# Experimental study of the complex dynamics of semiconductor lasers with feedback via symbolic time-series analysis

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## ABSTRACT

We investigate the symbolic dynamics of an excitable optical system under periodic forcing. Particularly, we consider the low-frequency fluctuation (LFF) dynamics of a semiconductor laser with periodically-modulated injection current and optical feedback. We use a method of symbolic time-series analysis that allows us to unveil serial correlations in the sequence of intensity dropouts. By transforming the sequence of inter-dropout intervals into a sequence of ordinal patterns and analyzing the statistics of the patterns, we uncover correlations among several consecutive dropouts and we identify definite changes in the dynamics as the modulation amplitude increases. We confirm the robustness of the observations by conducting the experiments with two different lasers under different feedback conditions. The results are also shown to be robust to variations of the threshold used for detecting the dropouts. Simulations of the Lang-Kobayashi (LK) model, including spontaneous emission noise, are found to be in good qualitative agreement with the observations, providing an interpretation of the correlations present in the dropout sequence as due to the interplay of the underlying attractor topology, the periodic forcing, and the noise that sustains the dropout events.

**Keywords:** Semiconductor lasers, optical feedback, ordinal analysis, symbolic dynamics.

## 1. INTRODUCTION

In the last few decades, semiconductor lasers were used as specially adapted tools to explore a great diversity of dynamical scenarios (see, for instance,[1,2] and references therein). When these lasers are under external perturbations, such as optical (and/or electronical) feedback, optical injection or current modulation, their output intensity can exhibit dynamical phenomena that vary from periodic dynamics to broad band chaos. A remarkable behavior observed when the laser is operated close to its threshold current, in the presence of moderate levels of optical feedback, is the appearance of apparently sudden and random intensity dropouts, known as low-frequency fluctuations (LFFs). The name comes from the fact that the dropouts appear with a frequency much lower than the other characteristic frequencies of the system (relaxation oscillations, external cavity resonance). LFF dynamics had attracted attention in the last decades both because of potential applications and because the mechanisms that trigger the LFFs involve the interaction of nonlinearity, delayed feedback and noise, which are present in many natural systems. In this sense, knowing better the LFF dynamics can improve our understanding of other real-world spiking systems, such as neurons.

This work deals with the complexity underlying the LFF phenomenon, in the specific case in which the laser is subject to current modulation. As the LLF dynamics is excitable, this system provides a controllable experimental setup to study the dynamics of a excitable system under periodic forcing. We use a symbolic method of time-series analysis, referred to as ordinal analysis,[3] to investigate the transition from the LFF dynamics of

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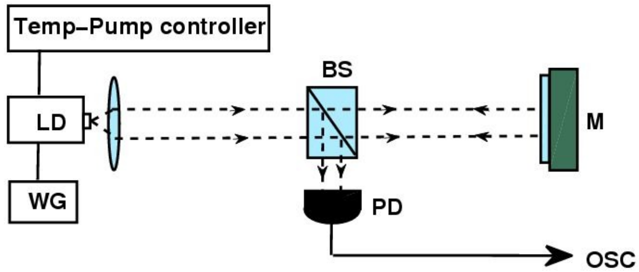


Figure 1. Experimental setup of the semiconductor laser with optical feedback. LD stands for laser diode, BS for beam-spliter, PD for photodetector, M for mirror, OSC for oscilloscope and WG for waveform generator.

the laser without modulation, in which the dropouts are much more stochastic and reveal only weak indications of an underlying deterministic attractor,[4] to the LFF dynamics of the modulated laser, which consists of more regular dropouts, with a periodicity related to the forcing period.[5] The increase of the modulation amplitude induces a gradual transition from mainly stochastic to mainly deterministic behavior, and our aim is to identify in this transition characteristic features that are signatures of the underlying topology of the phase space of the system.

We study experimentally and numerically the LFF spiking dynamics by considering long sequences of inter-dropout intervals, IDIs (*i.e.*, the time intervals between two consecutive intensity dropouts) and transforming the IDI sequences into sequences of symbolic patterns, referred to as Ordinal Patterns (OPs), or words.[6,7,8,9] By calculating the word probabilities, we unveil a nontrivial symbolic organization underlying the sequence of LFF intensity dropouts. The word probabilities depend on the modulation amplitude, and we identify clear changes in the most probable and less probable words, as the modulation amplitude increases. We perform experiments with two different lasers under different feedback conditions, and numerical simulations with the well known Lang-Kobayashi model.[10] We find a good agreement between the experiments and the numerical simulations of the model.

## 2. EXPERIMENTAL SETUP AND ORDINAL ANALYSIS METHOD

Figure 1 shows the experimental setup of a semiconductor laser (LD) with external optical feedback by a mirror (M). A 50/50 beam-splitter (BS) sends light to a photo-detector (PD) connected with a 1 GHz oscilloscope (OSC). Through a bias-tee in the laser head, a sinusoidal RF component from a leveled waveform generator (WG) is combined with a constant dc current. To confirm the robustness of the results, two different experimental setups are used: for a 650-nm-wavelength laser, travelling through free space, with 4.7 ns of time-delayed feedback, and for a 1550-nm-wavelength laser, travelling through an optical fiber, with 25 ns of time-delayed feedback.

We measure the experimental power output of the laser in the LFF regime and compute the inter-dropout intervals (see Figure 2). We use a method of symbolic time-series analysis, known as ordinal analysis,[3,6] to study the sequences of dropout events. This transforms the sequence of inter-dropout intervals (IDIs) into a sequence of symbolic ordinal patterns, or words, which consider $D$ consecutive IDIs ($D + 1$ consecutive dropouts). This method allows to infer signatures of determinism and stochasticity in the dynamics at an event level description, resulting in an efficient tool to reveal the hidden structure of the dynamics.

Words of length $D$ are defined by considering the relative length of $D$ consecutive IDIs. For $D = 2$ there are two OPs: $\Delta T_i < \Delta T_{i+1}$ gives word '01' and $\Delta T_i > \Delta T_{i+1}$ gives word '10', where $T_i$ is the time a dropout takes place. For $D = 3$ there are six OPs: $\Delta T_i < \Delta T_{i+1} < \Delta T_{i+2}$ gives '012', $\Delta T_{i+2} < \Delta T_{i+1} < \Delta T_i$ gives '210', etc. This symbolic transformation keeps the information about correlations present in the dropout sequence, but neglects the information contained in the duration of the IDIs. Figure 2(a) shows as examples one word of dimension $D = 2$ ('10') and one word of dimension $D = 3$ ('021').

Further insight can be gained by analyzing the frequency of occurrence of pairs of consecutive words. Specifically, for words of $D = 2$ we compute the probabilities of the four possible pairs ('01' $\rightarrow$ '01', '01' $\rightarrow$ '10', '10' $\rightarrow$

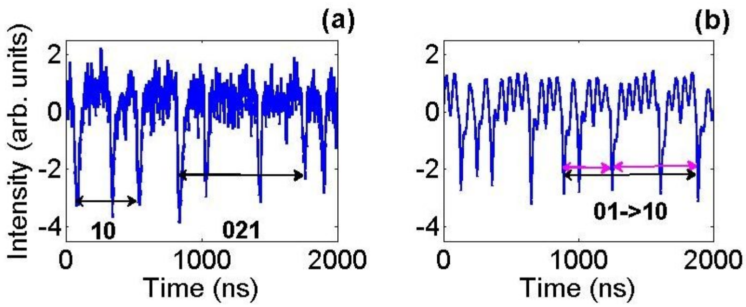


Figure 2. (a) Intensity output of the laser with optical feedback in the LFF regime. Words '10' for $D = 2$ and '021' for $D = 3$ are depicted as examples. (b) Intensity output of the laser with optical feedback and modulation. The transition from word '01' to word '10' is depicted as example.

'01', and '10' $\rightarrow$ '10'),[6,11] which are referred to as Transition Probabilities (TPs), and are normalized such that their sum is equal to one ($\sum_{i,j} TP_{i\rightarrow j} = 1$). The analysis of these TPs allows uncovering correlations among five consecutive dropouts, and thus, obtain information about the memory of the system in a longer time scale. Figure 2(b) shows as example the transition from word '01' to word '10'.

## 3. THE LK MODEL

The Lang and Kobayashi model[5] is a well stablish model that describes the slowly varying complex electric field amplitude $E$, and the carrier density $N$ of a semiconductor laser, with and without feedback. The rate equations of the LK model are

$$\frac{dE}{dt} = \frac{1}{2\tau_p}(1+\alpha)(G-1)E + \eta E(t-\tau)e^{-i\omega_0\tau} + \sqrt{2\beta_{sp}}\xi, \quad (1)$$

$$\frac{dN}{dt} = \frac{1}{\tau_N}(\mu - N - G|E|^2), \quad (2)$$

where $\tau_p$ and $\tau_N$ are the photon and carrier lifetimes respectively, $\alpha$ is the line-width enhancement factor, $G$ is the optical gain, $G = N/(1+\epsilon|E|^2)$ (with $\epsilon$ a saturation coefficient), $\mu$ is the pump current parameter, $\eta$ is the feedback strength, $\tau$ is the feedback delay time, $\omega_0\tau$ is the feedback phase, and $\beta_{sp}$ is the noise strength, representing spontaneous emission. For simulating the dynamics with current modulation, the pump current parameter is $\mu = \mu_0 + a\sin(2\pi f_{mod}t)$, where $a$ is the modulation amplitude, $f_{mod}$ is the modulation frequency, and $\mu_0$ is the dc current.

## 4. RESULTS

Figure 3 shows the probabilities of the different words of dimension $D = 2$ (first row) and $D = 3$ (second row) for different values of the modulation amplitude, for both sets of experiments (650 nm and 1550 nm), and the numerical simulations with the LK model. The dashed lines are the corresponding probabilities after having surrogated the data. The words from the surrogate data are, as expected, equally probable, as no correlations among the shuffled dropouts are expected. It is observed that, for low and high modulation amplitudes, words of $D = 2$ are equally probable, while words of $D = 3$ show correlations among three consecutive IDIs for high modulation amplitudes. This points to the need to consider words of $D = 3$ to unveil the structure of the dynamics.

For $D = 3$ a stochastic dynamics can be distinguished for low modulation amplitudes, while a deterministic dynamics is seen for high modulation amplitudes, finding a transition in the dynamics at around 1.6% for the 650 nm experimental data, at around 16% for the 1550 nm experimental data, and at around 4% for the LK model. In the deterministic regime, two groups of words are distinguished, one less probable ('012', '210') and one more probable ('021','102', '120', '201'). The less probable words are those which imply three consecutively increasing or decreasing IDIs. This can be understood in the following terms: strong enough modulation forces a rhythm in

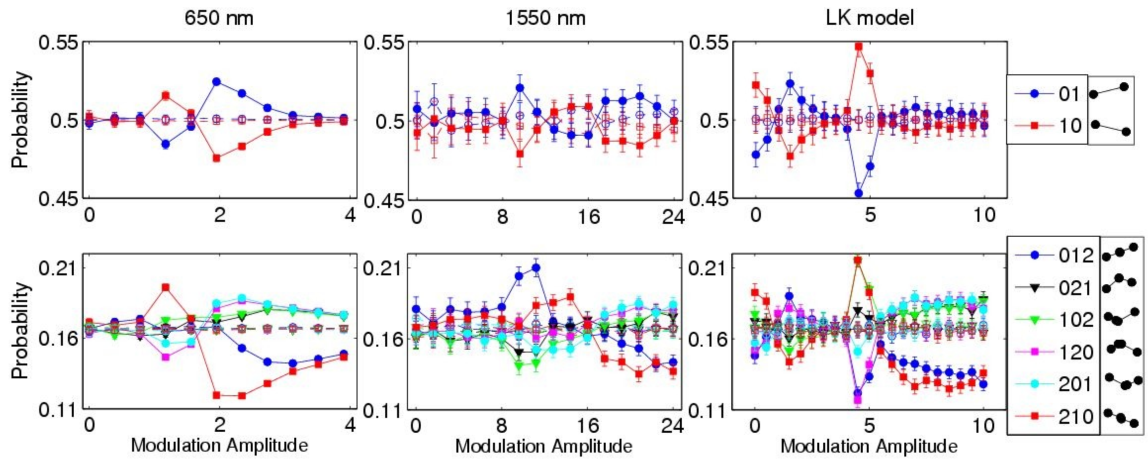


Figure 3. Words probabilities versus modulation amplitude, for D = 2 (first row) and D = 3 (second row), for the experimental data and the LK simulations.

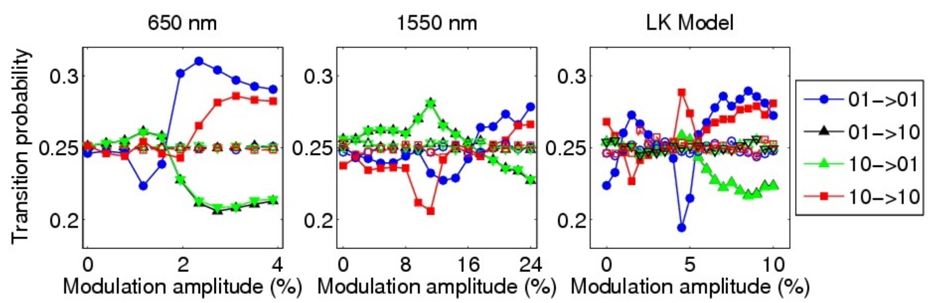


Figure 4. Transition probabilities versus modulation amplitude, for D = 2 (first row) and D = 3 (second row), for the experimental data and the LK simulations.

the LFF dynamics, and three consecutively increasing or decreasing intervals imply a loss of synchrony with the external rhythm, and thus, are less likely to occur. There is a good qualitative agreement between experiments and the LK model.

Figure 4 show the transition probabilities for the different values of the modulation amplitude, for the experimental data, and the numerical simulations. Results from these transition probabilities also show a change in the dynamics from low to high modulation amplitudes, at the same amplitude values as in the words probabilities. But these transitions consider five consecutive dropouts, revealing, thus, longer correlations than those seen with words of $D = 3$. Also a good qualitative agreement is found with this analysis, between experiments and the LK model.

## 5. INFLUENCE OF THE DETECTION THRESHOLD IN THE ORDINAL PATTERNS PROBABILITIES

One may question if and how the results shown above depend on the choice made for the dropout detection threshold. In the next we analyze the influence of the detection threshold in the probabilities of the ordinal patterns and verify that our results are robust to threshold variations. Figure 5 shows the probabilities vs. the detection threshold, for different fixed (without modulation) laser pump currents. It can be appreciated that, while the values of the probabilities vary with the threshold, the hierarchy and the clusters ('021'-'102' and '120'-'201') are robust and occur in a wide range of threshold values. While for too low (or too deep) thresholds, the probabilities vary significantly (as too many or too few spikes are detected), they are robust to threshold variations in a wide range of thresholds.

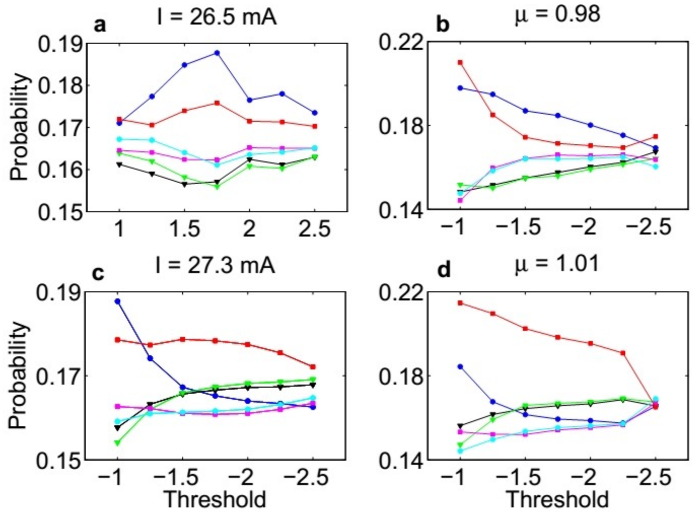


Figure 5. Probabilities of the words vs. the detection threshold is shown for different pump current values both, for the experimental (a, c) and the simulations (b, d) data. The color legend is the same as for the second row in Figure 3.

Most importantly, the variation of the probabilities with the threshold is qualitatively similar in the experimental and in the numerical data. The same hierarchy and clusters are seen. This is remarkable because the model used for the simulations is the simplest rate-equation model (assumes single-mode emission, neglects spatial and thermal effects, considers only optical noise, etc.) and the filter used to simulate the finite detection bandwidth is also a simple moving-average window.

While an optimal threshold could be defined for each pump current value (that is in the center of the plateau where the probabilities do not vary significantly with the threshold), for the sake of simplicity in our work we used a fixed threshold value, equal to -1.5, for detecting the spikes.

## 6. CONCLUSIONS

We study experimentally and numerically the symbolic dynamics of a semiconductor laser with optical feedback and current modulation in the LFF regime. By analyzing the time series of inter-dropout intervals using a symbolic transformation, we identify clear changes in the dynamics induced by the modulation. For weak modulation, the dynamics of the intensity of the laser is mainly stochastic, while for increasing modulation it becomes more deterministic showing, in this regime, correlations among several consecutive dropouts. The LK model is also tested and we find a good qualitative agreement with the experimental observations. The results are shown to be robust to variations of the dropouts detection threshold. We speculate that the symbolic behavior uncovered here is a fingerprint of the underlying topology of the phase space, and it is due to the interplay of noise-induced escapes from an stable external cavity mode, and the dynamics in the coexisting attractor.

## ACKNOWLEDGMENTS

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