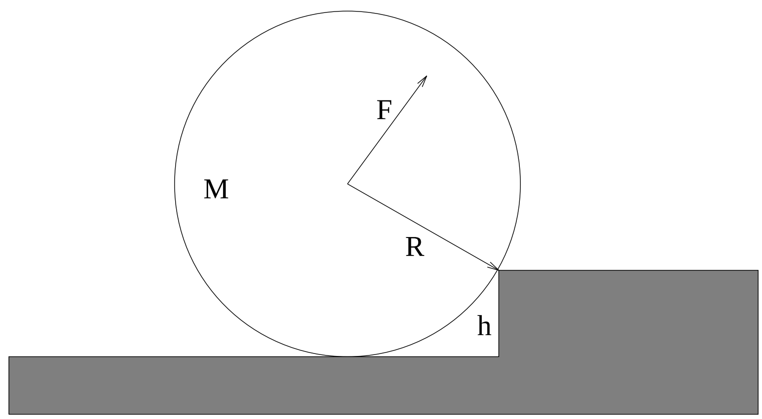
a) Find the force acting on a car of mass m a distance $r < R$ from the center of the planet.

b) Write Newton's second law for the car, and extract the differential equation of motion.

c) From this, find $r(t)$ for the car, assuming that it starts at $r_0 = R$ on one (e.g. the top) side at time $t = 0$.

d) How long does it take the car to get from one side of the moon to the other, starting from rest?

e) **Extra Credit**: Suppose a message capsule were fired in a circular orbit of radius R at the same time a message capsule were dropped through the tunnel. Which would arrive on the other side first?



66. A cylinder of mass M and radius R sits against a step of height $h = R/2$ as shown above. A force $\vec{F}$ is applied at right angles to the line connecting the corner of the step and the center of the cylinder. All answers should be in terms of M, R, g.

a) Find the minimum value of $|\vec{F}|$ that will roll the cylinder over the step if the cylinder does not slide on the corner.

b) What is the force exerted by the corner (magnitude and direction) when that force $\vec{F}$ is being exerted on the center?

67. a) A police siren emits sound at a primary frequency of 1000 Hz. Two police cars with sirens wailing are travelling towards you at 34 m/sec. What frequency do you hear? (Assume that the speed of sound in still air is 340 m/sec).

b) One of the two cars slows down to 31 m/sec. What **beat** frequency do you hear?

68. In the figure above, M_1 and M_2 are masses, and the pulley (a disk) has mass M and radius r. The plane is at an angle θ and has a coefficient of kinetic friction μ_k. Use g for the gravitational force.

a) Draw a force (free body) diagram for each mass. Don't forget the massive pulley.

b) Write Newton's second law, in the appropriate form, for each mass.

c) Solve for the acceleration of the system, assuming that it is moving initially to the right (so that static friction is irrelevant). Show all work.

69. A skier at an exhibition wants to loop the loop of radius R on a special (frictionless) ice track set up as shown. How high (in terms of R) must the starting slope be for the skier to get around the track if she needs to feel one "gravity" of normal-force acceleration at the top of the track to stay in control? (Note well: This means that you should have $N = mg$ (down) at the top of the track instead of $N = 0$ for the case where the skier barely loops the loop).

70. A spherical yo-yo is released from rest on a string of length L meter as shown. The spindle of the yo-yo has a radius of r and the radius of the yo-yo itself is $R > r$. The yo-yo has a mass M grams and for the purposes of this problem has the moment of inertia of the sphere, $I = \frac{2}{5}MR^2$.

a) Find the acceleration of the center of mass of the yo-yo.

b) Find the angular momentum of the yo-yo when it reaches the bottom of the string.

71. A straight, smooth (frictionless) transit tunnel is dug through a planet of radius R whose mass density ρ_0 is constant. The tunnel passes through the center of the planet and is lined up with its axis of rotation (so that the planet's rotation is **irrelevant** to this problem). All the air is evacuated from the tunnel to eliminate drag forces.

a) Find the force acting on a car of mass m a distance $r < R$ from the center of the planet.

b) Write Newton's second law for the car, and extract the differential equation of motion. From this you should be able to find $r(t)$ for the car, assuming that it starts at $r_0 = R$ on the North Pole at time $t = 0$.

c) How long does it take the car to get to the South Pole starting from rest at the North Pole?

72. Celebrating the end of their Physics 51 exam, three students decide to study Bernoulli's equation with their beer keg. First they pump it up so that there are 1.03 atmospheres of pressure inside the tank at the top (recall 1 atm $= 10^5$ N/m^2). Then one punches a **small** (Hint: Toricelli's Law) hole near the bottom of the keg, $H = 0.5$ meters below the surface of the beer, so that the beer comes out horizontally. The other two students (A & B) lie on their backs with their open mouths $h = 0.45$ meters below the hole and $X_A = 1.0$ meters and $X_B = 1.2$ meters (respectively) horizontally in the direction of the stream. Note that all relevant distances are shown on the figure (and that all distances shown are relevant!). Assume that $\rho_{\text{beer}} = 1$ gram/cc and that $g = 10$ m/sec^2 as usual (if you want the arithmetic to come out simple).

    Which student (A or B or neither) has solved the problem correctly (and is rewarded by getting the beer)? Note that the answer is completely unimportant, what matters is how you get it!

73. A certain heat engine uses an ideal gas as a medium and follows the cycle a-b: isothermal expansion at $T_1 = 600°$ K; b-c: constant volume cooling to a new temperature $T_2 = 300°$ k; c-d: isothermal compression to the original volume; d-a: constant volume heating back to T_1. This cycle is shown on the PV-diagram above. $V_a = V_d = 1$ liter, $V_b = V_c = 2$ liters and $P_a = 2$ atm.

a) Evaluate Nk for this engine, and find N.

b) How much work is done per cycle of this engine?

c) For **extra credit**, determine how much heat is absorbed (d-a and a-b) and rejected (b-c and c-d) by the engine and evaluate its efficiency. You will need to use the first law extensively. Note that even if you can't do all of this part, showing what you can do might get you some extra credit.

74. One way to reduce the cost of lifting mass into orbit is to use a linear accelerator to drive a payload up to escape velocity (or thereabouts) and then let it go. This way one doesn't have to lift the fuel used to lift the fuel used to lift the ... (almost all the fuel used in a rocket is used to lift fuel, not payload).

Assume that fusion energy has been developed and electricity is cheap, and that high temperature superconductors have made such a mass driver feasible. Your job is to do a first estimate of the design parameters.

A proposed plan for the mass driver is shown above. The track is 100 kilometers long and slopes gently upwards. The payload capsule has a mass of 2×10^3 kg (two metric tons). The head of the track is high in the Andes, $R = 6375$ kilometers from the center of the earth.

a) Neglecting air resistance, find the escape velocity for the capsule. Although bound orbits will not require quite as much energy, air resistance will dissipate some energy. Either way, this is a reasonable estimate of the velocity the driver must be able to produce.

b) Assuming that the capsule is started from rest and that a constant tangential force accelerates it, find the tangential force necessary to achieve escape velocity at the end of the track. Note: Ignore the normal force that the track must exert to divert it so that it departs at an upward angle.) From this find the acceleration of the capsule, in multiples of g. Is this acceleration likely to be tolerable to humans?

75. In the figure above, M_1 and M_2 are masses, and the pulley (a disk) has mass M and radius r. The plane is at an angle θ and has a coefficient of kinetic friction μ_k. Use g for the gravitational force.

a) Draw a force (free body) diagram for each mass. Don't forget the massive pulley.

b) Write Newton's second law, in the appropriate form, for each mass.

c) Solve for the acceleration of the system, assuming that it is moving initially to the right (so that static friction is irrelevant). Show all work.

Short Answer Questions

Answer the following short questions. Each answer can be an equation, and/or a sentence and/or a diagram. Long answers are **not** necessary or desirable. These are (for the most part) *not* like the ones likely to be on a 53 exam, but they are certainly important things to know! In fact, this section contains *most of the starting points* for the solution of problems you are likely to encounter, as well as the *basis for answering* most of the short answer, conceptual questions.

1. What is Newton's First Law?
2. What is Newton's Second Law?
3. What is Newton's Third Law?
4. What is the Work-Energy Theorem?
5. What "force" makes hurricanes spin anticlockwise in the southern hemisphere?
6. What is "centrifugal force"?
7. What is the "terminal velocity" of an object in free fall in air?
8. You may use $g = 10$ m/sec^2 (if you wish) throughout this exam. However, show that you know the correct value of g here: $g =$
9. What is Hooke's Law?
10. What is Kepler's first law?

11. What is Kepler's second law?

12. What is Kepler's third law?

13. When is the angular momentum of a system conserved?

14. Write the perpendicular axis theorem. Draw a picture to go with it, if it helps.

15. Write the parallel axis theorem. Draw a picture to go with it, if it helps.

16. What is the definition (either one) of gravitational potential?

17. What is the law of conservation of momentum?

18. What is the fully Generalized Work-energy Theorem (expressed with both conservative and nonconservative forces present)?

19. A golf ball is hit off of the tee. The club is in contact with the ball for a time $\Delta t \approx 10$ msec. How might you estimate the average force exerted by the club on the ball? (Assume that you know or can measure the ball's mass and its speed off of the club.)

20. Qualitatively sketch $P_{\rm av}(\omega)$ for a damped, driven harmonic oscillator with resonant frequency ω_0 and $Q = 10$.

21. What is the definition of Young's modulus? Draw a picture to illustrate the quantities involved.

22. What is Pascal's Law? A small picture would help.

23. What is Toricelli's Law? How is it derived?

24. What is the Venturi Effect? (Note: In the Physics 53 text, this isn't given by name, but it is covered as the "Venturi Tube" and is responsible for the lift of airplane wings and the function of spray mister bottles everywhere).

25. What is the law for thermal expansion of a material?

26. What causes lunar (or solar) tides? Are they (approximately!) the same on the side of the earth facing the moon and the opposite side? Which has the greatest effect on the tide, the sun or the moon?

27. Where is the center of mass of a collection of objects $m_1, m_2, m_3...$ located at $\vec{x}_1, \vec{x}_2, ...$?

28. Write the wave equation (the differential equation) for waves on a string with tension T and mass density μ. Identify all parts.

29. One measures sound intensity in decibels. What is a decibel? (Equation, please, and define all constants.)

30. What is Bernoulli's equation? What does it describe?

31. What is the "0th" Law of Thermodynamics?

32. What is the 1st Law of Thermodynamics?

33. What is the 2nd Law of Thermodynamics (any form, extra points for more than one).

34. What is the "3rd" Law of Thermodynamics.

35. What is the Equipartition Theorem?

36. When is a force a "conservative force"? What is being conserved?

37. What is Archimedes' Principle?

38. Suppose a heat engine is operating between two reservoirs: one is at $T_c = 300$ °K, the other is at $T_h = 600$ °K. What is the **highest** its efficiency could be?

39. What are the four "fundamental" forces of nature?

    (a)
    (b)
    (c)
    (d)

40. A pitcher throws a ball at home plate with backspin (that is, bottom of the ball is moving toward the direction of motion, the top away as it rotates). Does the pitch tend to rise or fall? Why? (Draw picture)

41. Roman soldiers (like soldiers the world over) marched in step – except when crossing bridges, when they broke the march and walked over with random pacing. Why?