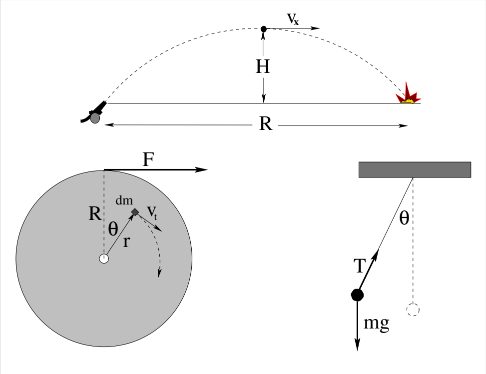
# Introductory Physics 141/151/161

## Self-Guided Learning Problems

Robert G. Brown
Summer, 2020



# Contents

# About These Problems

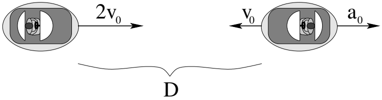
This is an experiment. Beamer allows me to make slides that will successively reveal lines of math-heavy text. This gives me a unique opportunity to build a collection of self-guided learning problems for physics that do what I've fantasized about doing for years now – present a problem, then provide a hint, then another hint, then another (or reveal a step, and then another step) until finally, the entire souution is presented, annotated.

Hopefully these problems will help students everywhere as they struggle to learn physics problems solving techniques and learn to "think like a physicist" as they do so.

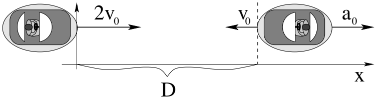
To use this resource, pick a problem or topic from the table of contents and go directly to it, or work your way through all the problems systematically. Work on a separate sheet of paper, and when you get stuck, page down through the frames to see (hopefully) where you went wrong.

Remember, the point is to **master these problems**, not just to get through them. Make sure that before you are done, you can do every problem **without looking, without hints, and without remembering the exact solution** but rather, understanding *how* to find it!

# Kinematics: Two Bumper Cars




- Two bumper cars are headed straight at one another, one travelling at $2v_0$ to the right, the other at speed $v_0$ to the left. When they are separated by a distance $D$, the car on the right slows down with a constant acceleration $a_0$. Does the right hand car manage to stop before being hit by the left hand car?

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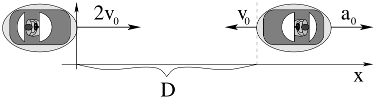


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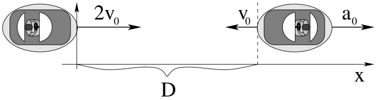


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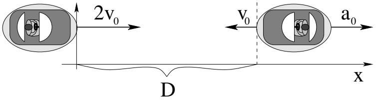


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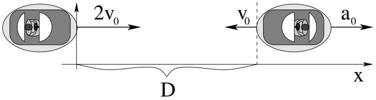


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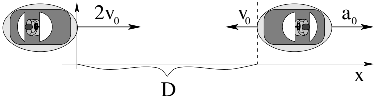


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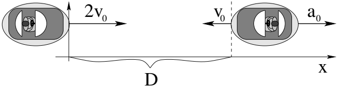


Solution: $x_l(t) = 2v_0 t, \quad v_l(t) = 2v_0$

$$x_r(t) = D - v_0 t + \frac{1}{2}a_0 t^2, \quad v_r(t) = -v_0 + a_0 t$$

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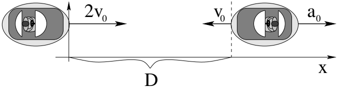


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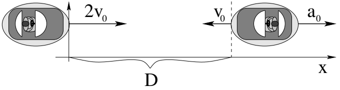


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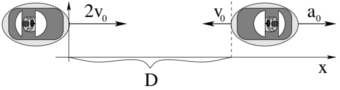


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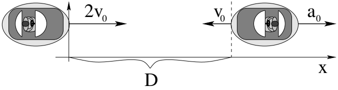
$$D < \frac{5v_0^2}{2a_0}$$

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It makes sense – larger $D$ makes it *less* likely to collide, larger $a_0$ makes it *less* likely they will collide, larger $v_0$ makes it *more* likely.

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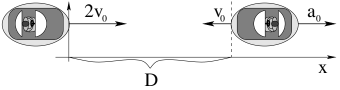


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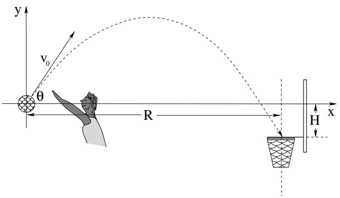
It makes sense – larger $D$ makes it *less* likely to collide, larger $a_0$ makes it *less* likely they will collide, larger $v_0$ makes it *more* likely. Knowing they collide, if we write:

$$x_l(t_c) = 2v_0 t_c = D - v_0 t_c + \frac{1}{2}a_0 t_c^2 = x_r(t_c)$$

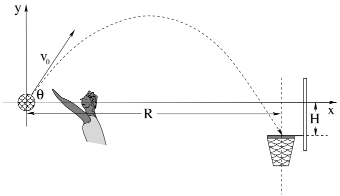
would let us find the time of collision and answer other questions about e.g. their relative velocity at that time. This is a simple quadratic equation for $t_c$.

# Kinematics: 2D Basketball Trajectory




- A basketball player shoots a jump hook at a (horizontal) distance $R$ from the basket, releasing the ball at a height $H$ above the rim as shown. To shoot over his opponent's outstretched arm, he releases the basketball at an angle $\theta$ with respect to the horizontal.

Find $v_0$, the **speed** he must release the basketball with (in terms of $H$, $R$, $g$ and $\theta$) for the ball to go through the hoop "perfectly" as shown. Assume that his release is on line and undeflected, at initial speed $v_0$ and that the acceleration of the basketball is $\vec{a} = -g\hat{j}$, ignoring drag.

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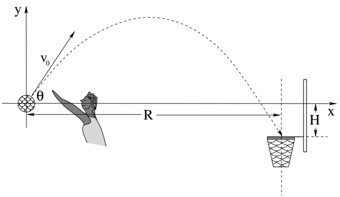


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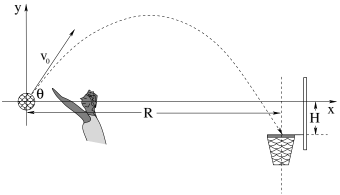


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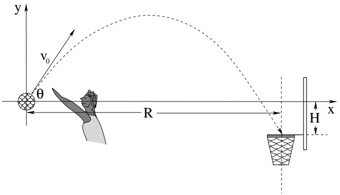


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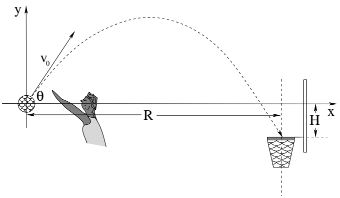


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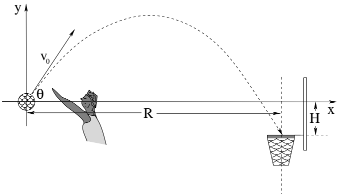


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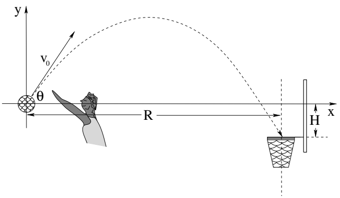


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The solution is on the next page. Don't advance until you are ready!

# Kinematics: 2D Basketball Trajectory-Solution

Initial Conditions:

$$a_x = 0, v_{0x} = v_0 \cos\theta, x_0 = 0 \text{ and } a_y = -g, v_{0y} = v_0 \sin\theta, y_0 = 0$$

Integrate:

$$x(t) = v_0 \cos\theta t \quad v_x(t) = v_0 \cos\theta \quad y(t) = -\frac{1}{2}gt^2 + v_0 \sin\theta t \quad v_y(t) = v_0 \sin\theta - gt$$

Find the **time** $t_b$ that the basketball reaches the horizontal position of the hoop:

$$R = v_0 \cos\theta t_b \Rightarrow t_b = R/(v_0 \cos\theta)$$

This must also be the time that the ball has exactly the height of the hoop:

$$-H = -\frac{1}{2}gt_b^2 + v_0 \sin\theta t_b \Rightarrow \frac{gR^2}{2v_0^2 \cos^2\theta} = R\tan\theta + H$$

And finally, we solve for $v_0$:

$$\boxed{v_0 = \sqrt{\frac{gR^2}{2(R\sin\theta\cos\theta + H\cos^2\theta)}}}$$

After doing the algebra, check the dimensions. Are they OK?

# Firing a Speargun


Hints:

- An underwater fisherman fires her speargun at a distant fish. The neutral-buoyancy spear leaves the gun at initial speed $v_0$ and experiences a *linear* drag force $F_d = -bv$ opposite to its velocity. Find $v(t)$ and $R$, the maximum range of the spear.

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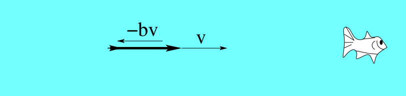


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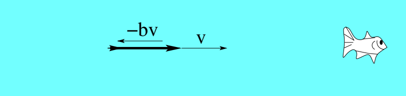
Hints:
First, draw a picture of the *general* motion and drag force.

# Firing a Speargun




- An underwater fisherman fires her speargun at a distant fish. The neutral-buoyancy spear leaves the gun at initial speed $v_0$ and experiences a *linear* drag force $F_d = -bv$ opposite to its velocity. Find $v(t)$ and $R$, the maximum range of the spear.

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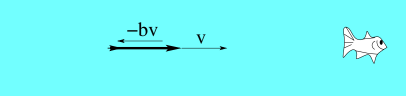


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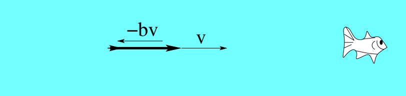


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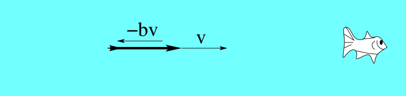


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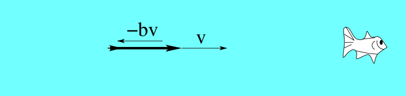


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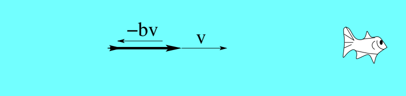


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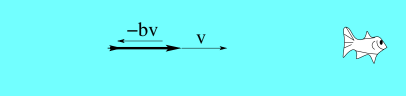


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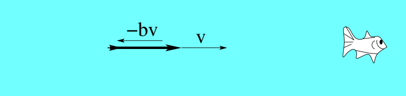

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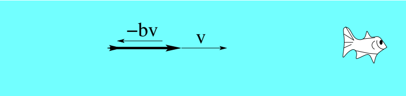


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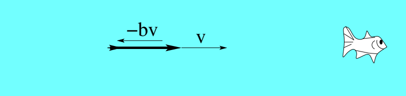


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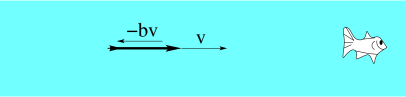


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$$\Rightarrow \ln\left(\frac{v}{v_0}\right) = -\frac{b}{m}t \Rightarrow e^{\left\{\ln\left(\frac{v}{v_0}\right) = -\frac{b}{m}t\right\}} \Rightarrow \frac{v}{v_0} = e^{-\frac{b}{m}t} \Rightarrow \boxed{v(t) = v_0 e^{-\frac{b}{m}t}}$$

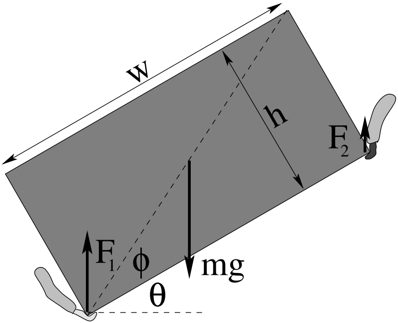
$$\frac{dx}{dt} = v_0 e^{-\frac{b}{m}t} \Rightarrow x(t) = \int_0^x dx = \int_0^t v_0 e^{-\frac{b}{m}t}dt = -\frac{mv_0}{b}\int_0^t e^{-\frac{b}{m}t}\left(-\frac{b}{m}\right)dt$$

$$x(t) = \frac{mv_0}{b}\int_{-bt/m}^{0} e^u du = \frac{mv_0}{b}\left(1 - e^{-\frac{b}{m}t}\right)$$

so the range $R$ is:

$$\boxed{R = x(t \to \infty) = \frac{mv_0}{b}}$$

# Static Equilibrium: Carrying A Box Up the Stairs

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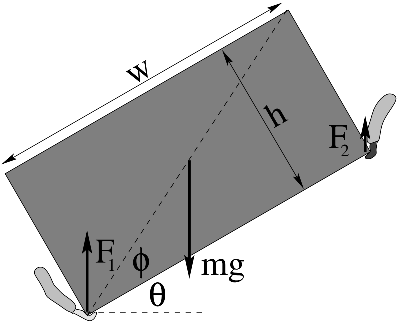


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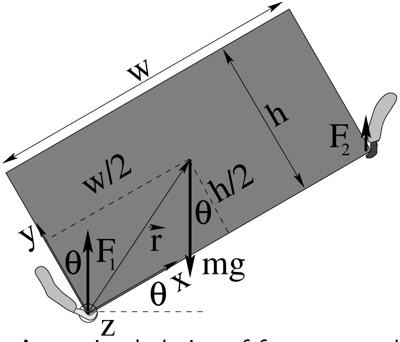


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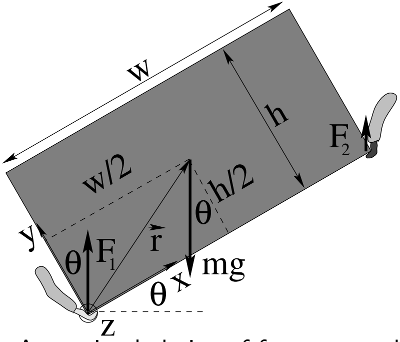


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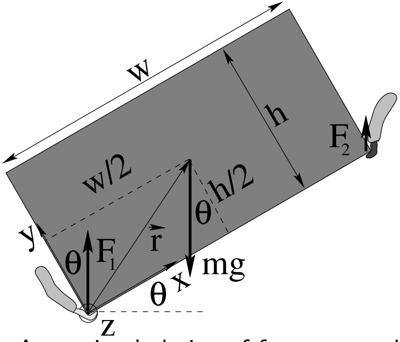


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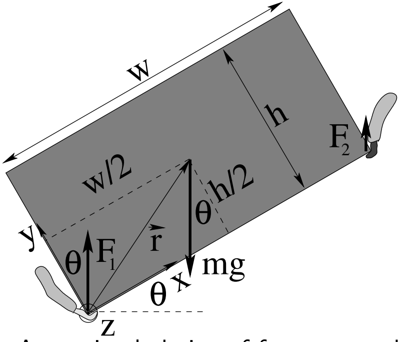


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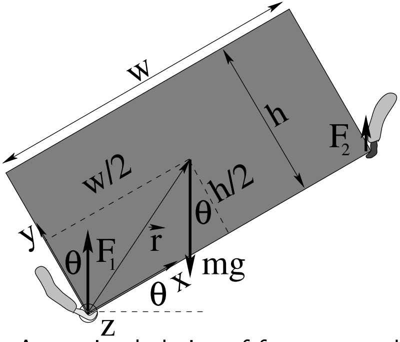


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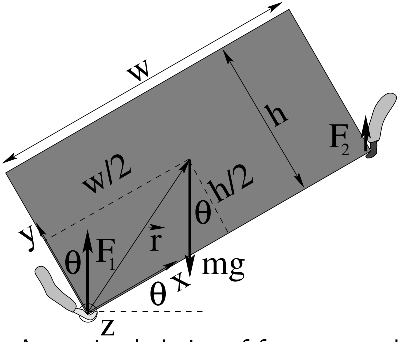


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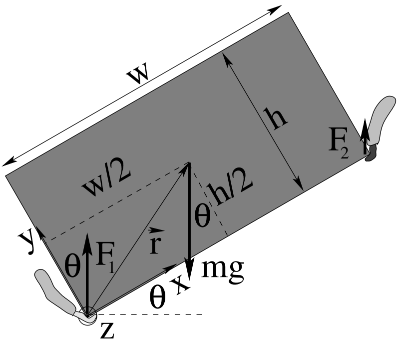


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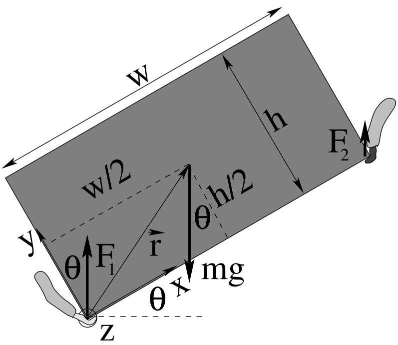


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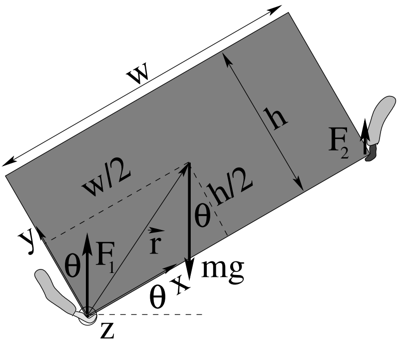


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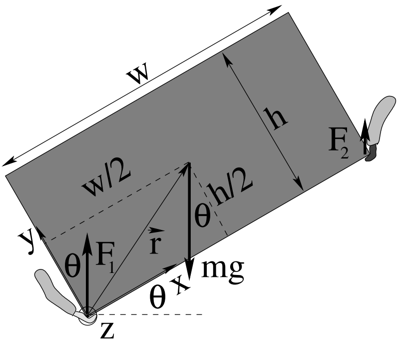


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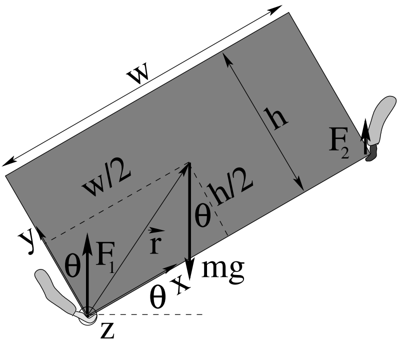


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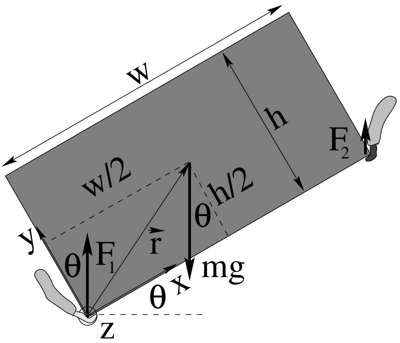


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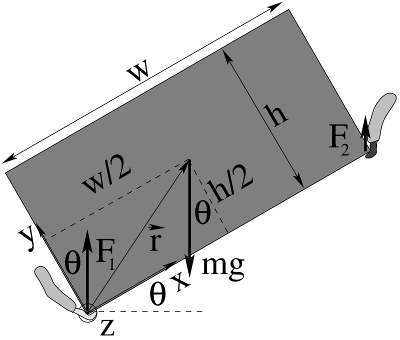


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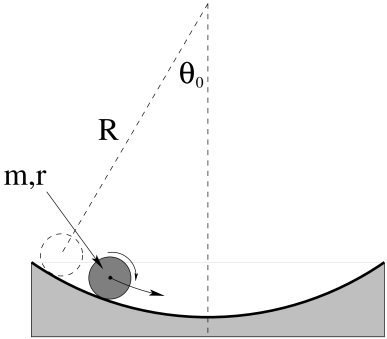
# Oscillations: A Rolling Pendulum




- A disk of mass $m$ and radius $r$ is gently set on a rough circular floor so that it makes an angle $\theta_0$ relative to a vertical through the center of curvature of the floor, with its center of mass a distance $R$ from the center of curvature as shown, and is released from rest at $t = 0$ so that it **rolls without slipping and oscillates**.
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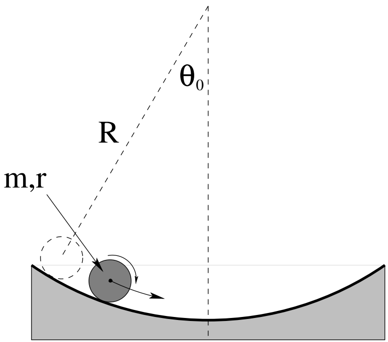


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Draw a force diagram on the rolling mass at a general angle $\theta$.

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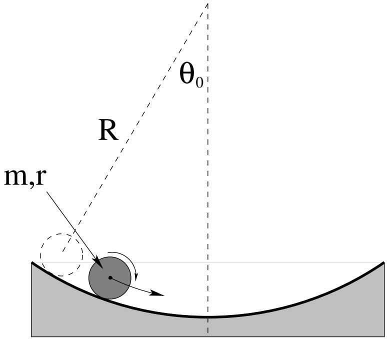


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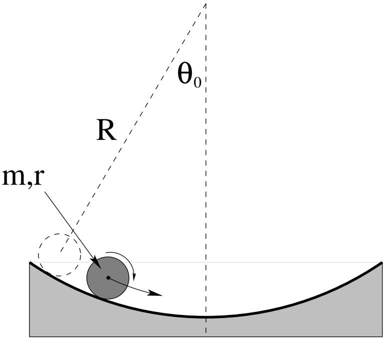


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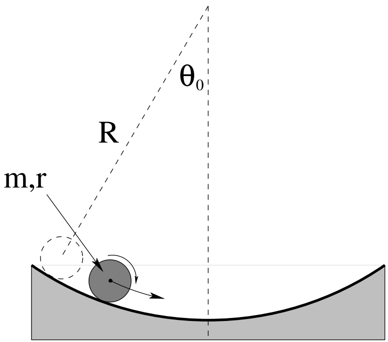


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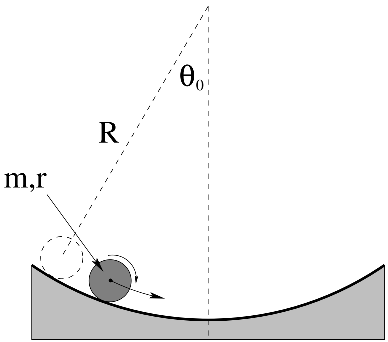


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* - The solution is on the next page. Don't advance until you are ready!

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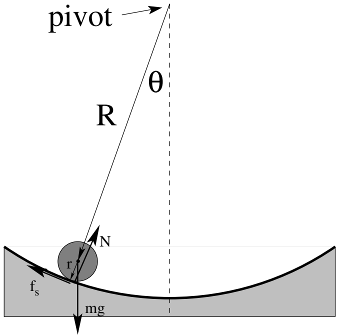


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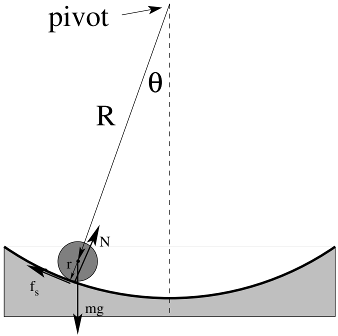


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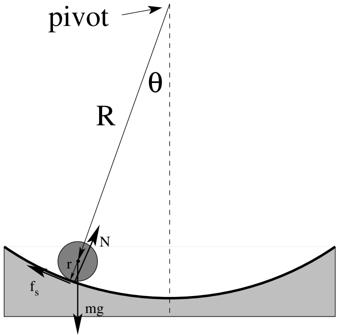


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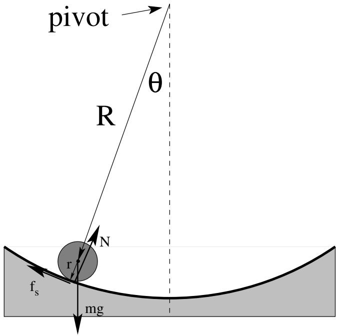


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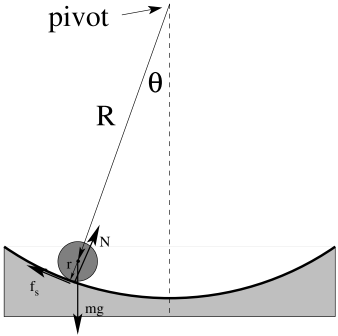


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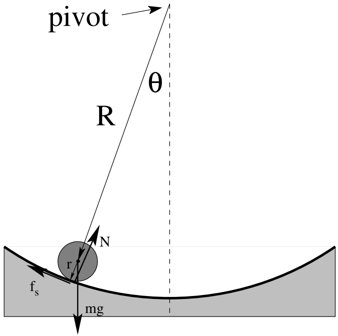


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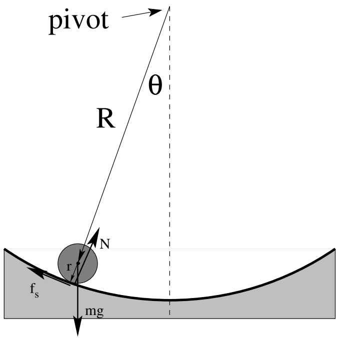


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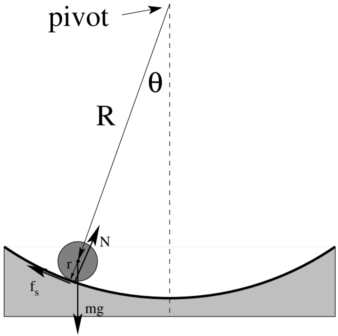


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make the small angle approximation and rearrange into $\frac{d^2\theta}{dt^2} + \frac{2g}{3R}\theta = 0$. Thus:

$$\boxed{\omega = \sqrt{\frac{2g}{3R}}} \qquad \boxed{\theta(t) = \theta_0\cos(\sqrt{\frac{2g}{3R}}t)} \qquad \boxed{f_s = -\frac{1}{2}ma_t = \frac{mg}{3}\theta_0\cos(\sqrt{\frac{2g}{3R}}t)}$$

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This too makes sense! At $t = 0$, the disk starts to roll *down to the right*, so $\Omega_{\rm disk}$ is into the page, positive. You should be able to trace each quarter cycle of its oscillation and see that everything is consistent and correct.

# The End

**Feedback Welcome**

Send Comments To: rgb at duke dot edu