# Review Problems
# for
# Elementary Physics 42

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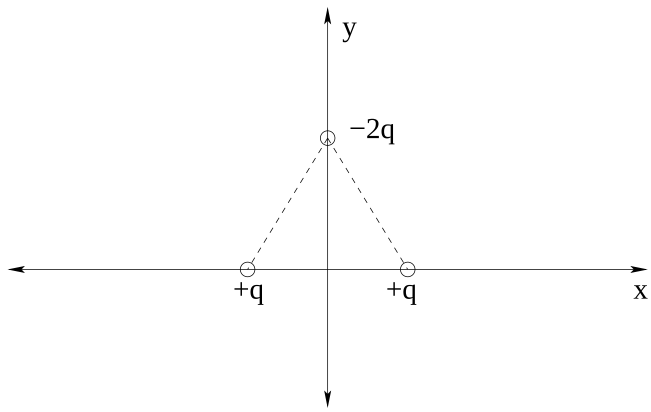
# Contents

This review guide is NOT sorted or made unique – it is the simple union of all the problems in my problem repository. Some problems that I give "a lot" (such as Maxwell's Equations) may be represented many times. Others may occur only once. To a certain extent this reflects their frequency of occurrence and hence the weight I give to the material being tested. This can be used as a clue for how best to spend your limited study time – if something is here three or four times in various forms it is a good idea to know how to do it. If it is here only once, you *might* get a way without knowing it, although of course you should try not to have to take that sort of risk.

Note well that I *always* make up a new problem or two for just about any exam I give, so this list is not and cannot be totally complete. Still, if you can do all of these problems you are likely to be able to do any of the new problems I make up (or at least do as well as any students on those problems).

This whole course emphasizes problem solving and derivation as a route to a deep conceptual understanding of the material. By the time you can work *all* of these problems *quickly* you will be well on your way to true mastery of the material.

Enjoy!




# 1 Electric Field

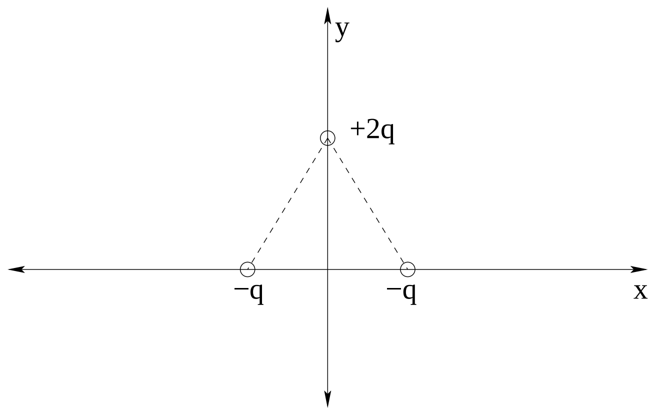
## 1.1

Charges of $+q$ are located at the two bottom corners of an equilateral triangle with sides of length $a$. A charge of $-2q$ is at that top corner. This arrangement of charge can be considered two dipoles oriented at $60^\circ$ with respect to one another.

a) Find the electric field (magnitude and direction) at an arbitrary point on the y-axis above/outside the triangle.

b) What are the first two terms in the binomial-theorem-derived series for the electric field evaluated *far* from the charges, i.e. – for $y >> a$[1]?

[1] Hint: since the net charge balances (=0), we expect no monopolar part (like $1/x^2$). Since the dipoles do not quite balance, we might see a dipolar part (like $1/x^3$). However, since the dipoles are not parallel we might expect to see a significant *quadrupolar* term that varies like $1/x^4$ as well.

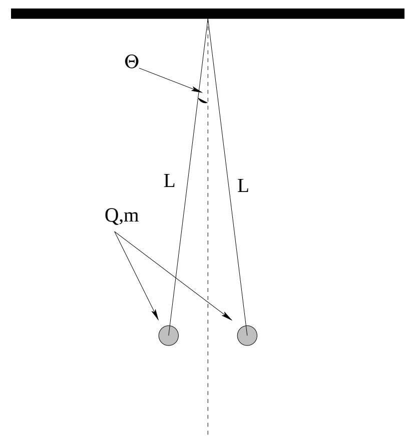



Charges of $-q$ are located at both $y = a$ and $y = -a$, and a charge of $+2q$ is located at $y = 0$ on the y-axis. This arrangement of charge can be visualized as two opposing dipoles.

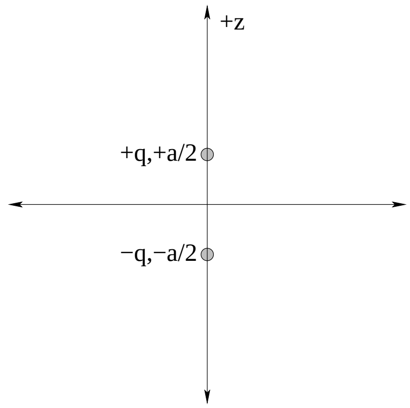
a) Find the electric field (magnitude and direction) at an arbitrary point on the x-axis.

b) What is *nonzero* term in the expansion of the electric field evaluated *far* from the charges, i.e. – for $x >> a$? Your answer should be a series of terms in inverse powers of $x$.

**1.2**




Two positively charged pith balls of mass $m$ each have a charge $Q$ and are suspended by insulating (massless) lines of length $L$ from a common point as shown. Assume that $L$ is long enough that $\theta$ at the top is a small angle. Find $\theta$ such that the pith balls are in static equilibrium in terms of $k$, $Q$, $m$, $L$ and of course $g$.

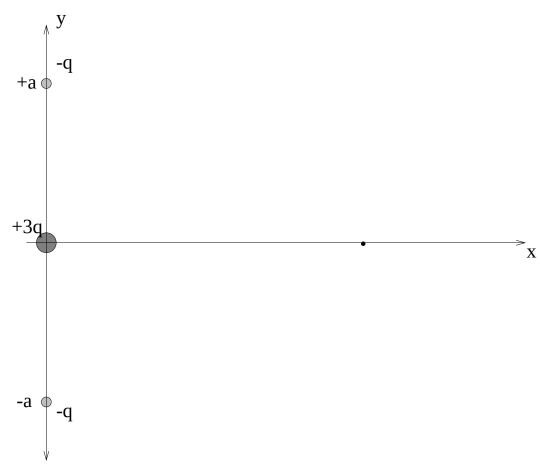



### 1.3

Charges of $\pm q$ are located at both $z = \pm a/2$, respectively. This arrangement forms a electric dipole.

a) Find both the electric potential and electric field (magnitude and direction) at an arbitrary point $z > a/2$ on the z-axis.

b) What is the first *nonzero* term in the expansion of the electric field evaluated *far* from the charges, i.e. – for $z \gg a/2$?

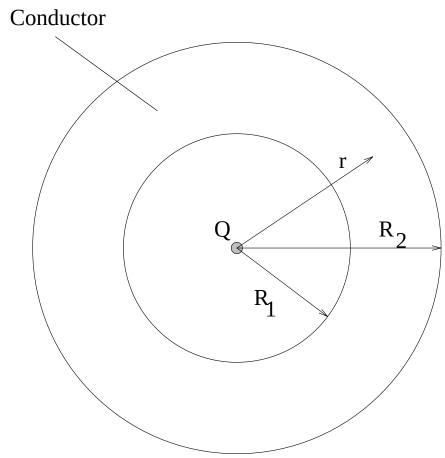



### 1.4

Charges of $-q$ are located at both $y = a$ and $y = -a$, and a charge of $+3q$ is located at $y = 0$ on the y-axis. This arrangement of charge can be visualized as two opposing dipoles plus a charge at the center.

a) Find the electric field (magnitude and direction) at an arbitrary point on the x-axis.

b) What are the first two *nonzero* terms in the electric field evaluated *far* from the charges, i.e. – for $x >> a$? Your answer should be a series of terms in inverse powers of $x$.

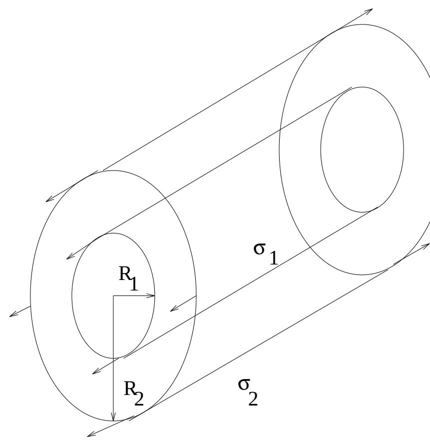



### 1.5

A conducting shell concentrically surrounds a point charge of magnitude $Q$ located at the origin. The inner radius of the shell is $R_1$ and the outer radius $R_2$.

a) Find the electric field $\vec{E}$ at all points in space (you should have three answers for three distinct regions).

b) Find the surface charge density $\sigma$ on the inner surface of the conductor. Justify your answer with Gauss's law.

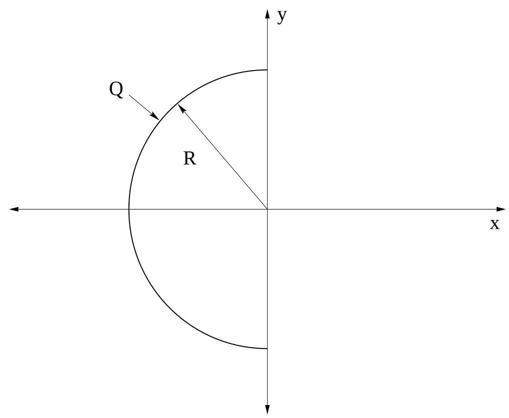



## 1.6

Two infinitely long, cylindrical conducting shells are concentrically arranged as shown above. The inner shell has a radius $R_1$ and the outer shell the radius $R_2$. The inner shell has a charge per unit area $\sigma_1$, and the outer shell a charge per unit area $\sigma_2$.

a) Find the electric field $\vec{E}$ at all points in space (you should have three answers for three distinct regions).

b) Find the surface charge density $\sigma_2$ (in terms of $\sigma_1, R_1, R_2$, etc.) that causes the field to vanish everywhere but in between the two shells. *Justify your answer with Gauss's law.*

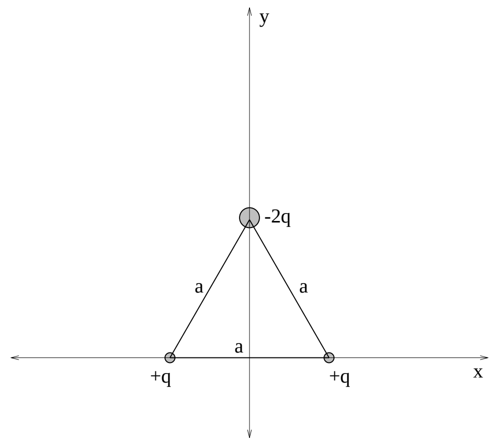



# 2 Electric Potential

## 2.1

A half-ring of total charge $Q$ and radius $R$ sits symmetrically across the $x$-axis around the origin as shown in the figure above.

1. Find the electric field at the origin (magnitude and direction) from direct integration.
2. What is the electric potential at the origin?



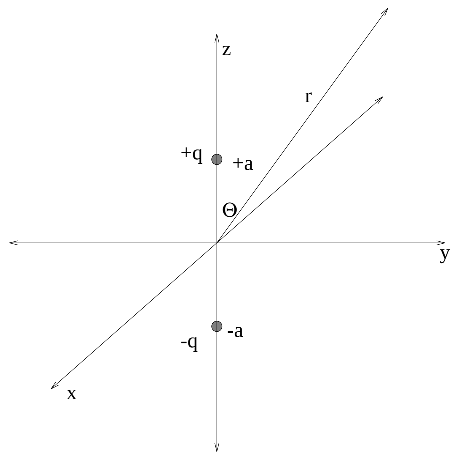

### 2.2

Charges of $+q$ are located at the two bottom corners of an equilateral triangle with sides of length $a$. A charge of $-2q$ is at that top corner. This arrangement of charge can be considered two dipoles oriented at $60°$ with respect to one another.

a) Find the electric field (magnitude and direction) at an arbitrary point on the y-axis above/outside the triangle.

b) What are the first two surviving terms in the binomial-theorem-derived series for the electric field evaluated *far* from the charges, i.e. – for $y >> a^2$?

[2] Hint: since the net charge balances (=0), we expect no monopolar part (like $1/x^2$). Since the dipoles do not quite balance, we might see a dipolar part (like $1/x^3$). However, since the dipoles are not parallel we might expect to see a significant *quadrupolar* term that varies like $1/x^4$ as well.




# 3 Electric Potential

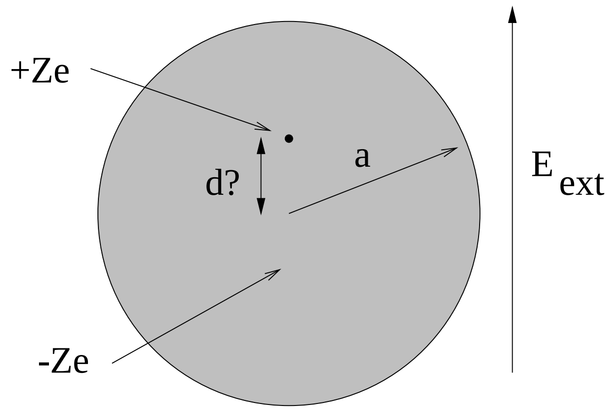
## 3.1

A point charge of $-q$ is located at $z = -a$ on the $z$-axis and a point charge of $+q$ is located at $z = +a$.

1. (10 points) Write down the potential at an arbitrary point in space in spherical coordinates $(r, \theta, \phi)$.

2. (10 points) What is the leading term in the expansion of the potential for $r \gg a$, expressed in terms of the dipole moment?

(Note: Obviously, I *really do* want you to learn to do this one, since I did a bunch of it in lecture and asked you to finish the rest on your own. You derive a very fundamental result that will be useful to you in the years to come....so much that I'll give two hints. One is use the law of cosines to determine the distance from e.g. $+q$ to the point of observation in terms of $r$, $a$, and $\theta$ only. Second is to use the binomial expansion to extract the leading order potential term(s) for each charge and add them until you get the overall leading order term that *survives* – the first term or two might well cancel.)

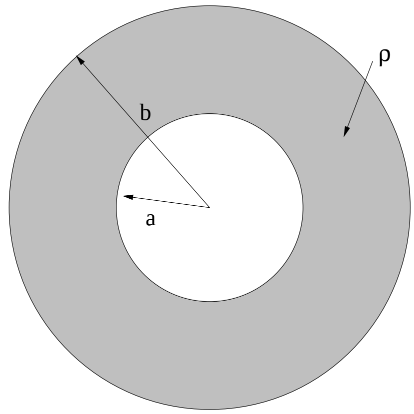



### 3.2

A simple model for an atom has a tiny (point-like) nucleus with charge $+Ze$ is located at the center of a uniform sphere of charge with radius $a$ and total charge -$Ze$. The atom is placed in a uniform electric field which displaces the nucleus as shown. Find:

a) The equilibrium separation $d$ of the nucleus from the center of the spherical electron cloud;

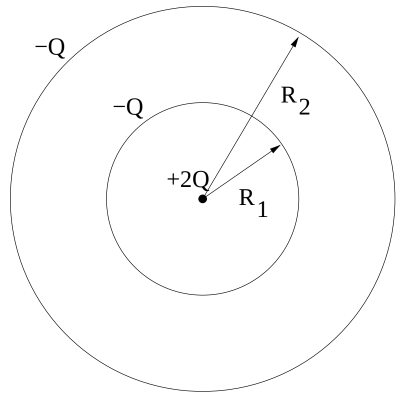
b) The approximate *average* polarization density (dipole moment per unit volume) of the polarized atom.




### 3.3

A spherical shell of inner radius $a$ and outer radius $b$ contains a uniform distribution of charge with charge density $\rho$.

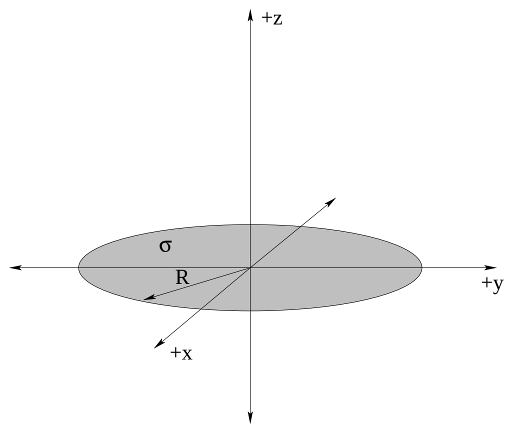
Find the field and potential at all points in space.




### 3.4

Two spherical shells with radii $R_1$ and $R_2$ respectively concentrically surround a point charge. The central point charge has magnitude $2Q$. Both the spherical shells have a charge of $-Q$ (each) distributed uniformly upon the shells.

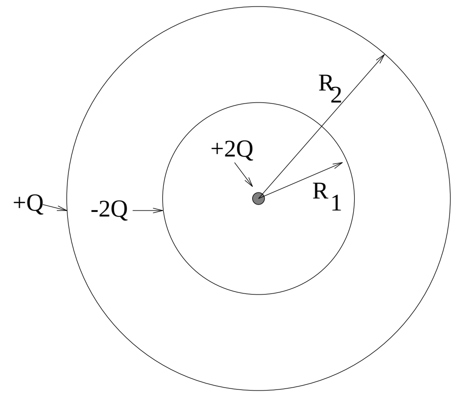
Find the field and potential at all points in space. Show your work – even if you can just write the answer(s) down for each region, briefly sketch the methodology used to get the answers.




### 3.5

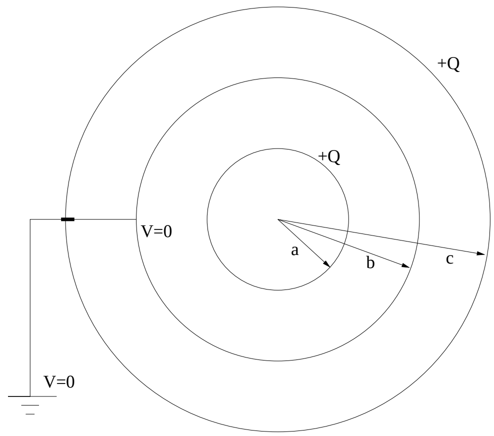
Find the electric potential on the $(z)$ axis of a disk of charge of radius $R$ with uniform surface charge distribution $\sigma$.

For extra credit (if you have time) find the electric field on the $z$-axis.




### 3.6

Find the potential $V(r)$ at all points in space for the arrangement of charge pictured above, where there is a point charge $+2Q$ at the origin, a charge uniformly distributed $-2Q$ on the inner shell (radius $R_1$), and a charge $+Q$ uniformly distributed on the outer shell (radius $R_2$). You will need three different answers for the three distinct regions of space.

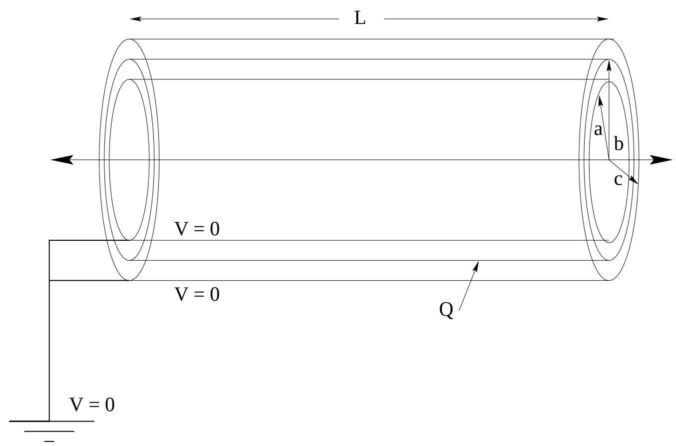



### 3.7

A charge of $+Q$ is placed on the innermost and outermost of three concentric conducting spherical shells. The middle shell is grounded via a thin wire that passes through an insulated hole in the outer shell and hence has a potential (relative to $\infty$) of 0.

a) Find the charge $Q_s$ on the middle shell in terms of $k$, $Q$, and the given radii $a$, $b$ and $c$.

b) Find the potential at all points in space (in each region where there is a distinct field). You may express your answers algebraically in terms of $Q_s$ to make life a bit simpler (and independent of your answer to part a).

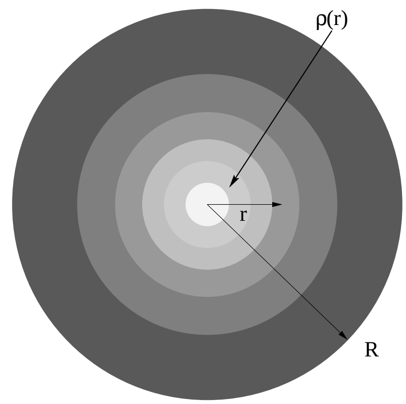



## 3.8

Three cylindrical conducting shells of radii $a < b < c$ and of length $L \gg c$ are placed in a concentric configuration as shown. The middle shell is given a total charge $Q$, and both the inner and outer shells are grounded (connected by a thin wire to each other and to something at a potential of "0"). Find:

a) The total charge on the *inner* shell, in terms of $a, b, c, L, Q$ and $k$.

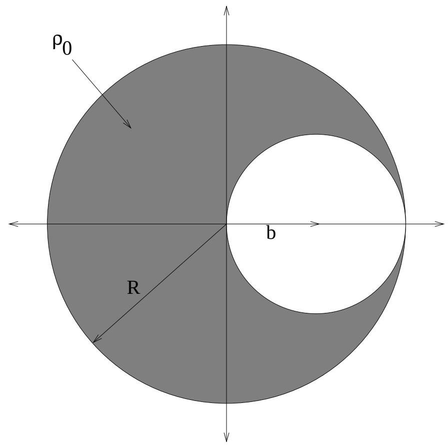
b) The potential on the middle shell. In what direction does the field point in between $a$ and $b$ and in between $b$ and $c$?




### 3.9

Find the electric field and electric potential at all points in space of a sphere with radial charge density:

$$\rho(r) = \rho_0 r \qquad r < R$$
$$\rho(r) = 0 \qquad r < R$$



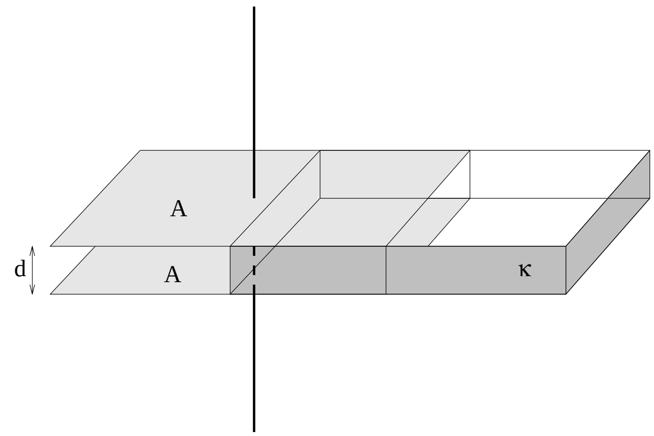

### 3.10

A sphere of uniform charge density $\rho_0$ has a hole of radius $b = R/2$ centered on $x = b$ cut out of it as shown in the figure.

a) Find the electric field **vector** inside the hole.

b) Find the electric potential at an arbitrary point $x > R$.

c) What is the limiting form of the electric potential as $x \to +\infty$? What kind of field does this correspond to?



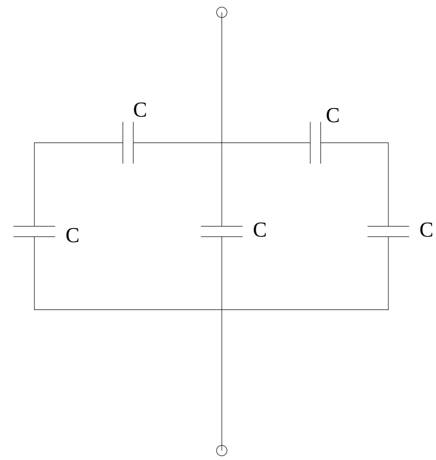

# 4 Capacitance

## 4.1

A parallel plate capacitor is constructed from two square conducting plates of with an area of $A$, separated by a distance of $d$. An insulating slab of thickness $d$ and a dielectric constant $\kappa$ is inserted so that it half-fills the space between the plates. Find:

1. (15 points) the capacitance of this arrangement;

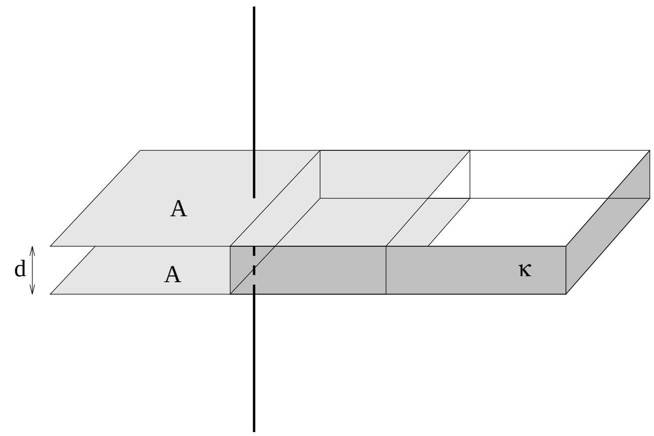
2. (5 points) the electrostatic force on the dielectric slab when a potential $V$ is maintained across the capacitor. Does it pull the dielectric in between the plates or push it out from between them?




## 4.2

1. Find the total effective capacitance between the two contacts (the round circles at the top and the bottom) of the arrangement of capacitors drawn above. Naturally, show all work.

2. If a potential $V$ is connected across the contacts, indicate the *relative* size of the charge on each capacitor. (It will probably be easiest if you give your answers in terms of $Q = CV$.)



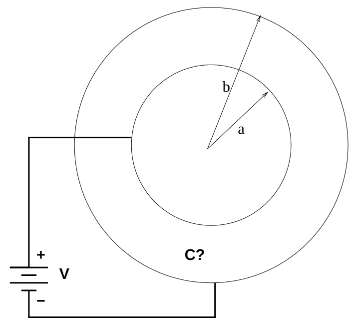

### 4.3

A parallel plate capacitor is constructed from two square conducting plates of with an area of $A$, separated by a distance of $d$. An insulating slab of thickness $d$ and a dielectric constant $\kappa$ is inserted so that it half-fills the space between the plates.

a) Find the capacitance of this arrangement. Clearly indicate the basic principles and definitions you are using, e.g. the definition of capacitance, the equation that defines the effect of a dielectric on the electric field and so forth.

b) Find the electrostatic force on the dielectric slab when a fixed charge of $\pm Q$ is placed on the two plates of the capacitor. If you cannot do this, for partial credit at least indicate on physical grounds whether the force pulls the dielectric in between the plates (trying to fill the space between them) or pushes it out from between them and explain your reasoning.

Hint: Consider the relation between force and potential energy.

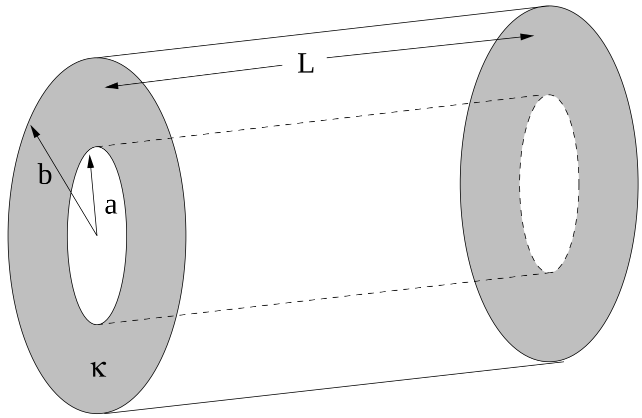



### 4.4

A spherical capacitor has inner radius $a$ and outer radius $b$

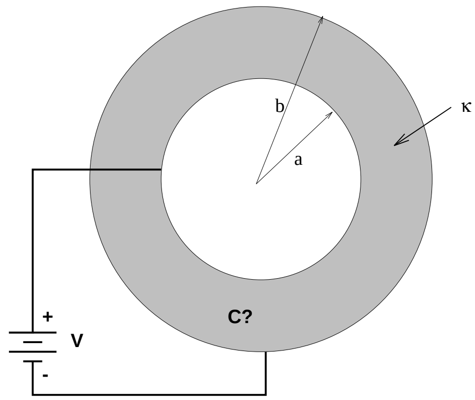
a) Find (derive!) its capacitance of this arrangement. *Show All Work!*

b) Show that when $b = a + \delta$ with $\delta \ll a$ the capacitance has the limiting form $C = \epsilon_0 A/\delta$ where $A$ is the area of the inner sphere and $\delta$ is the separation of the shells.



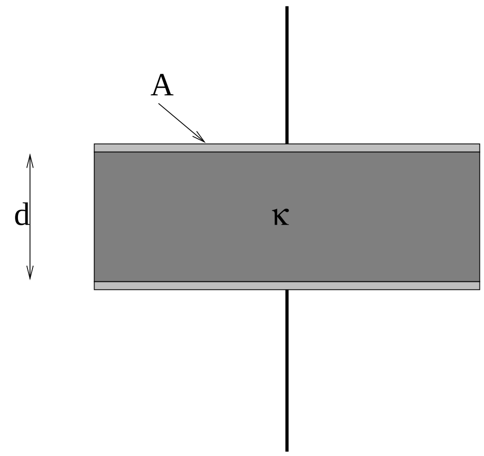

### 4.5

Find $\vec{\mathbf{E}}$ and $V$ at all points in space, and $C$ for the cylindrical capacitor drawn above, with inner radius $a$, outer radius $b$, length $L$, filled with a dielectric of dielectric constant $\kappa$.




## 4.6

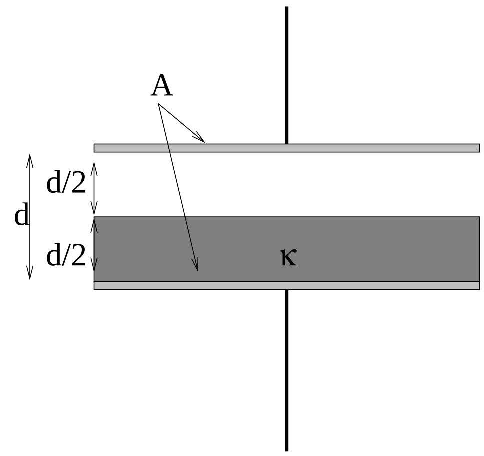
A spherical capacitor with inner radius $a$ and outer radius $b$ has the space in between filled with a dielectric with dielectric constant $\kappa$. Find the capacitance of this arrangement. *Show All Work!*




**4.7**

A parallel plate capacitor has cross sectional area $A$ and separation $d$. A dielectric material with dielectric constant $\kappa$ of thickness $d$ and area $A$ fills the space in between as shown.

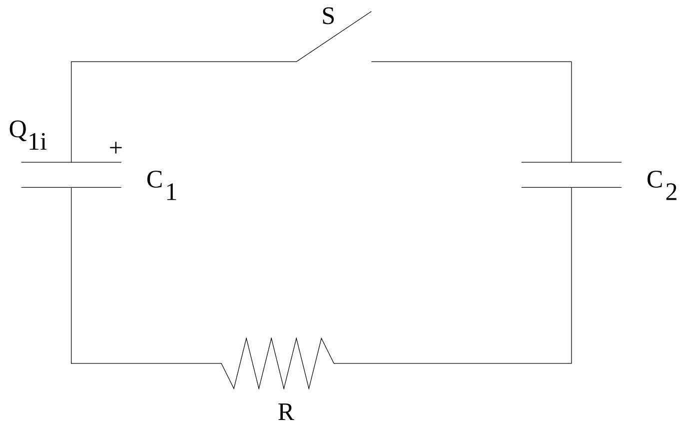
Suppose a potential $V$ is connected across this capacitor. Find the electric field inside the dielectric, the free charge (and hence the capacitance), and the bound charge on the surface of the dielectric. *Show all work!*




### 4.8

A parallel plate capacitor has cross sectional area $A$ and separation $d$. A dielectric material with dielectric constant $\kappa$ of thickness $d/2$ and area $A$ half fills the space in between as shown.

Find the capacitance of this arrangement. *Show all work!*.




# 5 Resistors and RC Circuits

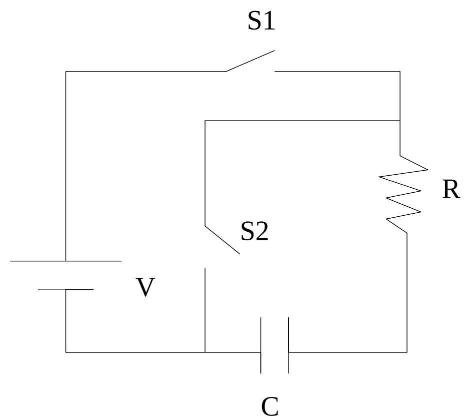
## 5.1

A pair of capacitors $C_1$ and $C_2$ is connected as shown, with a resistance $R$ in between them. Initially, the first capacitor carries a total charge $Q_{1i}$ and the second one is uncharged, $Q_{2i} = 0$. At $t = 0$ the switch is closed. Find:

a) The equilibrium ($t = \infty$) charges on the two capacitors, $Q_{1f}$ and $Q_{2f}$.

b) Using Kirchoff's laws for this arrangement, find the time constant for the equilibration process. Note that you do NOT have to solve the DE, just formulate it with $dt$ and some arrangement of $R$, $C_1$, and $C_2$ on the other side.

c) For extra credit, either solve the DE (it is integrable, although a bit messy) or GUESS what its solution is, based on your answers to a) and b). To do the latter, try visualizing what $Q_1(t)$ and $Q_2(t)$ will formally look like – it is just a matter of setting the various constants so that the asymptotic (final) and initial conditions are correctly represented and the approach to those conditions has the right time dependence.



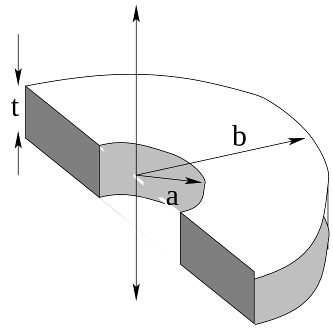

## 5.2

In the circuit above, R = 100Ω and C = 1$\mu$Farad, and V = 10 volts. The capacitor is initially uncharged. To simplify arithmetic to the finger and toe level, answers given algebraically in terms of powers of $e$ are acceptable – no calculators should be strictly necessary although you can smoke 'em if you got 'em.

a) At time $t = 0$, switch 1 is closed. What is the charge on the capacitor as a function of time?

b) At time $t = 300$ microseconds, switch 1 is opened and switch 2 is closed. What is the voltage across the capacitor as a function of time.

c) At time $t = 500$ microseconds (from $t = 0$ in part a) switch 2 is opened. How much energy is stored in the capacitor at that time?




**5.3**

A printed circuit board contains a resistor formed of a semicircular bend of resistive material (resistivity $\rho$) of thickness $t$, inner radius $a$ and outer radius $b$ as shown in the figure above. Copper traces maintain a constant voltage $V$ across the semicircular resistor. Find (in terms of the givens):

a) the resistance $R$ of the resistor – be sure to indicate the basic formulae you are starting from for partial credit in case you can't quite get the integral right (it is like the integral in a homework problem);

b) the energy dissipated as heat in the resistor in $s$ seconds. Again, if you cannot find $R$ in terms of the givens (or have little confidence in your answer), you may use $R$ and other given quantities to answer b) for at least partial credit.

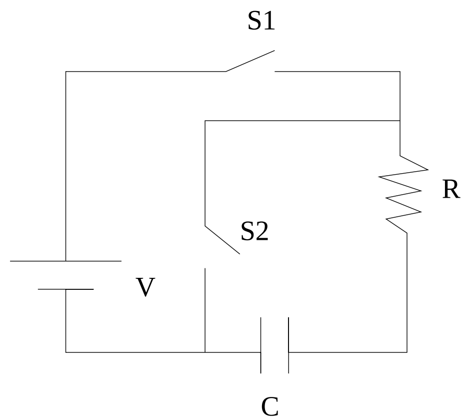
**Check the Units of Your Answers!**

## 5.4

You are given a box containing a digital meter that measures current. It reads 100.0 when a current of 0.1 mA is passed through it. It has a resistance of 10 Ω. The box also contains assorted resistors in powers of 10 Ω, e.g. $\ldots \mathrm{R}_{-6} = 10^{-6}\Omega$, $\mathrm{R}_{-5} = 10^{-5}\Omega$, $\ldots$, $\mathrm{R}_6 = 10^6\Omega$, ... with at least ten resistors available at each size. The resistors are only good to 10%, though, so there is no point in trying make combinations with more than one significant digit out of different powers. Use this material to design:

a) An ammeter that makes the digital scale read Amps (that is, read (approximately) 100.0 when a current of 100.0 A is flowing into it). Draw it, label all parts, and show your reasoning.

b) A voltmeter that makes the digital scale read Volts (that is, read 100.0 with it is placed across 100 volts). Draw it, label all parts, and show your reasoning.




### 5.5

In the circuit above, assume $R$, $C$, and $V$ are given. Derive all your answers for up to five bonus points, but you may just give the answers below for full credit. All answers may be expressed in terms of powers of $e$.

a) At time $t = 0$, switch 1 is closed. What is the *charge* on the capacitor as a function of time in terms of the given quantities.

b) After a very long time ($\gg RC$) switch 1 is opened and switch 2 is closed. What is the *voltage* across the capacitor as a function of time.

c) At time $t = 2RC$ switch 2 is opened. How much energy is stored in the capacitor at that time?

**5.6**

A large Leyden jar (capacitor) is surrounded by dry air so that the net resistance between its charged and grounded terminal is approximately $10^{1}0\ \Omega$. It is charged up to 50,000 volts by a Wimshurst generator (at which time it contains 0.005 Coulombs of charge). It is then disconnected and left there by a negligent physics instructor. 33 1/3 minutes later, an astrophysics professor comes into the room and, seeking to move the jar, grabs the ungrounded, charged, central terminal. How much charge seeks ground through this hapless soul's body? How much stored energy is dissipated in the process? (You can solve this algebraically if you have no calculator handy.)

Ouch! These are not unrealistic parameters. Leyden jars can be very, very dangerous for hours after they are charged up.

# 6 Magnetic Forces

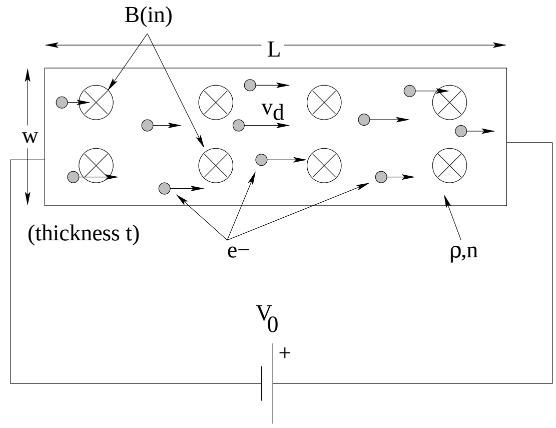
## 6.1

A particle of charge $q$ and mass $m$ has momentum (magnitude) $p = mv$ and kinetic energy $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$. If the particle moves in a circular orbit of radius $r$ perpendicular to a uniform magnetic field of magnitude $B$, show that:

- $p = Bqr$
- $K = \frac{B^2q^2r^2}{2m}$
- The angular momentum magnitude $L = Bqr^2$.

## 6.2

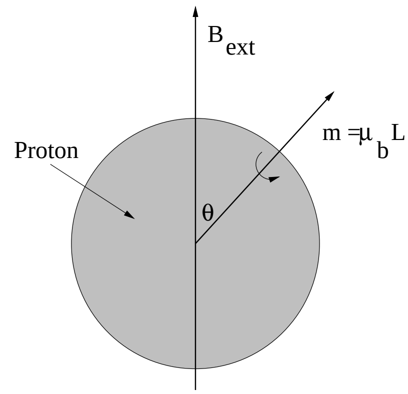
A beam of particles with velocity $\vec{v}$ enters a region of uniform magnetic field $\vec{B}$ that makes a small angle $\theta$ with $\vec{v}$. Show that after a particle moves a distance $\frac{2\pi m}{qB} v\cos(\theta)$ measured along the direction of $\vec{B}$, the velocity of the particle is in the same direction as it was when it entered the field. (Look familiar:-)




### 6.3

a) A rectangular metal strip of length $L$, width $w$, and thickness $t$ sits in a uniform magnetic field $B$ perpendicular to the strip and into the page as shown. The material has resistivity $\rho$ and a free (conducting) *electron* (charge $q = -e$) density of $n$. A voltage $V_0$ is connected across the strip so that the *electrons* travel from left to right as shown.

find an expression for the Hall potential (the potential difference across the strip from top to bottom) in terms of the given quantities. Note (as a set of recipe/hints) that you'll have to start by relating $I$ (the current in the strip) to the givens, and then translate that into a form involving the drift velocity $v_d$. From $v_d$ and your knowledge of magnetic forces you should be able to determine the electric field and then the potential across the strip in the steady state.

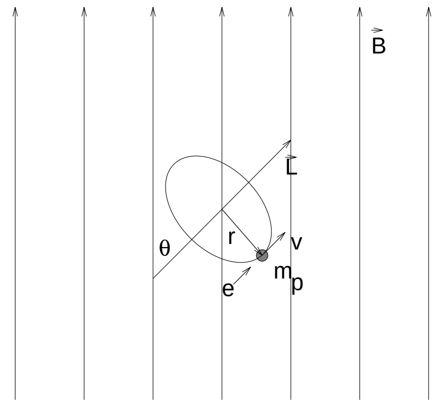



## 6.4

A proton (charge $+e$) with mass $m_p$ has an intrisic angular momentum given by $\vec{L}$ and a magnetic moment given by $\vec{m} = \mu_B \vec{L}$. When the proton is placed in a uniform magnetic field of strength $B$ so that $\vec{L}$ makes an angle of $\theta$ with $\vec{B}$, the angular momentum precesses around $\vec{B}$.

a) Find the angular frequency $\omega_p$ with which the angular momentum precesses. Indicate the direction of precession on the figure above (into or out of page, as drawn).

b) For extra credit, **derive** $\mu_B$. One way you might proceed is to simply derive $\vec{m}$ and $\vec{L}$ separately for the proton, assuming uniform mass and charge distribution and a common angular velocity $\vec{\omega}$. A better way to proceed might be to relate $dm_z$ (along the axis of rotation) to $dL_z$ (ditto) assuming axial symmetry so that $\vec{L}$ is parallel to $\vec{\omega}$.

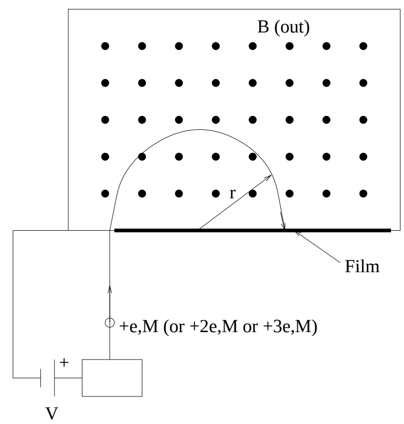



**6.5**

A proton (charge $+e$) with mass $m_p$ is in a circular orbit of radius $r$ such that it has an angular momentum given by $\vec{L}$. The orbiting proton has a magnetic moment $\vec{m}$ parallel to its angular momentum. When the orbiting proton is placed in a uniform magnetic field of strength $B$ so that $\vec{L}$ makes an angle of $\theta$ with $\vec{B}$, the angular momentum precesses around $\vec{B}$. Find:

a) The magnetic moment of the orbit in terms of $\vec{L}$;

b) The angular frequency with which the angular momentum precesses.

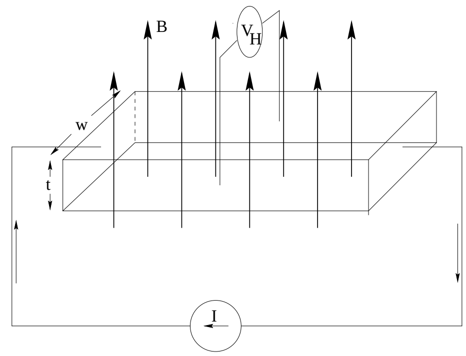



### 6.6

In the mass spectrograph above, the goo in the source chamber contains molecules of mass $M$ that are ionized to have charges of $+e$, $+2e$ or $+3e$ at the source. The particles then fall through a potential of $V$ and enter the uniform $B$ field in the box.

a) Derive an expression for the radius $r$ at which a fragment of charge-to-mass ratio of $m/q$ hits.

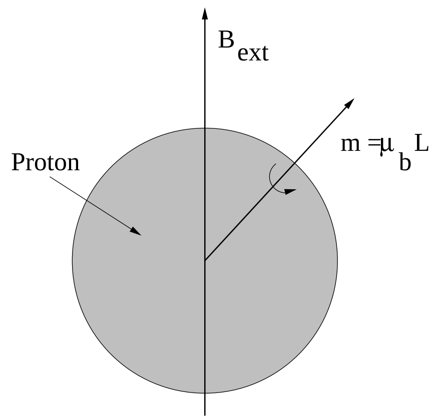
b) Use this expression to find $r$ for each of the three possible ionization charges, and draw a picture of the bars produced on the film to a reasonable scale.




Voltage $V_H$ measured across w
Magnetic Field B perpendicular to strip as shown
Current I measured through strip in direction shown

### 6.7

The apparatus for measuring the Hall effect is shown above. Consider a charge carrier $q$ (to keep you from having to mess with the negative charge on the real charge carriers – electrons) moving through the apparatus in a material with an unknown $n$ charge carriers per unit volume. **Derive** an expression for $n$, given $I$, $V_H$, $t$, $B$, $w$ and $q$. Note that I'd have to consider you moderately insane to have memorized this result (I certainly haven't) but by considering the strip to be a region of self-maintaining crossed fields and relating the current to the drift velocity you should be able to get it fairly easily.

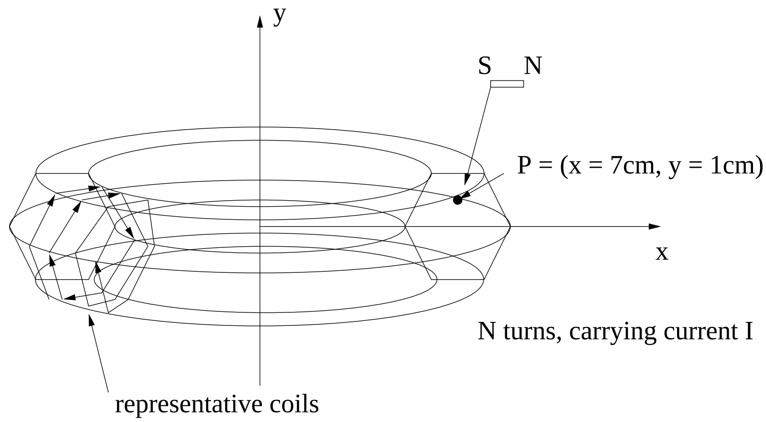



**6.8**

A proton (charge $+e$) with mass $m_p$ has an intrisic angular momentum given by $\vec{L}$ and a magnetic moment given by $\vec{m} = \mu_B \vec{L}$. When the proton is placed in a uniform magnetic field of strength $B$ so that $\vec{L}$ makes an angle of $\theta$ with $\vec{B}$, the angular momentum precesses around $\vec{B}$. Find:

a) The angular frequency with which the angular momentum precesses. Indicate the direction of precession on the figure above (into or out of page, as drawn).

b) For extra credit, indicate how one might *change* the angle $\theta$ with a time-dependent magnetic field.

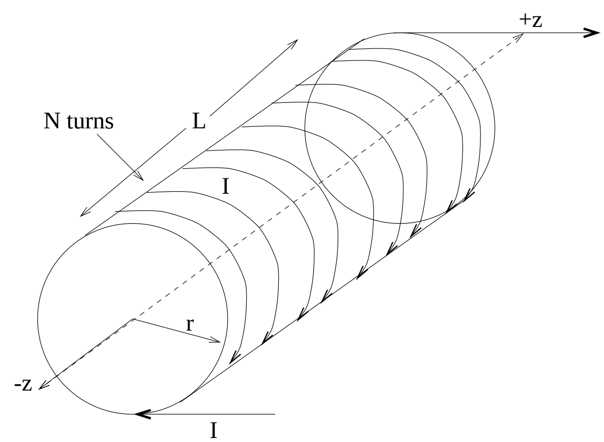



# 7 Magnetic Fields and Ampere's Law

## 7.1

a) Find the magnetic field at the point labelled $P$ in the toroidal solenoid pictured above, when $N = 1000$ turns and $I = 10$ amps. Indicate its direction on the picture.

b) If a bar magnet (magnetic dipole with length $l$ and pole strength $q$) is placed so that its south pole is at $P$ and its north pole is oriented outward along x (as shown) what is the approximate *direction* of the net force on the dipole?




## 7.2

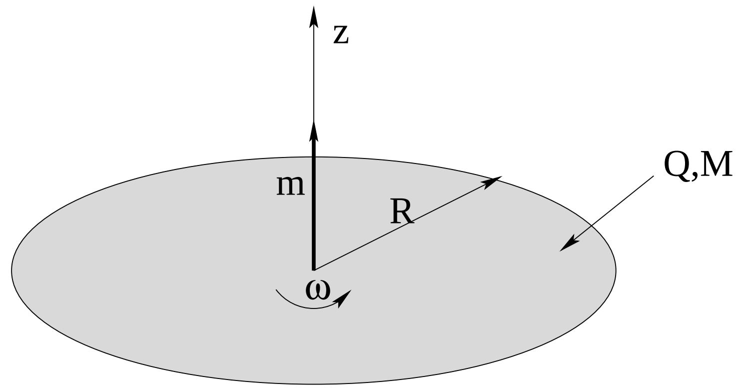
A solenoid is built of length $L = 0.1$ meter with $N = 10^4$ turns and a radius of $r = 1$ cm as drawn. A current $I = 5\text{A}$ is driven through the solenoid.

a) Find the magnetic field $\vec{B}$ inside the solenoid, neglecting end effects (magnitude and direction, given the direction of current flow drawn). Show algebraically how you got your numerical answer in SI units. Draw the direction of the field lines in on your picture.

b) Find the self-inductance of the solenoid.

**7.3**

Using the Biot-Savart law, find the magnetic field at an arbitrary point on the $z$-axis of a circular loop or radius $a$ carrying a current $I$ counterclockwise around the $z$-axis and centered on the origin in the $x$-$y$ plane. Draw the arrangement in the space provided above, of course. Show all work – don't just write the answer down even. Then find the field in the limit that $z \gg a$ and show that it is the familiar field of a (magnetic) dipole on its axis.



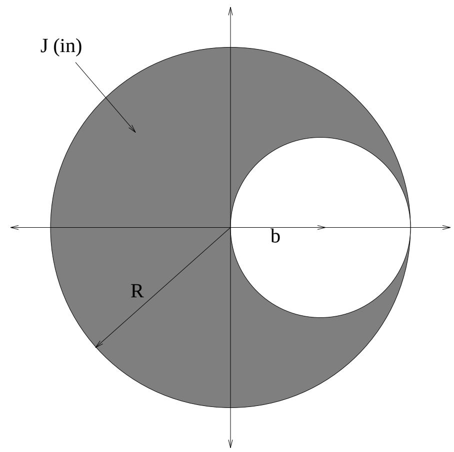

**7.4**

A flat disk of radius $R$ with uniform surface charge density $\sigma_q = \frac{Q}{\pi R^2}$ and surface mass density $\sigma_m = \frac{M}{\pi R^2}$ is rotating at angular velocity $\omega$.

Show that its magnetic moment $\vec{m} = \mu_B \vec{L}$ with $\mu_B = \frac{Q}{2M}$.

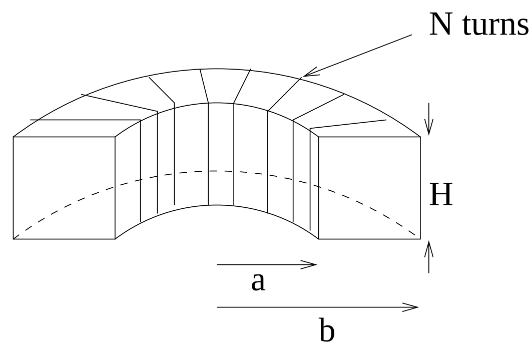
Hints: So many ways to do this one. One I showed you this morning – consider the relation between $\sigma_m$, differential angular momentum about the $z$-axis of a tiny chunk of the disk of area $dA$, and stuff like $\omega$, $T$, $v$, $r$. Then consider the relation between $\sigma_q$ and the differential $z$-component of the magnetic moment of the *same* tiny chunk of the disk. The desired result can be derived either as a differential relation (and nominally integrating) or by doing the simple integrals over the disk.

Or you can just evaluate the two (like you did for a homework problem or two) and divide. This isn't difficult either (and you'll find yourself doing the same actual $r$-integral twice with different constants).




### 7.5

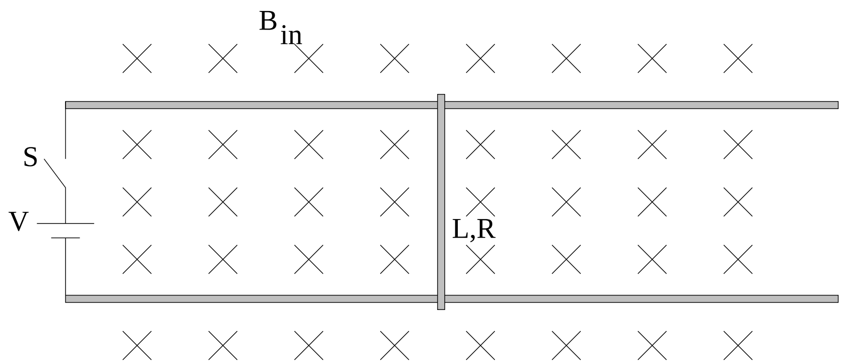
A cylindrical long straight wire of radius $R$ has a cylindrical long straight hold of radius $b = R/2$ and carries a current density of $\vec{\mathbf{J}}$ into the page as drawn. Find the magnetic field (magnitude and direction) at an arbitrary point inside the hole.




# 8 Inductance and Faraday's Law

## 8.1

Find the self-inductance $L$ of a toroidal solenoid with a rectangular cross-section (height $H$, inner radius $a$, outer radius $b$) and $N$ turns.




### 8.2

A rod of mass $m$, resistance $R$ and length $L$ is sitting at rest on frictionless rails in a magnetic field as shown. At $t = 0$, the switch $S$ is closed and a voltage $V$ applied across the rails. Show all work while deriving the following results, clearly indicating the physical law used and reasoning process. Neatness and clarity count.

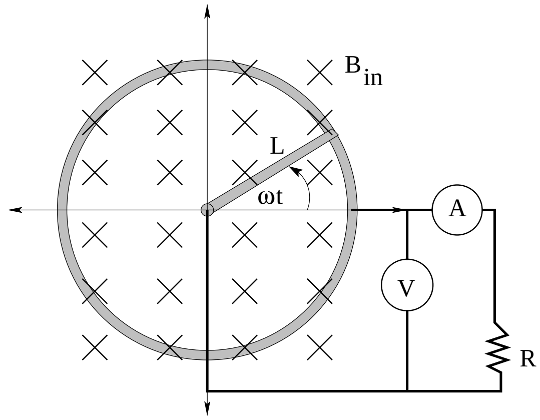
a) What is the net voltage across the resistance $R$ as a function of $|\vec{v}|$?

b) What is the current $I$ in the loop as a function of $\vec{v}$?

c) What is the force $\vec{F}$ on the rod as a function of $\vec{v}$?

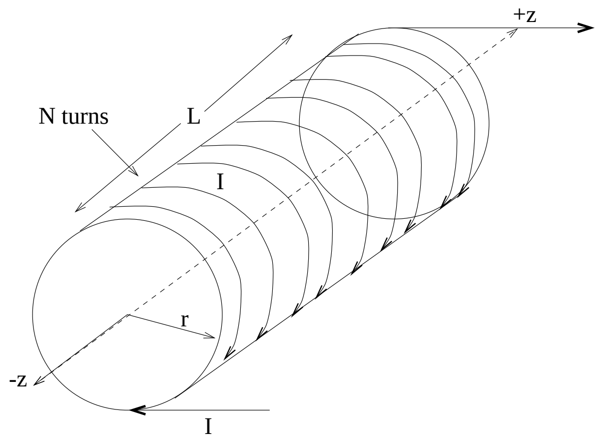
d) What is the terminal velocity of the rod as $t \to \infty$?

10 points of extra Credit: Solve the first order, linear, ordinary, inhomogeneous differential equation and find the velocity of the rod $\vec{v}(t)$ as a function of time. Draw a qualitatively correct curve showing this function and show how it corresponds to your answer to d).




**8.3**

A rod of length $L$ is pivoted at one end and swings around at an angular frequency $\omega$ with its other end sliding along a circular conducting track. A magnetic field $B_{\mathrm{in}}$ is oriented perpendicular to the plane of rotation of the rod as shown. The pivot point and the outer ring are connected by (fixed) wires across a resistance $R$ with a voltmeter and ammeter inserted in the circuit as shown. What do the voltmeter and ammeter read?




## 8.4

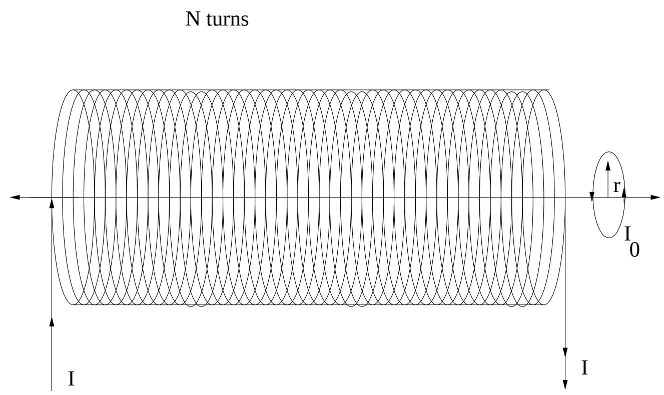
A solenoid is built of length $L$ with $N$ turns and a radius of $r$. A current $I$ is driven through the solenoid. **Derive from basic laws and definitions**:

a) The magnetic field $\vec{B}$ inside the solenoid, neglecting end effects (magnitude and direction, given the direction of current flow drawn).

b) The magnetic flux $\phi_m$ through the solenoid, as a function of I?

c) What is the self-inductance of the solenoid?

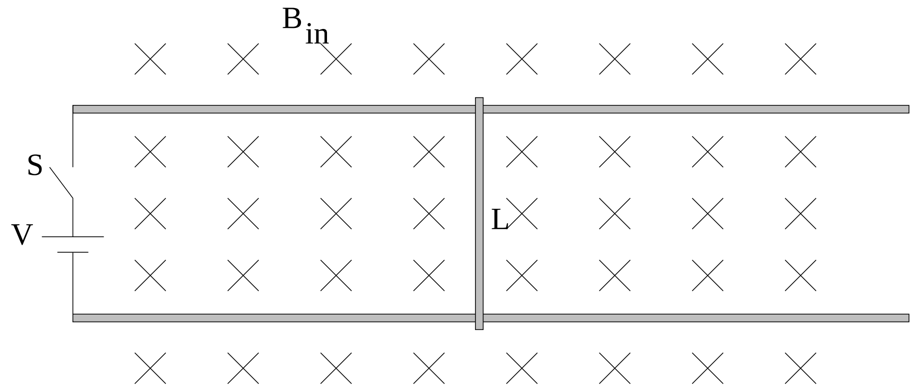
You might prefer to draw your own picture(s) to facilitate the work.




**8.5**

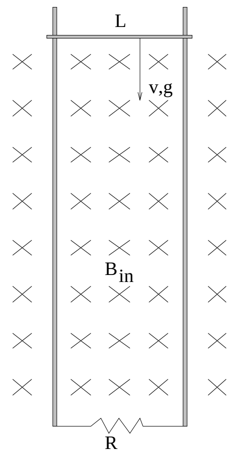
a) Find the magnetic field inside the solenoid pictured above, when $N = 1000$ turns and $I = 10$ amps, assuming that it is "much longer than its diameter". Indicate its direction on the picture.

b) If a current loop of radius $r$ carrying a current $I_0$ is placed near the end of the solenoid as shown, should it experience a force? Why or why not?




**8.6**

A rod of mass $m$, resistance $R$ and length $L$ is sitting at rest on frictionless rails in a magnetic field as shown. At $t = 0$, the switch $S$ is closed and a voltage $V$ applied across the rails. Find the velocity of the rod as a function of time from a combination of Newton's laws, Ohm's law, and Faraday's law. If you are clueless (in spite of this being a homework problem and very similar to a quiz problem) at least tell me what all those laws and the terminal velocity *are* for most of the credit.



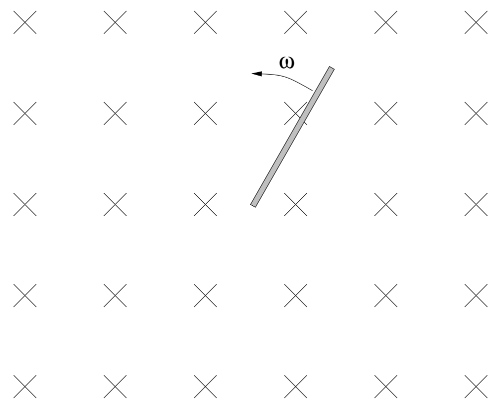

### 8.7

A rod of length $L$ and mass $m$ slides on frictionless conducting guides down vertical rails, connected at the bottom, that enclose a uniform magnetic field of magnitude $B$ as shown, starting at rest at $t = 0$. The loop formed by the rod and rails has a total resistance of $R$. **Gravity** makes the rod fall. Find:

a) The current $I(v)$ induced in the rod when the speed of the rod is $v$ (down). Indicate the direction on the figure above.

b) The **net** force on the rod as a function of $v$.

c) For extra credit, solve the resulting equations of motion for $v(t)$ and/or determine the terminal velocity of the rod.



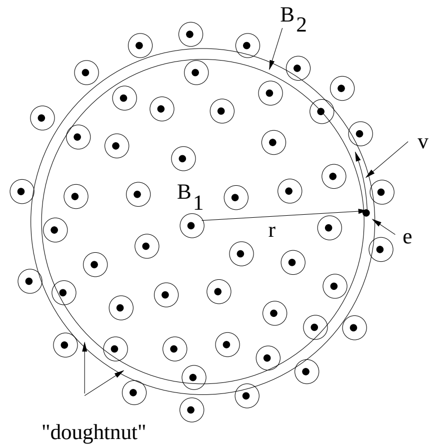

## 8.8

A conducting bar of length $L$ rotates at an angular frequency $\omega$ in a uniform, perpendicular magnetic field as shown.

a) Find the potential developed between the central end of the rod and a point at radius $r$ on the rotating rod.

b) Find the field associated with this potential difference (magnitude and direction) at the radius $r$.

c) Discuss the qualitative distribution of charge in the rod, assuming that it is in equilibrium (has been rotating for a long time).




### 8.9

A Betatron (pictured above with field out of the page) works by increasing a uniform magnetic field in such a way that electrons of charge $e$ and mass $m$ inside the "doughnut" tube are accelerated by the $E$-field produced by induction from the average time-dependent magnetic field $B_1(t)$ inside $r$ (via Faraday's law) while the average magnitude of the magnetic field at the radius $B_2(t)$ bends the electrons around in the constant radius circle of radius $r$.

This problem solves for the "betatron condition" which relates $B_1(t)$ to $B_2(t)$ such that both things can simultaneously be true.

a) First, assuming that the electrons go around in circles of radius $r$ and are accelerated by an $\vec{E}$ field produced by Faraday's law from the average field $B_1$ inside that radius, solve for that induced $E$ field in terms of $B_1$ and $r$.

b) Second, assuming that the electrons are bent into a circle of radius $r$ by the average field at that radius, $B_2$, relate $B_2$ to the momentum $p = mv$ and charge $e$ of the electron, and the radius $r$.

c) Third, noting that the force $F$ from the $E$-field acting on the electron with charge $e$ in part a) is equal to the time rate of change of $p$ in the result of b) substitute, cancel stuff, and solve for $dB_1/dt$ in terms of $dB_2/dt$. If you did things right, the units will make sense and the relationship will only involve dimensionless numbers, not $e$ or $m$.

Cool! You've just figured out how to build one of the world's cheapest electron accelerators! Or perhaps not....

# 9 Maxwell's Equations and Light

## 9.1

Invent and compare spaceships (draw them in the blank space above) that are driven according to the following (ideal) criteria. The actual source of power is e.g. a small fusion plant onboard the spaceship.

a) Suppose a spaceship is powered by a laser that emits 1000 Watts in a beam 1 $\mathrm{cm}^2$ in cross-sectional area. What is the recoil force (per KW) exerted by the laser?

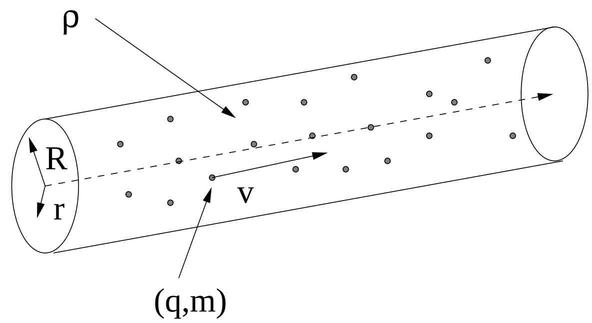
b) Suppose instead the spaceship is powered by throwing mass. If it throws 1000 small beads per second, each with mass $m = 1$ gram and with a kinetic energy of 1 Joule per bead (so the power required to operate it is still 1000 Watts), what is the average force (per KW) exerted by the mass-driver?

## 9.2

We'll decide whether or not the sun is a viable source of energy in this problem. Draw your figures above.

a) The sun produces $4 \times 10^{26}$ Watts of power and is $1.5 \times 10^{11}$ meters away from the earth. Estimate the energy that, on a clear day, strikes a solar panel 1 meter square in one hour when sunlight is incident on it at an angle of 45°. Use any reasonable assumptions you like about absorption or reflection at the panel surface and by the atmosphere.

b) If one could store and recover only 1% of that energy in later use, how long could you run a 100 Watt light bulb for in the evening from an hour of sunlight during the day?

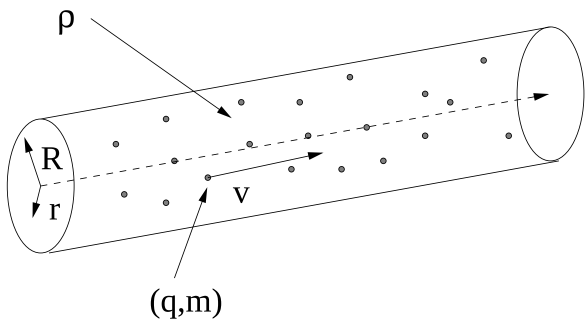



### 9.3

**Beam Dynamics:** Each part of this problem (a and b) will be graded separately. You do not need to get the first part right to do the second part, but obviously you need to get both parts right to get the extra credit.

A cylindrical beam of particles each with charge $q$ and mass $m$ has a uniform initial (charge) density $\rho$ and radius $R$. Each particle in the beam is initially travelling with velocity $v$ parallel to the beam's axis. We will discuss the stability of this beam by examining the forces on a particle travelling in the beam at a distance $r < R$ from the axis (the center of the cylinder).

a) Find the force on a particle at radius $r$ caused by the other particles in the beam. You will need to use Gauss's law to calculate the electric field at radius $r$. Describe your work, and do not skip steps; show that you understand Gauss's law. Make a sketch as needed. .

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b) Find the magnetic force on a particle at radius $r$ caused by the other particles in the beam. Use Ampere's law to calculate the magnetic field. Describe your work, and do not skip steps; show that you understand Ampere's law. Make a sketch.

c) (5 points extra credit) At what beam velocity do the forces in a) and b) exactly balance? Given the unbalanced electric force *in the rest frame of the particles* from a), offer a hypothesis that can explain both measurements.

# 10 AC Circuits

## 10.1

Draw a *series* LRC circuit above with an alternating voltage $V_0 \cos(\omega t)$, clearly labeling the (presumably given) $L$, $R$, and $C$.

a) Draw the phasor diagram that represents Kirchhoff's rule for the voltages around the loop.

b) Draw the phasor diagram for the impedance $Z$ and write down its value in terms of the givens. Also indicate the value of the phase angle $\delta$ in terms of the givens.

c) What is the resonant frequency $\omega_0$ for the circuit in terms of the givens?

d) Draw a semi-quantitatively correct graph of the average power $P(\omega)$ delivered to the circuit for $Q = 10$, clearly indicating the location of $\omega_0$. At the very least, the graph scales should be arguably consistent with the value of $Q$.

## 10.2

Draw a *parallel* LRC circuit above with an alternating voltage $V_0 \cos(\omega t)$, clearly labeling the (presumably given) $L$, $R$, and $C$.

a) Draw the phasor diagram that represents Kirchhoff's rule for the currents around the loop. What is the form of the total current as a function of time?

b) Draw the phasor diagram from which the impedance $Z$ can be determined and write down its value in terms of the givens. Also indicate the value of the phase angle $\delta$ in terms of the givens.

c) What is the resonant frequency $\omega_0$ for the circuit in terms of the givens?

d) Does the average power delivered to the circuit depend on $\omega$? Why or why not?

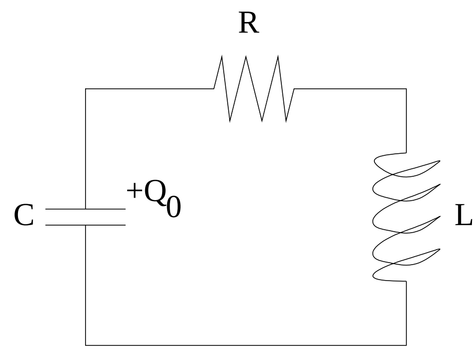
### 10.3

(20 pts)

You are building an FM radio ($f \approx 100$ MHz) and have a power supply and circuitry that generates annoying harmonics in the low frequencies (especially 60 Hz, but also AM stations around 1 MHz contribute) that contaminate your high frequency output and causes your signal to "buzz'. Naturally, you have a parts box that contains resistors and capacitors. The range of resistors available runs from 1 Ohm through 100,000 Ohms (to one significant digit – don't bother with resistances like 3.845 Ohms as their rated value is generally accurate only to 10% or so anyway – call it 4 Ohms instead), and you have capacitors that range from 1 microfarad to 1 picofarad, but only in multiples of ten (e.g. $10^{-6}$ farads, $10^{-7}$ farads, ..., $10^{-12}$ farads).

Design a "high pass" filter built from one resistor and one capacitor (where you get to choose suitable values for $R$ and $C$ as well as their arrangement) that will output more than half the input voltage for all frequencies greater than 10 MHz but strongly attenuates the output voltage for frequencies more than a bit less than this, and derive the expression (for your circuit) for $V_{\text{out}}/V_{\text{in}}$ as a function of $R$, $C$ and $\omega$. Draw the circuit in the space above, of course, clearly indicating where $V_{\text{in}}$ and $V_{\text{out}}$ go.

(Hints and Notes: This *was* a homework problem, so you *should* know what a high pass filter is. If you don't remember exactly, consider a series combination of $R$ and $C$ and think about what happens to the voltage drops across each one as a function of $\omega$. Your "output voltage" will come from a parallel connection across one or the other.)




## 10.4

At time $t = 0$ the capacitor in the $LRC$ circuit above has a charge $Q_0$ and the current in the wire is $I_0 = 0$ (there is no current in the wire). Find $Q(t)$, and draw a qualitatively correct picture of $Q(t)$ in the case that the oscillation is only weakly damped. Show all your work. Remember that $Q(t)$ is real.

## 10.5

Draw a series LRC circuit connected across an alternating voltage source. Suppose $R = 40\Omega$, $C = 0.2\mu$f, $L = 0.80m$H, and the frequency of the applied voltage is $\omega = 10^5$ radians/second. What is:

a) The impedance $Z$ of the circuit?

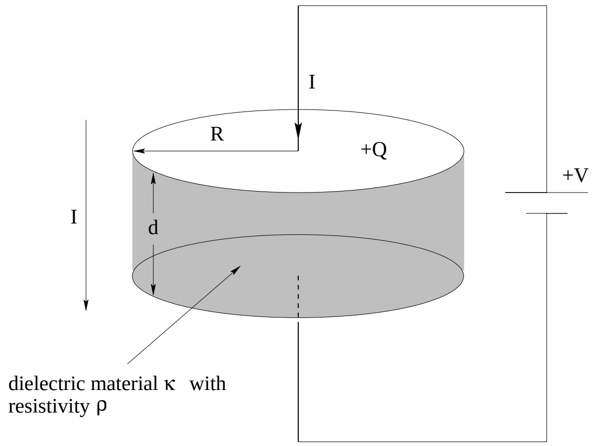
b) The resonant frequency of the circuit, $\omega_0$?

## 10.6

Draw a parallel LRC circuit connected across an alternating voltage source. Suppose you are given $R$, $C$, $L$, and an applied voltage $V_0 \sin(\omega t)$. What is:

a) The impedance $Z$ of the circuit?

b) The power $P(t)$ dissipated by the resistor?




### 10.7

This is basically problem 64 from your homework. Our archetypical model for a resistor is drawn above: two circular conducting plates (metal contacts) with radius $R$, separated at a distance $d$ by a material with resistivity $\rho$.

a) In a steady state situation where a DC voltage $V$ is applied as shown, find the field $\vec{E}$ inside the resistive material.

b) Find the current density $\vec{J}$ inside the resistive material.

c) From Ampere's law, find the magnetic field as a function of $r$ in the region between the plates.

d) From your answers to a) and c), find the Poynting vector $\vec{S}$ (magnitude and direction) as a function of $r$ in the region in between the plates.

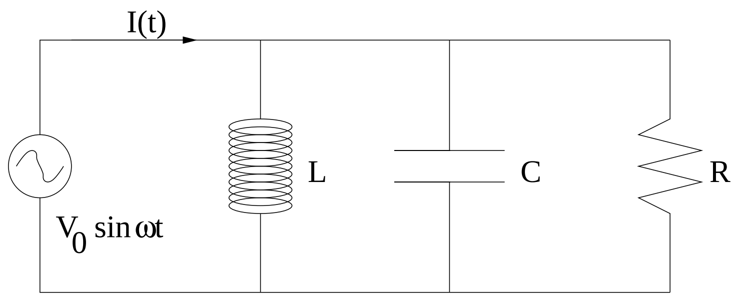
e) NOW show that:

$$\oint_A \vec{S} \cdot \hat{n} dA = -I^2 R$$

where $A$ is the outer surface of the resistor and $\hat{n}$ is its outward-directed normal unit vector.

Thus the heat that appears in the resistor *can* be thought of as the electromagnetic field energy that flows in through its outer surface!

Solution Page

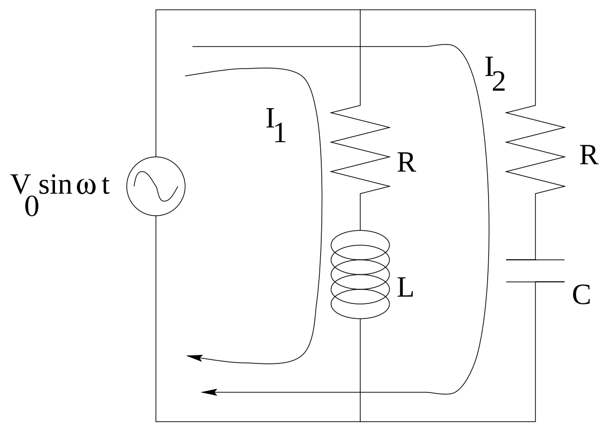



**10.8**

A *parallel* LRC circuit connected across a variable AC voltage source $V = V_0 \sin(\omega t)$ is drawn above. Find (in terms of $L, R, C, V_0, \omega$ and any quantities you define in terms of these such as $\chi_L$ or $\chi_c$):

a) The current $I_{L,R,C}(t)$ in each element of the circuit. Don't forget the phase shifts (if any), and note that we are getting *current* from *voltage* – be careful!

b) Now find the current $I(t)$ in the primary supply wire (as shown in the figure above) with all terms, e.g. any required phase $\delta$, and the impedance $Z$ defined and (at the end) numerically evaluated. You will probably need to draw the appropriate phasor diagram to help you figure this out unless you managed to memorize the entire $LRC$ circuit chapter results.




**10.9**

a) Draw (two) qualitatively correct phasor diagrams that show the voltage drops and gains for each of the two loops shown. Be sure to correctly indicate the phases of the currents $I_1$ and $I_2$ relative to the phase of the applied voltage and the voltage drop across each element.

b) Write the Kirchoff's Law (Voltage) for each of the two loops shown that corresponds to your phasor diagram.

c) From a) and b), find the impedance of each loop $Z_1$ and $Z_2$, the current phase of each loop $\delta_1$ and $\delta_2$, and write down an expression for $I_1(t)$ and $I_2(t)$. Try to work neatly enough that I can grade this.

d) For extra credit, use Kirchoff's Law (current) to find the total impedance of the circuit, the total current provided by the voltage, and the total power provided by the voltage.

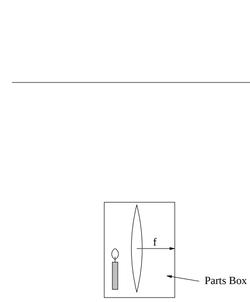
# 11 Geometric Optics and Polarization

## 11.1

Indicate, with pictures and/or a short descriptions, how light is polarized by absorption, by reflection, and by scattering. Derive and explain the formula for the Brewster angle (telling us what the Brewster angle is).

### 11.2

Derive Snell's Law. You may use either the wave picture (that I gave in class) or the Fermat principle (which was on your homework). For a bit of extra credit, do it both ways. Be sure to give the definition of index of refraction.



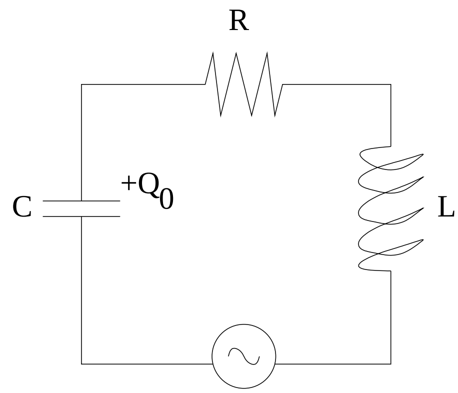

### 11.3

You have a candle and a lens with a focal length of 15 cm. You wish to cast a real image of the candle upon a screen. You want the image size (magnitude) to be exactly two times the size of the actual candle.

a) Find $s$ and $s'$ such that this kind of image can be formed.

b) Carefully place the components on the figure above and draw a ray diagram to locate the image, to scale, in agreement with your answers to a. Be sure to include the 3 rays that uniquely specify the image location.

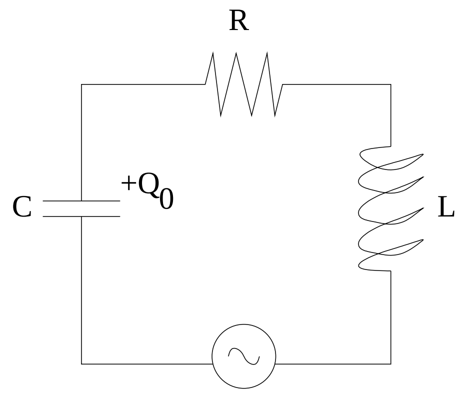
Don't burn yourself on the candle.




### 11.4

The $LRC$ circuit above is connected to an alternating voltage $V_0 \sin(\omega t)$ and the circuit run until it is in a steady state. Find or show:

1. The impedance $Z$ of the circuit.
2. The current $I(t)$ in the circuit, in terms of the givens. Be sure to define all terms used e.g. $\phi$.
3. Draw a *qualitatively* correct figure of $P_{\rm av}(\omega)$ (the average power as a function of $\omega$) for the oscillator if it has $Q = 10$ and a resonant frequency $\omega_0$.




### 11.5

The $LRC$ circuit above is connected to an alternating voltage $V_0 \sin(\omega t)$ and the circuit run until it is in a steady state.

1. Write Kirchoff's voltage rule for this circuit loop.
2. Draw the phasor diagram for the voltage, noting that the current must be in phase with the voltage across the resistor.
3. From this phasor diagram and the relations between maximum current, reactance or resistance of the circuit elements, and the maximum voltage drop across them, deduce and draw the phasor diagram for the impedance $Z$ of the circuit.
4. Draw a *qualitatively* correct figure of $P_{\rm av}(\omega)$ (the average power as a function of $\omega$) for the oscillator if it has $Q = 10$ and a resonant frequency $\omega_0$.

### 11.6

It is sunset on a clear day. You are wearing your trusty polaroid sunglasses. You look straight overhead and the sky is somewhat dark. You slowly turn your body to the right (continuing to look up) and the sky gradually lightens to become maximally bright. At this moment your body is facing(circle correct answer):

1. North
2. East
3. South
4. West
5. East or West
6. North or South
7. Cannot tell from information given

Draw a set of diagrams and write a paragraph or two showing ALL OF THE PHYSICS that explains your answer – include descriptions BOTH of the polaroid sunglasses themselves and the light scattering off of molecules in the atmosphere overhead.

### 11.7

Light propagates down a light fiber by reflecting off the walls.

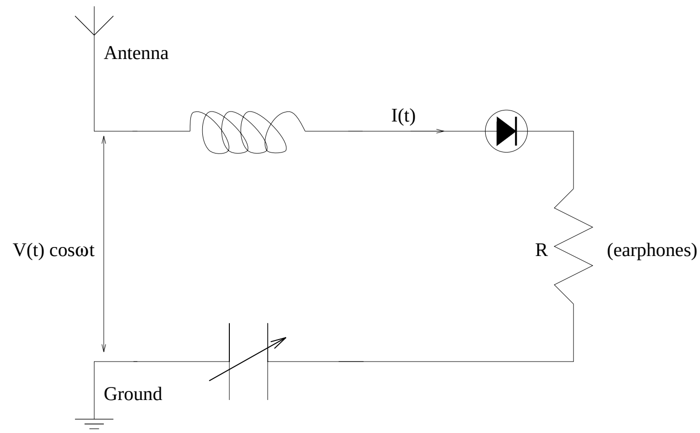
a) Assuming that the fiber has an index of refraction of $n = 1.4$, what is the critical angle of incidence such that light will remain trapped in the fiber?

b) Indicate how you *think* light might be polarized in the fiber after propagating a few meters (and bouncing several times off the walls). Show the polarization direction(s) in cross section. Indicate WHY you think the light would be polarized that way.

**11.8**

a) Unpolarized light is incident on the surface of diamond (n = 2.4). Some of the light is reflected from the diamond; the rest penetrates the diamond surface and is refracted. Find the angle at which the reflected light is *completely* polarized and indicate the direction of polarization on a suitable figure.

b) Diamond is interesting for another reason. It "traps light" and reflects it internally many times as it bounces from facet to facet. Explain how a diamond (with n = 2.4) traps more light more than a piece of glass (n = 1.5). Your answer should be at least partly quantitative.

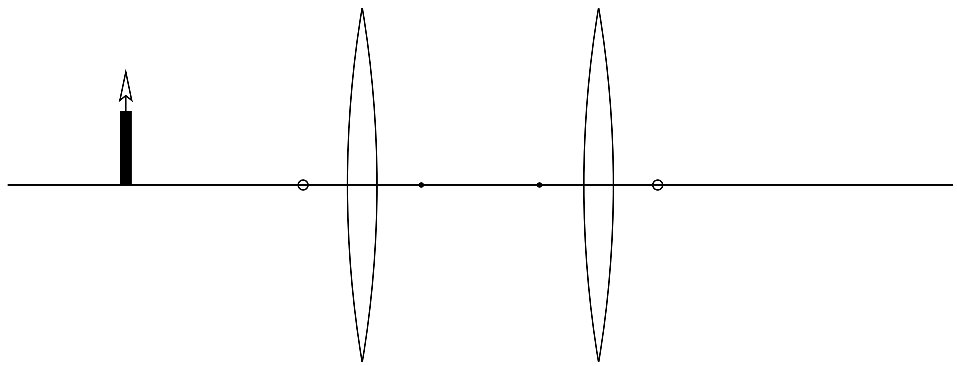



## 11.9

One kind of simple crystal radio consists of the *series* LRC circuit drawn above. The antenna-to-ground connection represents an amplitude-modulated AC voltage source $V = V(t)\cos(\omega t)$ where $\omega = 2\pi \times 10^6$ radians/sec. The diode (in series with the earphone of resistance $R$) is a circuit element that only lets current flow in the direction of the arrow – a one-way gate. The capacitor can be varied to tune the radio.

a) How should the capacitor be set (with respect to the value of $L$) to tune the radio to deliver the maximum current through the earphones (resistance $R$) and what is that maximum current? Assume that the diode has negligible resistance and capacitance and that the amplitude-modulation is slow relative to $\omega^{-1}$.

b) Describe qualitatively, with a suitable picture or figure, how the diode allows the amplitude-modulated signal ($V(t)$) to be extracted from the carrier frequency $\omega$.



# 12 More Optics (mostly)

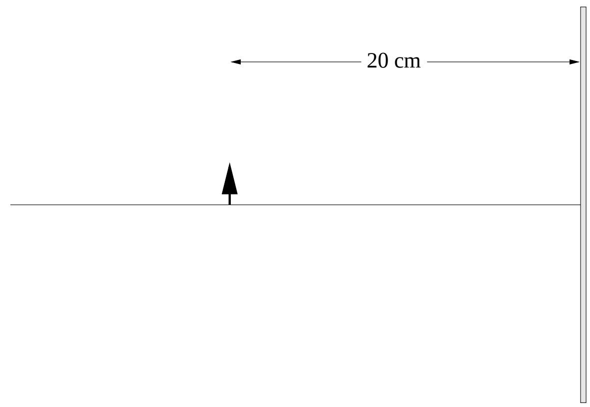
## 12.1

A candle 20 cm high is placed 40 cm in front of the center of a thin lens. This lens has a focal length of 10 cm. A second thin lens, also with a focal length of 10 cm, is placed 40 cm from the first. Find:

a) The location $s'$ of the image due to the **first** lens and its magnification. Indicate whether the image is real or virtual.

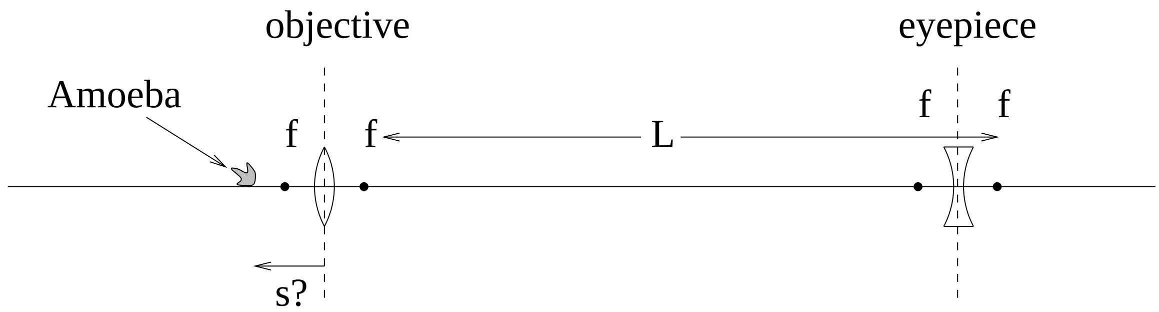
b) The location $s''$ of the image (of the image of the first lens) of the **second** lens. Find the overall magnification, and indicate if the final image is real or virtual.

c) Draw a ray diagram to locate the image in agreement with your answers to a and b. Be sure to include and label 3 rays that uniquely specify the image locations.




### 12.2

There is an object 20 cm away from a screen. Using a concave mirror, I would like to throw an image of this object upon the screen that is three times larger than the object itself. Find the location of the mirror (with respect to object and screen and the focal length of the mirror necessary to accomplish this. Draw the corresponding ray diagram.




**12.3**

The arrangement of lenses that makes up a “Gallilean” compound microscope is pictured above. The focal lengths of the objective and eyepiece lenses are $f_o = 1$ cm and $f_e = -1$ cm. The tube length is $L = 20$ cm.

a) Find $s$ (the object distance from the objective lens) such that the final image viewed by the eye is in focus (at infinity, as imaged by the eyepiece).

b) Draw the ray diagram from which you can find the overall magnification. NOTE WELL the tube length goes to the second (negative) focal point of the eyepiece. Why?

c) From this diagram, find the overall magnification. Explain each part (that is, what are the separate roles of the objective and eyepiece).

d) What is the advantage of this kind of microscope compared to one with two converging lenses?

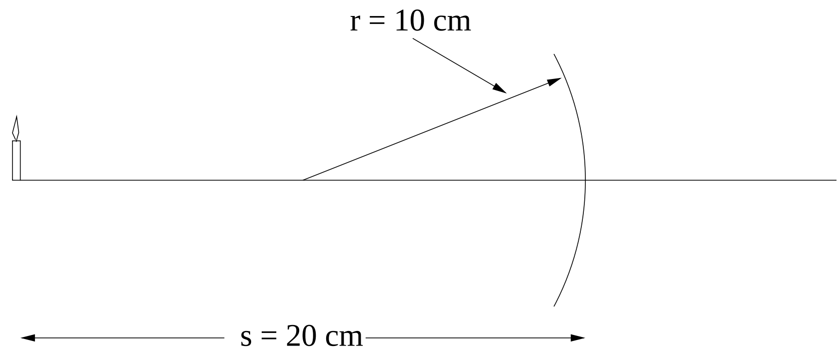
### 12.4

Draw below a Gallilean telescope (one built with a converging primary lens and a *diverging* eyepiece lens). Draw it to scale so that the overall angular magnification is $M = 10$. Derive its magnification in terms of $f_p$, $f_e$, and any other parameters you think necessary. Remember, $f_e$ is *negative* for a Gallilean telescope. Draw on the diagram the rays used to derive the magnfication (which are tricky for a diverging eyepiece, so be careful).

## 12.5

a) Design a microscope with a tube length $\ell = 10$ cm and a magnification of 500. Draw it below to scale. Derive its magnification in terms of $f_o$ (the objective lens) , $f_e$, $\ell$, and any other parameters you think necessary. You may pick $f_o$ and $f_e$ to have any "sensible" values, and can make the microscope invert the image or not as you wish.

b) Determine where (that is, the actual position in cm) one has to place the object in front of the objective lens in order for the relaxed, normal eye to view its image at infinity through the eyepiece. Note that this answer will depend (obviously) on $f_o$ and other parameters, so the number answer is less important than the algebra (which is what will be checked).




### 12.6

The mirror above has a radius of curvature $r = 10$ cm. A candle is placed at $s = 20$ cm as shown. Find:

- The focal length of the mirror (draw the focal point in on the diagram above).
- The location $s'$ of the image in centimeters.
- The magnification of the image.
- State whether the image is real or virtual, erect or inverted.
- Draw the ray diagram for this arrangement using the three "named" rays used for both lenses and mirrors as shown in class. Obviously it should validate your answers to the above.

## 12.7

A small object is placed 15 cm from a diverging mirror with focal length -5 cm. Determine:

1. The image distance $s'$.
2. The magnification $m$.
3. The kind of image (erect/inverted, real/virtual).

Draw a **ray diagram** for the arrangement using (and labelling!) the three standard rays covered in class to locate the image. It should at least approximately correspond to your numerical results above.

## 12.8

A physics professor hands you a box that contains the following material: lens A with $f_A = 100$ cm, lens B with $f_B = 200$ cm, lens C with $f_C = -2$ mm, lens D with $f_D = 5$ mm and lens E with $f_E = -5$ mm. There are also 4 meter sections of PVC pipe that fit each lens and that can be cut with a handy hacksaw, sleeves that nest the PVC pipe sections together, some glue, focus gears (that can be used to move the eyepiece lens small distances along its axis), and things like that.

a) Create a rough design in the space above for a refracting telescope with an angular magnification $M = -200$, made using this material and equipment. Clearly indicate the lenses you use and their arrangement in the tube(s).

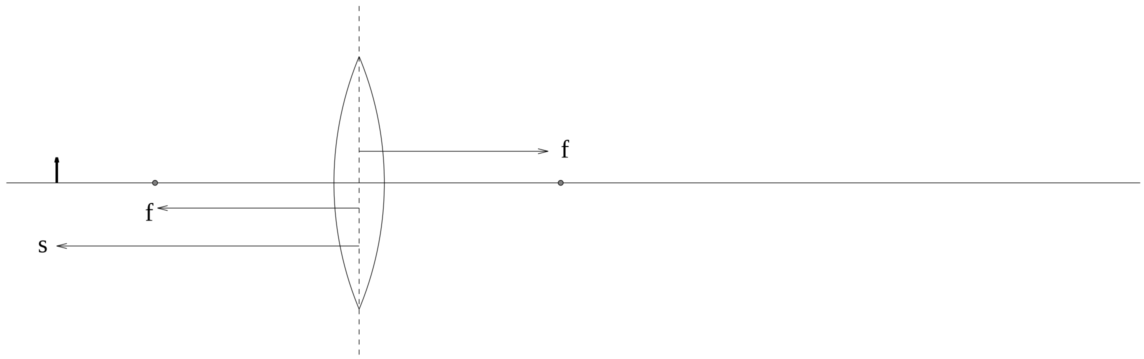
b) Draw below a simple ray diagram from which the angular magnification of a general refracting telescope of the sort you design can be evaluated. It need not be precisely to scale.

Note that this telescope will only be used to look relatively *distant* objects.

**12.9**

A physics professor hands you a box that contains the following material: (mounted) lens A with $f_A = 10$ cm, lens B with $f_B = 1$ cm, lens C with $f_C = 5$ mm and lens D with $f_D = -2$ mm. There is also a piece of tubing 15 cm long that fits the lens mounts exactly and can be cut to any length you like with the enclosed hacksaw, a focus gear (that can be used to move the objective lens mount small distances along its axis in the tube), glue, screws, a slide/tube mounting bracket, and things like that.

Create a rough design in the space above for a simple microscope with a magnification of $M = -500$ using this material and equipment. Clearly indicate the lenses you use and their arrangement in the tube.

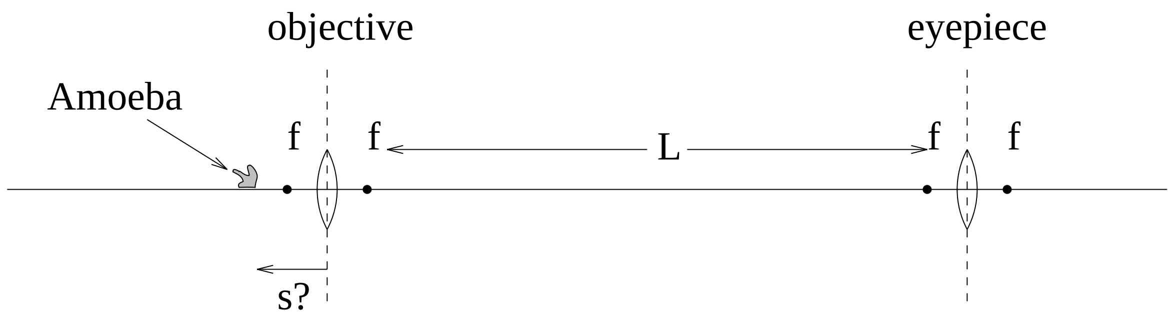



## 12.10

A candle 10 cm high is placed 75 cm in front of the center of a thin lens. The lens has a focal length of 50 cm.

a) Find the location $s'$ of the image, its magnification, and indicate whether the image is real or virtual.

b) Draw a ray diagram to locate the image in agreement with your answers to a. Be sure to include the 3 rays that uniquely specify the image location.



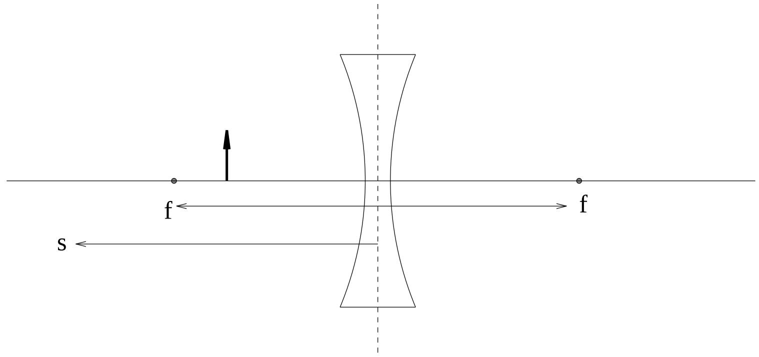

**12.11**

The arrangement of lenses that makes up a compound microscope is pictured above. The focal lengths of the objective and eyepiece lenses are $f_o = f_e = f = 1$ cm. The tube length is $L = 19$ cm.

a) Find $s$ (the object distance from the objective lens) such that the final image viewed by the eye is in focus (at infinity, as imaged by the eyepiece).

b) Draw the ray diagram from which you can find the overall magnification (try to use a straight edge to do this).

c) From this diagram and your knowledge of the separate purposes of the two lenses, find the overall magnification. Explain each part (that is, what are the separate roles of the objective and eyepiece).




### 12.12

A candle 20 cm high is placed 60 cm in front of the center of a thin lens. The lens has a focal length of -80 cm.

a) Find the location $s'$ of the image, its magnification, and indicate whether the image is real or virtual.

b) Draw a ray diagram to locate the image in agreement with your answers to a. Be sure to include 3 rays that uniquely specify the image location.


# 13 Physical/Wave Optics

## 13.1

Two vertical slits of width 1500 nanometers (nm) are separated (center to center) by a distance of 2500 nm and illuminated by light of wavelength 500 nm. The light which passes through is then projected on a distant screen. Find:

a) The location (angles $\theta$) of all **diffraction minima**.

b) The location of all **interference minima**.

c) The location of all **interference maxima**.

d) Finally, draw a properly proportional figure of the resulting interference pattern between 0 and $\pi/2$ (on either side), indicating the maximum intensity in terms of the central maximum intensity that would result from a single slit.


### 13.2

All angles in the parts a-c may be expressed by means of tables of inverse trigonometric functions of simple fractions, e.g. $\cos^{-1}(1/2)$, $sin^{-1}(2/7)$, etc.

Two vertical slits of width $a = 1200$ nanometers (nm) are separated (center to center) by a distance of $d = 3000$ nm and illuminated by light of wavelength $\lambda = 600$ nm. The light which passes through is then projected on a distant screen. Find:

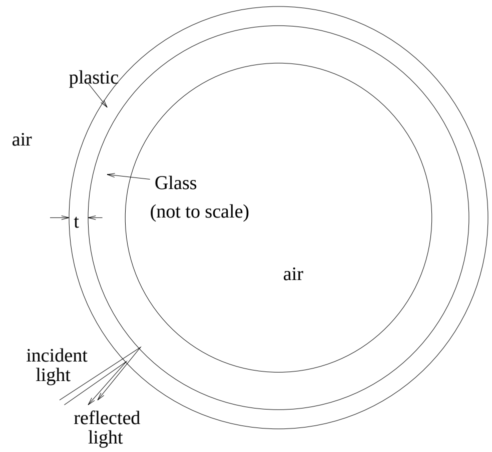
a) The location (angles $\theta$) of all **diffraction minima**.

b) The location of all **interference minima**.

c) The location of all **interference maxima**.

d) Finally, draw a properly proportional figure of the resulting interference pattern between 0 and $\pi/2$ (on either side), indicating the maximum intensity in terms of the central maximum intensity that would result from a single slit.

e) For five points of extra credit, write down the algebraic expression for $I(\theta)$ in terms of $I_0$ (the central intensity of a single slit), defining all variables used (like $\phi$ and $\delta$) in terms of $a$, $d$, $\lambda$ and $\theta$.



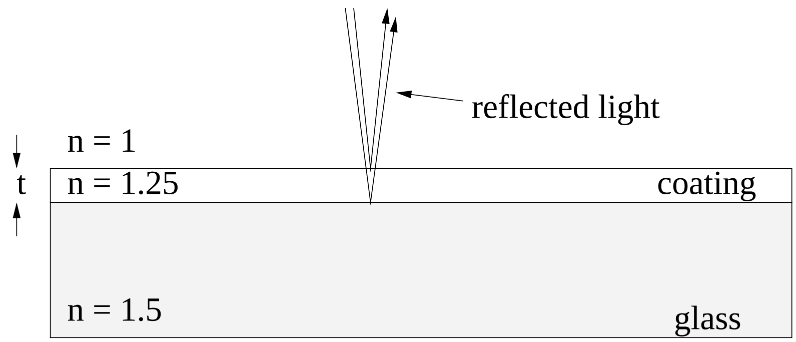

**13.3**

A Christmas tree ornament is constructed by vapor-depositing a thin, transparent film (with $n = 1.25$) on a "thick" ($\sim$ 2 mm) spherical glass ($n = 1.5$) bubble as drawn schematically above. The thin plastic film is not quite uniform in thickness, and this variation produces brilliant streaks of color in the reflected light.

a) What does the light reflected from the ornament look like where $t \sim 0$ (or $t \ll \lambda$, at any rate). Explain the physics behind your answer with a single sentence and/or diagram.

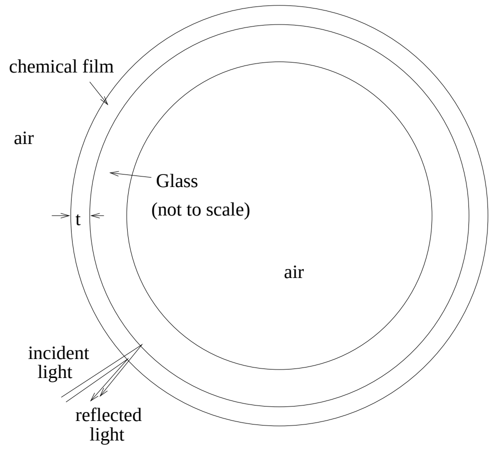
b) At what thickness $t \sim \lambda > 0$ of the film will the reflected light first have a constructive interference *maximum* at $\lambda = 550$ nm (where $\lambda$, recall, is the wavelength in free space where $n = 1$)?

c) At that thickness, will any other visible wavelengths have an interference maximum or minimum? Justify your answer – just 'yes' or 'no' (even if correct) are incorrect.




**13.4**

You would like to eliminate the reflected light from a flat glass pane for perpendicularly incident light of wavelength 550 nm. The index of refraction of the glass is $n_g = 1.5$, and the index of refraction of the coating material to be used is $n_c = 1.25$. What minimum thickness $t$ of the coating material will have the desired effect? (Try to show your reasoning, and don't forget "details".)




**13.5**

A Christmas tree ornament is constructed by vapor-depositing a chemical film (with n = 1.7) on a "thick" ($\sim$ 2 mm) spherical glass (n = 1.5) bubble as drawn schematically above. The thin chemical film is not uniform in thickness, and its variation in the range 0-2 microns (micrometers) produces brilliant streaks of color in the reflected light.

a) What is the smallest (nontrivial) mean thickness $t$ of the film such that reflected light to has a constructive interference *maximum* in the center of the visible spectrum ($\lambda$ = 400-700 nm in free space where $n = 1$).

b) When the film first starts to deposit on the glass (and has a thickness $t$ of only a few nanometers) does the film on the bulb turn shiny (constructively reflecting all wavelengths) or transparent (destructively reflecting all wavelengths)? Explain.


### 13.6

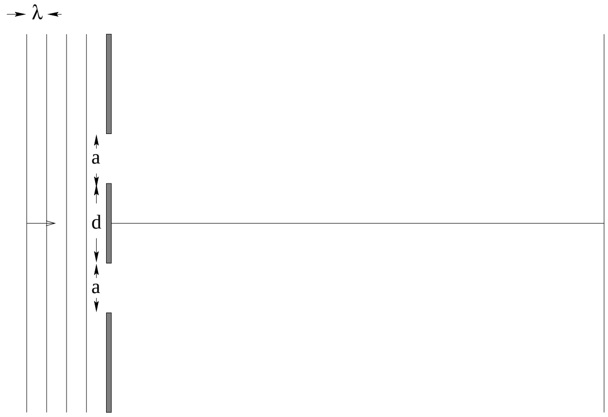
Light with wavelength $\lambda = 700$ nm passes through two slits of a width $a = 1400$ nm. The centerpoints of these two slits are separated by a distance of $d = 3500$ nm. The light then travels a long distance and falls on a screen. It is *not* necessary (for once) to derive or justify the equation(s) you use below, but if you do you will get partial credit even if your numerical answers are wrong.

a) Write down (or derive) the algebraic formula for the intensity of the combined interference-diffraction pattern for this arrangement.

b) Write down (or derive) the formula from which the angles at which diffraction minima occur can be found, and apply it to find *all* these angles (put them in a table).

c) Write down (or derive) the formulas from which the angles at which interference maxima and minima occur, and apply them to find **the first three of each** (only) (put them in a table).

d) Draw a qualitatively correct picture of the expected diffraction/integration pattern $I(\theta)$.



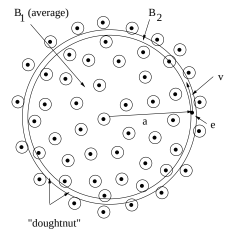

### 13.7

Light with wavelength $\lambda = 300$ nm passes through two slits of a width $a = 900$ nm. The centerpoints of these two slits are separated by a distance of $d = 2700$ nm. The light then travels a long distance and falls on a screen. It is *not* necessary (for once) to derive or justify the equation(s) you use below, but if you do you will get partial credit even if your answers are wrong.

a) Write down (or derive) the formula from which the angles at which diffraction minima occur can be found, and apply it to find *all* these angles (put them in a table).

b) Write down (or derive) the formulas from which the angles at which interference maxima and minima occur, and apply them to find the first three of each (put them in a table).

c) Draw a qualitatively correct picture of the expected diffraction/integration pattern $I(\theta)$.




## 13.8

A Betatron is pictured above (with field out of the page). It works by increasing a non-uniform magnetic field $\vec{B}(r)$ in such a way that electrons of charge $e$ and mass $m$ inside the "doughnut" tube are accelerated by the $E$-field produced by induction (via Faraday's law) from the "average" time-dependent magnetic field $B_1(t)$ inside $a$, while the magnitude of the magnetic field at the radius $a$, $B_2(t) = |\vec{B}(a,t)|$, bends those same electrons around in the circle of (constant) radius $a$.

This problem solves, in simple steps, for the "betatron condition" which relates $B_1(t)$ to $B_2(t)$ such that both things can simultaneously be true.

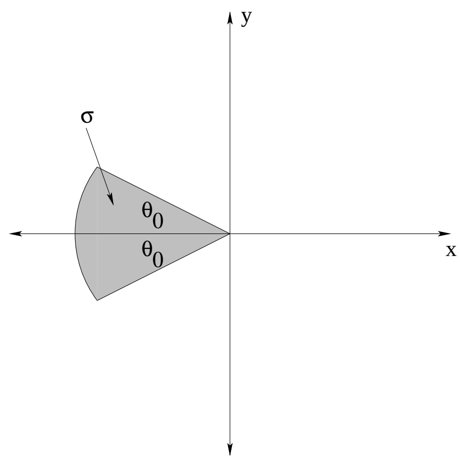
a) The electrons go around in circles of radius $a$ and are accelerated by an $\vec{E}$ field produced by Faraday's law. We will define the (magnitude of the) average field $B_1$ by $\phi_m = B_1(\pi a^2) = \int_{(r<a)} \vec{B}(r) \cdot \hat{n} dA$. What is the induced $E$ field (tangent to the circle) in terms of $B_1$ and $a$?

**(Problem continued on next page!)**

b) The electrons (at their instantaneous speed $v$ tangent to the circle) are bent into the circle of radius $a$ by the field $B_2$. Relate $B_2$ to the magnitude of the momentum $p = mv$, the charge $e$ of the electron, and the radius $a$.

c) The force $\vec{F}$ from the $E$-field acting on the electron with charge $e$ in the direction of its motion is equal to the time rate of change of the magnitude of its momentum $p$ (if Newton did not live in vain). Substitute, cancel stuff, and solve for $\frac{dB_1}{dt}$ in terms of $\frac{dB_2}{dt}$. If you did things right, the units will make sense and the relationship will only involve dimensionless numbers, not $e$ or $m$.

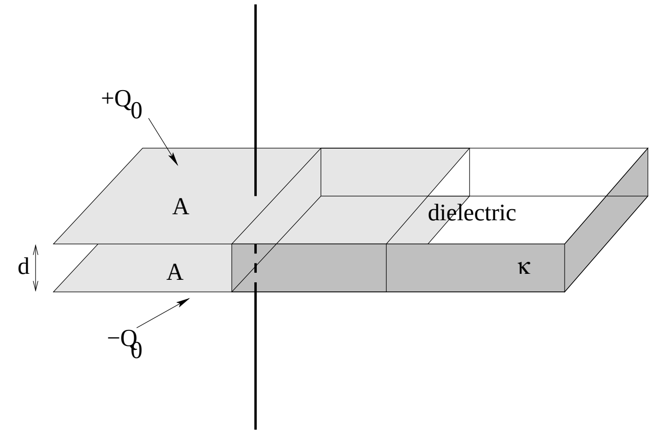
Cool! You've just figured out how to build one of the world's cheapest electron accelerators! Or perhaps not....




## 14 Misc

### 14.1

A pie-shaped wedge of uniform charge per unit area $\sigma$ is arranged as shown. Find the electric field at the origin (magnitude and direction).



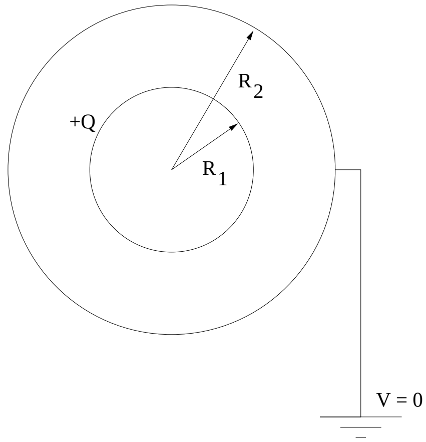

**14.2**

A parallel plate capacitor is constructed from two square conducting plates of with an area of $A$, separated by a distance of $d$. An insulating slab of thickness $d$ and a dielectric constant $\kappa$ is inserted so that it half-fills the space between the plates as shown. Find:

1. The capacitance of this arrangement;

2. The electrostatic force on the dielectric slab when a the capacitor carries a total charge (fixed) $Q_0$ on the top plate and $-Q_0$ on the bottom plate.

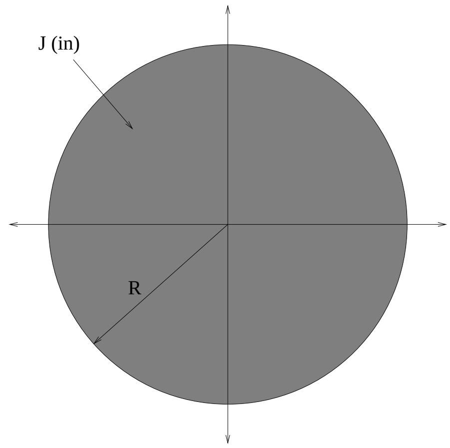
3. For extra credit, would the direction of the force be the same if a constant voltage $\Delta V$ were applied across the plates? Indicate *why* if you try to answer this one.




## 14.3

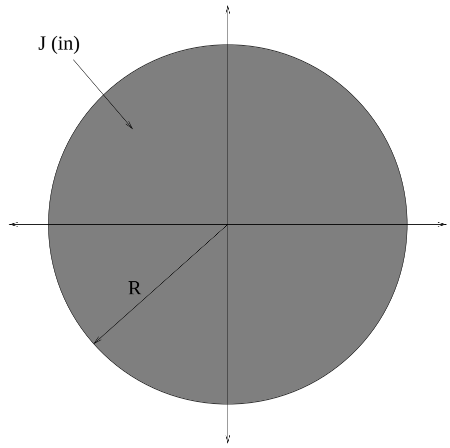
Two concentric spherical conducting shells of radii $R_1$ and $R_2$ are arranged as shown. The inner shell is given a total charge $+Q$. The outer shell is grounded (connected to a conductor at zero potential) as shown.

Find the potential and field at all points in space. Show all work – don't just write down answers even if you can "see" what the answers must be.



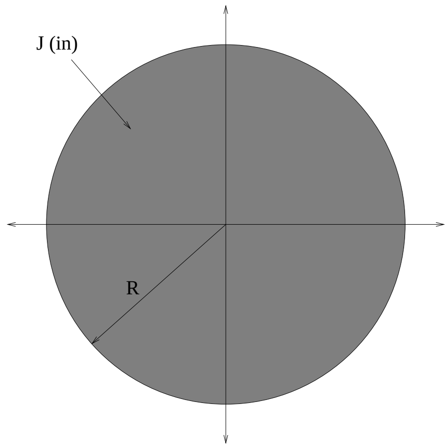

## 14.4

A cylindrical long straight wire of radius $R$ has a cylindrical long straight hold of radius $b = R/2$ and carries a current density of $\vec{\mathbf{J}}$ into the page as drawn. Find the magnitude of the magnetic field for arbitrary $r > R$ (region I) and $r < R$ (region II). Indicate the direction of the magnetic field in these regions on the diagram above.

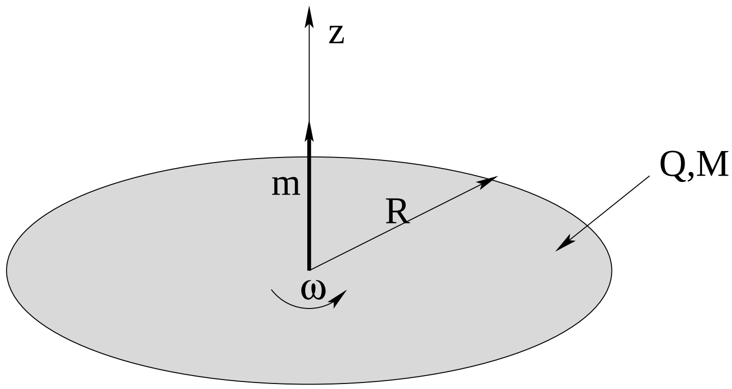



### 14.5

A cylindrical long straight wire of radius $R$ carries a current density of $\vec{\mathbf{J}}$ into the page as drawn. Find the magnetic field (magnitude and direction) at arbitrary points inside and outside the wire. Show all work and clearly label the law or rule used to find the answer.



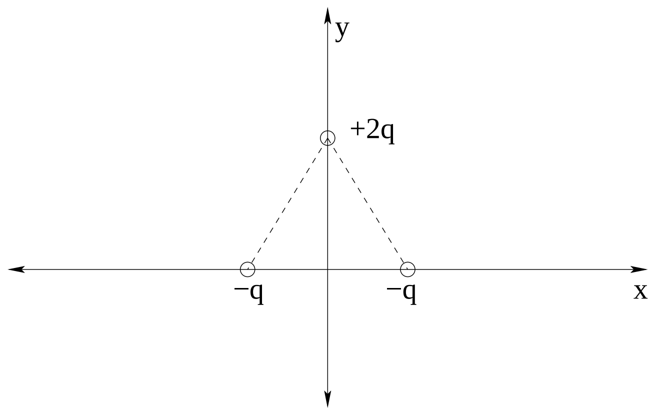

## 14.6




### 14.7

A flat disk of radius $R$ and mass $M$ with uniform surface charge density $\sigma_q = \frac{Q}{\pi R^2}$ is rotating at angular velocity $\omega$ about the $z$-axis as shown.

Find its magnetic field at an arbitrary point on the $z$-axis.




### 14.8

Charges of $-q$ are located at both $y = a$ and $y = -a$, and a charge of $+2q$ is located at $y = 0$ on the y-axis. This arrangement of charge can be visualized as two opposing dipoles.

a) Find the electric field (magnitude and direction) at an arbitrary point on the x-axis.

b) What is *nonzero* term in the expansion of the electric field evaluated *far* from the charges, i.e. – for $x >> a$? Your answer should be a series of terms in inverse powers of $x$.

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