Optimal pump profile designs for broadband SBS slow-light systems

Ravi Pant\textsuperscript{1}, Michael D. Stenner\textsuperscript{1,2}, Mark A. Neifeld\textsuperscript{1,2}, and Daniel J. Gauthier\textsuperscript{3}

\textsuperscript{1} College of Optical Sciences, University of Arizona, Tucson, Arizona, 85721 USA. \textsuperscript{2} Department of Electrical and Computer Engineering, University of Arizona, Tucson, Arizona, 85721 USA. \textsuperscript{3} Department of Physics and the Fitzpatrick center for Photonics and Communication Systems, Duke University, Durham, North Carolina, 27708 USA.

\texttt{rpant@email.arizona.edu}

Abstract: We describe a methodology for designing the optimal gain profiles for gain-based, tunable, broadband, slow-light pulse delay devices based on stimulated Brillouin scattering. Optimal gain profiles are obtained under system constraints such as distortion, total pump power, and maximum gain. The delay performance of three candidate systems: Gaussian noise pump broadened (GNPB), optimal gain-only, and optimal gain+absorption are studied using Gaussian and super-Gaussian pulses. For the same pulse bandwidth, we find that the optimal gain+absorption medium improves the delay performance by 2.1 times the GNPB medium delay and 1.3 times the optimal gain-only medium delay for Gaussian pulses. For the super-Gaussian pulses the optimal gain-only medium provides a fractional pulse delay 1.8 times the GNPB medium delay.

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1. Introduction

All-optical processing such as data synchronization and optical buffering is required for high-speed optical networks. This has led to intense research in optically controlled tunable pulse delay and slow-light pulse propagation. Many studies performed to date have generated tunable pulse delay using the stimulated Brillouin scattering (SBS) process in optical fibers to create gain features. Pulse delay in these systems is tuned by varying the gain. Although slow-light pulse delay using SBS gain in optical fibers provides easy integration with existing optical communication systems, it is plagued by the inherently small linewidth of Brillouin gain. The inherent half-width at half-maximum (HWHM) linewidth of Brillouin gain in optical fibers is ~ 25MHz and therefore restricts the data rate to several Mbps. The bandwidth of Brillouin gain systems must therefore be increased as the pulse bandwidth increases.

Several methods of SBS pump modulation have been shown to increase the bandwidth of Brillouin gain systems. A simple way to broaden the Brillouin gain spectrum is to modulate the pump laser using a sinusoidal signal of fixed frequency. This results in a pair of sideband
SBS gain lines whose strength and frequency separation can be tuned using the amplitude and frequency of the modulating signal. Slow-light systems based on multiple gain lines have been experimentally demonstrated and shown to improve the delay performance of single-line gain systems under maximum gain and distortion constraints.\textsuperscript{9–14} Because the HWHM linewidths of all gain sidebands generated via this method will be \( \simeq 25 \text{MHz} \), many closely space lines are required in order to achieve large delay and low distortion as the data bandwidth increases. Such a pump comprising many closely spaced lines can be interpreted as a continuous pump. The height and frequency spacing between different pump spectral components determine the shape of the pump profile. In the recent past, several studies have been performed to arbitrarily extend the Brillouin gain bandwidth using a broadband pump.\textsuperscript{15–19,21}

Recently, a combination of Brillouin gain and absorption resonances was used to design a zero-gain slow light medium.\textsuperscript{17,23} Slow-light pulse delay has also been demonstrated using two widely separated anti-stokes absorption resonances in highly nonlinear fiber (HNLF).\textsuperscript{22} Such media are sometimes called “nearly transparent” due to their resemblance to electromagnetically induced transparency (EIT). Recently a nearly transparent medium was combined with a gain-based slow-light medium and demonstrated improved pulse delay compared to a gain-based system for a pulse width of 30 ns.\textsuperscript{20,21}

The approaches described above combine multiple SBS gain (and in some cases absorption) lines to create a broadband profile to achieve the delay of broadband pulses. In this paper, we present a technique for determining the \textit{optimal} pump profiles for broadband SBS slow-light systems under distortion and system resource constraints such as total pump power and maximum intensity gain. The delay performance of three candidate systems: Gaussian noise pump broadened (GNPB), optimal gain-only, and optimal gain+absorption are studied under these constraints. These results not only provide useful system designs for creating slow-light devices, but they also serve as upper bounds on achievable slow-light delay using SBS gain and absorption given reasonable system constraints.

\section{Gain-only medium}

In this section, we describe the pulse propagation through a Brillouin gain medium designed using a modulated pump. In an optical fiber, gain/absorption resonances can be created using the SBS process, which occurs as a result of the interaction between an optical pump of frequency \( \omega_p \) and a probe signal of frequency \( \omega_0 \) via an acoustic field with phonon frequency \( \Omega_B \). Probe gain occurs when \( \omega_0 = \omega_p - \Omega_B \) and absorption occurs when \( \omega_0 = \omega_p + \Omega_B \). This interaction among the pump, probe, and acoustic wave can be described by the coupled-mode equations.\textsuperscript{24}

For a monochromatic pump under the undepleted pump approximation, probe signal propagation using the coupled-mode equations simplifies to:\textsuperscript{24}

\[
\frac{\partial E(\omega)}{\partial z} = jk(\omega)E(\omega),
\]

where \( E(\omega) \) is the probe signal field and \( k(\omega) \) is the complex wave vector characterized by a Lorentzian profile:\textsuperscript{9–14}

\[
k(\omega) = \frac{g_0P_0\gamma/(2A)}{[\omega - (\omega_p - \Omega_B)] + j\gamma}.
\]

Here \( \gamma/(2\pi) \) is the HWHM linewidth, \( g_0L = g_0P_0L/A \) is the intensity gain exponent at the pulse carrier frequency, \( L \) is the length of the fiber, \( A \) is the effective mode area of the optical fiber, and \( P_0 \) is the pump power at frequency \( \omega_p \). From equation (1), we note that the probe signal propagation can be described by the transfer function approach for a monochromatic pump under the undepleted pump approximation.
For a modulated pump, probe signal propagation derived using the coupled-mode equations and undepleted pump approximation yield terms arising from the interference among the pump spectral components. This interference affects the gain experienced by the probe signal depending on the ratio of the pump laser coherence length \( L_{coh} \) relative to the fiber length \( L \). If \( L/L_{coh} \ll 1 \), interference among the pump spectral components affects the steady state gain depending upon the constructive or destructive interference. The maximum possible coherence length \( L_{coh}^{max} \) occurs for the simplest modulated pump that consists of two pump frequencies separated by the laser linewidth \( \Delta \nu_{laser} \). This results in \( L_{coh}^{max} = c/n\Delta \nu_{laser} \). For a two-line pump, Lichtman et al. demonstrated that the interference affects the gain if \( L/L_{coh} \leq 0.15 \). Using \( L/L_{coh}^{max} = 0.15 \) and typical laser linewidths of 100-500 kHz for commercially available lasers implies an interaction length range of 60-300 m for interference effects to become important. This interaction length range defines the minimum fiber length \( L_{min} \) required to avoid the interference effects. Many of the slow-light systems use \( L \geq 1 \) km to achieve the desired gain and therefore, interference terms can be neglected. Under these conditions, the gain experienced by the probe signal is determined by the convolution \( (P(\omega) \otimes k(\omega)) \) of the pump spectrum \( P(\omega) \) and the intrinsic Lorentzian SBS profile \( k(\omega) \). For the gain profile given by \( P(\omega) \otimes k(\omega) \), propagation of the probe signal using the coupled-wave equations simplifies to equation (1) with \( k(\omega) \) replaced by \( P(\omega) \otimes k(\omega) \), which is the transfer function approach. In our study, we satisfy the above conditions and therefore, use the transfer function approach for propagating the probe signal through the slow-light medium.

Because the intrinsic HWHM linewidth of Brillouin gain in optical fibers is \( \approx 25 \) MHz, it must be substantially increased to accommodate the GHz bandwidth signals typically used in optical communication systems. The required gain broadening has already been experimentally demonstrated using both multi-line pump and broadband pump and has shown improved delay performance. A broadband pump can be generated by directly modulating the pump laser with an arbitrary waveform generator (AWG), as shown in Fig. 1. Recently, Brillouin gain broadening has been demonstrated using a broadband pump generated by modulating the pump laser. We are interested in designing arbitrary gain profiles to maximize the delay performance under real-world operating constraints.

2.1. Constraints

In our study, we consider constraints on the maximum intensity gain, total pump power, and distortion. The constraint on maximum usable gain arises to avoid the initiation of pump depletion and nonlinear effects. In the pump depletion region, delay saturates and may in fact become smaller with increasing gain. In our study, we choose a practical value for the maximum
gain constraint, \( \max \omega |g(\omega)|L \leq 7 \), where \( g(\omega) \) is the imaginary part of the complex gain profile obtained by convolving the pump spectrum with the Lorentzian profile.\(^{14,15}\)

The constraint on total available pump power arises due to engineering considerations such as EDFA gain limitations, laser current limitations, total device power consumption, etc. We consider the normalized pump power constraint \( [g_0 L/(2A)] \int P(\omega) d\omega \leq 200 \), where \( P(\omega) \) is the pump power spectral density. For a single mode fiber with parameters: \( g_0 = 5 \times 10^{-11} \text{m/W}, A=50 \mu \text{m}^2 \), and total pump power \( \int P(\omega) d\omega \leq 300 \text{mW} \) requires a fiber of length 1.35 km. We choose a pump power of 300 mW assuming an EDFA with a typical gain of \( \sim 25 \text{dB} \) and a pump laser with an output power of 1 mW.

The amplitude and phase spectral response of the slow-light medium determines the distortion. For minimum pulse distortion, we want a uniform amplitude response and linear phase response over the finite pulse bandwidth.\(^{9-12}\) We define the distortion \( D_c \) as the 2-norm of the pulse-spectrum-weighted deviation of the slow-light system transfer function from the ideal system transfer function, over the finite pulse bandwidth \( B \)

\[
D_c = \text{norm}_2[S(\omega)(T_{SL}(\omega) - T_{ideal}(\omega))]_{\omega=0}^{\omega=B},
\]

where \( T_{SL}(\omega) \) is the transfer function of the slow-light system, \( T_{ideal}(\omega) \) is the ideal system transfer function, \( S(\omega) \) is the pulse power spectrum, \( B \) is the half-width at 1/e point for the signal spectrum, and \( \text{norm}_2 \) provides the root mean square of the deviation. The ideal system transfer function provides pure delay without effecting the pulse shape and is defined as

\[
T_{ideal}(\omega) = A \exp[i\phi_1 L + (\omega_0 - \omega)(\phi_0 L)],
\]

where \( A, \phi_0, \) and \( \phi_1 \) are the uniform amplitude, phase shift, and linear slope, respectively, of the ideal medium. Parameters \( A, \phi_0, \) and \( \phi_1 \) are optimized to fit the best ideal system transfer function to the real system transfer function such that the distortion constraint is satisfied and the delay is maximized.\(^{9-12}\) Because the distortion measure \( D_c \) is pulse-spectrum-weighted it is useful for designing the optimal profiles for single pulses. However, for designing the optimal profiles for a random pulse sequence and modulation formats such as non-return-to-zero (NRZ), differential phase shift keying (DPSK), etc. other distortion metrics such as eye-opening can be used.\(^{13,14}\)

In this study, we choose \( D_c \leq 0.25 \).

For the results reported here, we study both Gaussian pulses with pulse amplitude \( E(t) = \exp(-t^2/2T_0^2) \) and super-Gaussian pulses with pulse amplitude \( E(t) = \exp(-t^4/2T_0^4) \), where \( T_0 \) is the 1/e intensity half-width.

### 2.2. Optimization

To design the optimal gain-only profile at a given pulse bandwidth, we start with a random pump vector consisting of \( N \) samples as shown in Fig. 2a and convolve it with the Lorentzian profile. The real part of the resulting complex wave number is then used to calculate the group index profile \( n_g(\omega) \). We optimize the pump profile to maximize the signal pulse-spectrum-weighted average group index \( J = \int_{\omega=-B}^{\omega=B} S(\omega)n_g(\omega) d\omega \).

Figure 2a shows the initial random pump vector consisting of \( N = 1000 \) samples. In order to design a broadband pump with spectral width \( \simeq \text{GHz} \) using \( N = 1000 \) samples, the frequency spacing between these samples is several MHz. The resulting optimal pump power spectrum (dash) \( P(\omega) \) for a super-Gaussian pulse spectrum is shown in Fig. 2b along with the optimized pump profile for the GNPB (solid) medium. For the GNPB medium, we optimize the spectral width and height of the gain profile by optimizing the Gaussian pump spectral width and height. We note that the optimal pump profile is nearly uniform over the pulse bandwidth. By contrast, in order to achieve a nearly-flat gain profile over the pulse bandwidth, the GNPB pump must be much broader, placing pump power in the wings where it does little good. We will discuss below how this affects the maximum achievable pulse delay for the GNPB medium.
Fig. 2. (a) Initial random pump profile and (b) pump profiles for the GNPB (solid) and the optimal gain-only (dash) medium optimized using the super-Gaussian pulse spectrum and $T_{\text{pulse}}=166$ ps.

Figures 3a and 3b show the gain and refractive index profiles, respectively, obtained using the pump profiles shown in Fig. 2b. The GNPB gain profile shown in Fig. 3a must be broader compared to the optimal gain-only profile in order to satisfy the distortion constraint that tries to make the gain uniform over the pulse bandwidth. From Fig. 3a we also note that the optimal gain-only profile achieves the maximum gain of 7, whereas the GNPB medium has a gain of 6.2. Later we will show that this happens because the GNPB medium satisfies the maximum pump power constraint ($\int P(\omega)d\omega = 300$ mW) at the pulse bandwidth $Bw=5$ GHz, whereas for the optimal gain-only medium $\int P(\omega)d\omega = 300$ mW is satisfied at $Bw=7$ GHz, where $Bw=1/T_{\text{pulse}}$ and $T_{\text{pulse}}=2T_0$ is the pulse full-width at 1/e intensity. The optimal profiles shown in Figs. 2b, 3a, and 3b are obtained for a bandwidth of 6 GHz, which corresponds to the average of the bandwidths at which the GNPB medium and optimal gain-only medium reaches the maximum power constraint ($\int P(\omega)d\omega =300$ mW).

Figure 3b. shows the refractive index profiles obtained using the pump profiles shown in Fig. 2b. The slope of the refractive index profile depends on the gain profile via the Kramers-Kronig relation. For the optimal gain-only medium, the sharp rising edges of the gain profile result in large index slope as shown in Fig. 3b as compared to the GNPB medium. Therefore, we expect the delay to be larger for the optimal gain-only medium.

The approach described above—optimizing the spectrum at $N=1000$ points starting from a random vector—provides a high degree of confidence that the result is indeed optimal subject to the constraints. However, a 1000-parameter optimization is prohibitively time-consuming. Furthermore, performing the optimization repeatedly for several different pulse bandwidths and constraint values reveals that the resulting media are qualitatively similar. Therefore, one can dramatically reduce computation time by approximating them with a well-chosen parametric form.

Because the optimal pump profiles are roughly uniform over the pulse bandwidth, a super-Gaussian function provides a good approximation as shown in Fig. 4. Here, the super-Gaussian pump has the form $P_{sg}(\omega) = x_1 \exp\left(-\left((\omega - (\omega_0 + \Omega_B))/x_2\right)^3\right)$, where the parameters $x_1$, $x_2$, and $x_3$ have been optimized subject to the same metric and constraints as were the $N=1000$ parameters for the arbitrary pump. To determine the quality of this approximation in terms of delay and distortion, we compare the optimal gain and refractive index profiles with the gain and refractive index profiles obtained using the super-Gaussian pump. The gain and refractive index profiles using the optimized super-Gaussian pump spectrum matches well with the optimal profiles over the pulse bandwidth as shown in Figs. 3a and 3b, respectively, resulting in the same
pulse delay of 252 ps as for the optimal gain-only medium. Therefore, a super-Gaussian(SG) pump can provide delay performance similar to the optimal profiles.

Figures 5a and 5b show the output pulses for a Gaussian input pulse and a super-Gaussian input pulse, respectively, for the GNPB and optimal gain-only medium at $Bw = 6 \text{ GHz}$ ($T_{\text{pulse}} = 166 \text{ ps}$). For the Gaussian input pulse, the full-width half-maximum (FWHM) output pulse widths are 270 ps for both the GNPB and optimal gain-only media. Although the output pulse widths are same for the two media, the absolute pulse delay for the GNPB and optimal gain-only media are 167 ps and 252 ps respectively. Thus, the optimal gain-only medium provides a fractional delay improvement of $1.5 \times$ compared to the GNPB medium. For a super-Gaussian input pulse, FWHM output pulse widths for the GNPB and optimal gain-only media are 242 ps and 270 ps. In this case, the output pulse width for the optimal gain-only medium is 1.1 times the output pulse width for the GNPB. The fractional pulse delay for the optimal medium outperforms the fractional delay for the GNPB medium by a factor of 1.8. Thus, optimal design provides 50%-80% delay improvement over the GNPB medium with similar output pulse widths for the two media.
3. Gain+Absorption medium

In the previous section, we described how pump modulation can create a nearly arbitrary gain profile at the Stokes frequency. If we choose the pump frequency \( \omega_p \) such that \( \omega_p = \omega_0 - \Omega_B \) we can create an absorption resonance at the anti-Stokes signal frequency. The combination of an absorption resonance and a gain resonance can be used to design a zero-gain slow light medium. In these media, gain and absorption pumps at frequencies \( \omega_0 + \Omega_B \) and \( \omega_0 - \Omega_B \) respectively are used to create the gain and absorption resonances at the signal frequency \( \omega_0 \). Arbitrary control of the gain and absorption line shapes (as in the previous section) provides even finer control of the medium properties and allows for zero net gain at the signal frequency \( \omega_0 \). Recently, a slow-light medium consisting of a nearly transparent medium and a gain resonance demonstrated improved pulse delay compared to a gain-based system for a pulse width of 30 ns. Thus, combining the gain-only medium with a “nearly transparent” medium is a logical extension for increasing the slow-light pulse delay of a gain-based broadband slow-light system. Design of this “gain+absorption” medium requires the design of gain and absorption pumps.

For designing the optimal absorption pump profile, we use the peak absorption constraint of \( |g_a(\omega) L| \leq 7 \), where \( g_a(\omega) \) is the imaginary part of the absorption profile obtained by convolving the optimal absorption pump spectrum with the Lorentzian profile. The pump power constraint is modified to include the absorption pump \( \int (P_g(\omega) + P_a(\omega)) d\omega \leq 300 \text{ mW} \) while keeping the peak gain and distortion constraints the same as for the optimal gain-only medium design. Here, \( P_g(\omega) \) and \( P_a(\omega) \) are the power spectral densities for the optimal gain and absorption pumps respectively. The constraint \( |g_a(\omega) L| \leq 7 \) is required to prevent the gain associated with the Stokes resonance of the absorption pump from reaching the SBS threshold. This constraint of 7 on the gain exponent and absorption exponent is very conservative as compared to the gain exponent of 25 for SBS threshold in optical fibers. The gain pump profile for the optimal gain+absorption medium is similar to the optimal pump profile for the gain-only medium, whereas the resulting absorption pump profile gives rise to a nearly transparent medium at \( \omega_0 \). Figure 6a shows the optimal absorption pump power spectrum \( P_a(\omega) \) for the gain+absorption medium using the super-Gaussian pulse shape. The resulting gain+absorption medium is shown in fig. 6b. As for the optimal gain-only medium, we find that the optimal absorption pump profile can be approximated by a weighted sum of super-Gaussian functions as demonstrated in Figs. 6a and 6b.
At small pulse bandwidths, integrated pump power for the optimal gain-only pump satisfies \( \int P_g(\omega) d\omega < 300 \text{ mW} \). Thus, additional pump power can be used to create absorption resonances at the edges of the gain resonance while maintaining the overall integrated pump power constraint \( \int [P_g(\omega) + P_a(\omega)] d\omega \leq 300 \text{ mW} \). As the pulse bandwidth increases however, the optimal gain-only pump can alone satisfy \( \int P_g(\omega) d\omega = 300 \text{ mW} \). Therefore, for a given pump power constraint, gain+absorption medium improves the delay performance compared to the optimal gain-only medium at smaller pulse bandwidths. The profiles in Figure 6 are obtained for a super-Gaussian pulse shape using pulse bandwidth \( B_w = 4 \text{ GHz} \) at which additional pump power is available for creating the absorption resonances.

Fig. 6. (a) Optimal absorption pump profile (solid) and super-Gaussian fit (dash) and (b) gain+absorption profile (solid) and super-Gaussian fit (dash) for gain+absorption medium using super-Gaussian pulse spectrum and \( T_{\text{pulse}} = 250 \text{ ps} \).

Fig. 7. Output pulses for (a) Gaussian (solid) input pulse and (b) super-Gaussian (solid) input pulse for the GNPB medium (dash), optimal gain-only medium (dash-dot), and optimal gain+absorption medium (dot) for \( T_{\text{pulse}} = 250 \text{ ps} \).

Figures 7a and 7b show the output pulses for the Gaussian and super-Gaussian input pulses, respectively, at \( B_w = 4 \text{ GHz} \) (\( T_{\text{pulse}} = 250 \text{ ps} \)). The FWHM output pulse width for the Gaussian input pulse, when it propagates through the GNPB medium, is 1.92 times the FWHM input pulse width. The optimal gain-only and gain+absorption media have FWHM output pulse widths \( \sim 2.1 \) times the input pulse width. Although all three media generate output pulse widths close to twice the input pulse width, the fractional pulse delay for the optimal
gain+absorption medium is 1.95 times that of the GNPB medium and 1.22 times that of the optimal gain-only medium. Thus, for similar pulse distortion and system constraints, the optimal designs significantly outperform the GNPB pulse delay. For the super-Gaussian input pulse, the FWHM output pulse widths for the GNPB medium, optimal gain-only medium, and optimal gain+absorption medium are 360 ps, 410 ps, and 400 ps respectively. We note that for super-Gaussian input the optimal gain+absorption medium produces slightly smaller output pulse width compared to the optimal gain-only medium. Thus, optimal gain+absorption medium can improve both the delay performance and distortion performance compared to the optimal gain-only medium. Although in most cases the output pulse width for the optimal media is 1.07-1.12 times the output pulse width for the GNPB medium, using the optimal media provide a significant delay performance improvement (50%-110%) compared to the GNPB medium. Thus, under the same distortion and system constraints, optimal designs can provide significant delay improvement.

4. Delay-Bandwidth Results

In the previous sections, we presented results based on specific example pulses. In this section, we describe the delay and constraint behavior of these media as functions of signal bandwidth. Figures 8a, 8c, and 8e show the optimized fractional pulse delay, gain at pulse carrier frequency $\omega_0$, and total pump power, respectively, as functions of the input pulse bandwidth ($B_w$) for super-Gaussian pulses. These quantities are plotted for the optimized super-Gaussian pump (lines) and the fully optimized pump (markers) obtained using N=1000 samples. For Gaussian pulses the corresponding data are plotted in Figs. 8b, 8d, and 8f.

The fractional pulse delay vs. bandwidth plots in Figs. 8a and 8b exhibit two distinct regions. The region to the left of the delay maximum corresponds to $P_{\text{gain}} = \int P_g(\omega) d\omega \leq 300$ mW for the GNPB and the optimal gain-only medium. From Figs. 8c and 8d we see that the gain profile achieves the maximum peak gain in this region. To the right of the delay maximum, the maximum pump power constraint is reached and any further increase in bandwidth requires a decrease in gain and thus reduced delay. Table I lists the maximum fractional pulse delay and corresponding optimal pulse bandwidths for the optimal gain-only and GNPB media using Gaussian and super-Gaussian signal pulses. Note that the optimal design provides better delay performance and increases the optimal bandwidth.

Table 1. Maximum fractional delay and respective optimal pulse bandwidth

<table>
<thead>
<tr>
<th>Medium</th>
<th>$S(\omega)$</th>
<th>$\Delta T/T_{\text{pulse}}$</th>
<th>$BW_{\text{pulse}} = 1/T_{\text{pulse}}$ (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>optimal gain-only</td>
<td>super-Gaussian</td>
<td>1.54</td>
<td>6</td>
</tr>
<tr>
<td>GNPB</td>
<td>super-Gaussian</td>
<td>0.94</td>
<td>5</td>
</tr>
<tr>
<td>optimal gain-only</td>
<td>Gaussian</td>
<td>1.62</td>
<td>7</td>
</tr>
<tr>
<td>GNPB</td>
<td>Gaussian</td>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

For the optimal gain+absorption medium, absorption resonances near the edges of the gain spectrum cause increased delay at lower bandwidths. This happens because more power is available to the absorption pump at lower bandwidths as shown in Figs. 8e and 8f. As pulse bandwidth increases, the gain profile tries to achieve the maximum gain and uses all the pump power. As a result, the absorption pump power ($P_{\text{abs}} = \int P_a(\omega) d\omega$) reduces to zero as shown in Figs. 8e and 8f. The optimal gain+absorption medium thus acts as the optimal gain-only medium at high bandwidth. In this region, the fractional pulse delay for the optimal gain+absorption medium is lower-bounded by the fractional delay of the optimal gain-only medium. However, the optimal gain-only medium still performs better than the GNPB medium.
Fig. 8. Fractional pulse delay, central gain exponent, and total pump power for the GNPB (solid), optimal gain-only (circles), and optimal gain+absorption (diamonds and squares) medium as a function of pulse bandwidth for super-Gaussian (a,c,e) and Gaussian pulses (b,d,f).
For the optimal gain+absorption medium, maximum fractional pulse delays of 2.1 and 1.9 occur for the Gaussian and super-Gaussian pulses, respectively, over the range of bandwidths studied.

Although our designs are based on single pulse spectra we also analyze their delay-distortion performance using random pulse sequences. The distortion performance of the output bit stream is characterized using the eye-opening-penalty (EOP) described by

\[ EOP = -10 \log_{10} \left( \frac{EO_{out}}{EO_{in}} \right), \]

where \( EO_{out} \) and \( EO_{in} \) are the maximum eye-openings of the output and input pulse streams respectively. Figures 9a and 9b show some example eye diagrams for the GNPB and the optimal super-Gaussian pump, respectively for \( T_{\text{pulse}} = 166 \) ps and a return-to-zero (RZ) super-Gaussian pulse sequence. We note that the EOP for the GNPB and optimal super-Gaussian pump are 1.94 and 2.36 respectively. These EOP values correspond to the eye-openings of 64% and 58% respectively for the GNPB and optimized super-Gaussian pump. We use the optimal super-Gaussian pump for eye-diagram generation because sudden transitions near \( (\omega - \omega_0)T_0 = 1 \) in the phase and amplitude response of the optimal gain-only media and small fluctuations in the optimal gain-only pump as shown in Fig. 3 degrade the eye-opening. The eye-opening for the optimal gain-only pump is 36%. Therefore, for pulse trains the optimal pump profile should be designed using eye-opening as the distortion metric. The output eye-opening degrades due to the inter-symbol-interference (ISI) between successive pulses. For threshold detection and assuming a thermal-noise-limited detector, the bit-error-rate (BER=1/2erfc(Q/\sqrt{2})) is related to the eye-opening via the Q-factor \( (V_1 - V_0)/(\sigma_1 + \sigma_0) \), where \( V_1 \) and \( V_0 \) are the minimum value for 1-bits and maximum value of 0-bits, respectively, at the maximum eye-opening instant. The noise standard deviations for 1’s and 0’s are represented by \( \sigma_1 \) and \( \sigma_0 \) respectively. For a thermal-noise-limited detector with signal-to-noise ratio (SNR) of 35 dB the EOP occurs due to ISI only for both media and results in BER less than \( 10^{-12} \). Therefore, these designs for single pulse spectra give tolerable data-fidelity for random pulse sequences.

![Eye Diagrams](image)

Fig. 9. Output eye diagrams for (a) GNPB medium and (b) optimal gain-only medium using a super-Gaussian RZ pulse sequence and 1/e FWHM pulse width \( T_{\text{pulse}} = 166 \) ps.

The dashed lines in Figs. 9a and 9b show the maximum eye-opening instant for the input and output pulse streams. Note that the separation between these time instants is larger for the optimized super-Gaussian pump as compared to the GNPB medium. When a NRZ pulse stream was used, the EOP increases resulting in reduced data-fidelity. Recently, other modulation formats such as DPSK have been studied for designing the slow-light systems due to their better spectral efficiency. To improve the data-fidelity for random pulse sequences and different modulation formats such as NRZ, DPSK, etc. one can design the optimal profiles for pulse trains using distortion metrics such as eye-opening, power penalty, etc.

We observe that optimizing the pump profile uses the system resources judiciously and im-
proves the delay performance under realistic system constraints.

5. Conclusion

We have presented a methodology for designing optimal pump profiles for improving the delay performance of broadband SBS slow-light systems. For Gaussian pulses, the maximum fractional pulse delay of the optimal gain+absorption medium is 2.1 and 1.3 times the maximum fractional pulse delay of the GNPB and the optimal gain-only media, respectively. These results demonstrate the value of optimization to improve the delay performance under various resource constraints. Delay, gain, and total pump power vs. bandwidth plots show similar trends and delay improvement for the Gaussian and super-Gaussian pulse spectra demonstrating the usefulness of this approach for different pulse spectra.

6. Acknowledgement

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