Fast Causal Information Transmission in a Medium With a Slow Group Velocity

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It is widely believed that the velocity of information, \( v_i \), encoded on an optical pulse is equal to the group velocity, \( v_g \), at least when \( v_g \) is less than the speed of light in vacuum, \( c \). On the other hand, several authors suggest that \( v_i = c \), although the size of the signal traveling at this velocity may be small, thereby making it difficult to measure. Here, we measure \( v_i \) for pulses propagating through a resonant “slow-light” medium where \( v_i = 0.006c \). We find \( v_i = 1.03c^{0.595} \), or that \( v_i \approx 168v_g \), clearly demonstrating that the speed of information cannot be generally described by \( v_g \), but is characterized by its own velocity.

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Brillouin also made a surprising prediction: He found that \( v_g = c \), when \( v_g < c \). One might think that the group velocity, defined for a narrow bandwidth pulse, would not be a useful concept for a pulse containing a step discontinuity in the field amplitude. Despite this apparent contradiction, his prediction seems to hold for many cases of interest, leading to the belief that it is generally applicable [2–6].

Oughstun and Sherman [9] extended Brillouin’s work and find that the precursors can be much larger. For the case when \( \omega_0 \) is set to an atomic resonance, they predict that the precursors are the dominant part of the transmitted pulse, peaking immediately after the pulse front and hence one would expect that a straightforward measurement will reveal that \( v_i = c \).

Recently, Chiao and collaborators [10] proposed that information is contained only in points of nonanalyticity on electromagnetic waveforms, which travel at \( c \), and therefore that \( v_i = c \) [10,11]. An example of a nonanalytic point is a discontinuity in the waveform or one of its derivatives. Recently, Parker and Walker [12] suggest that the very act of encoding information on a waveform necessarily creates points of nonanalyticity. For these proposals to be practical, the precursors generated by the point of nonanalyticity must be large enough to be detected.

The primary purpose of this Letter is to measure \( v_i \) for pulses propagating in a resonant slow-light medium for which \( v_g \ll c \). We encode one bit of information on a pulse by making a rapid unpredictable change in the pulse amplitude. We estimate the velocity of this bit by measuring the moment when it is first possible to reliably detect the bit. By comparing the propagation of pulses through the slow-light medium and vacuum, we find that \( v_i \approx c \), demonstrating that resonant pulse propagation can give rise to results that are contrary to the conventional wisdom.

We prepare the slow-light medium using an amplifying resonance, realized by inducing large atomic coherence in a laser-driven potassium vapor as shown in Fig. 1(a). The potassium vapor is contained in an uncoated Pyrex cell of...
length $L = 19.2$ cm and heated to obtain an atomic number density of $6.5 \times 10^{13}$ atoms/cm$^3$. A linearly polarized coherence-preparation laser beam (frequency $\omega_d$ set to 1.91 GHz to the high-frequency side of the center of the potassium $4S_{1/2} \leftrightarrow 4P_{3/2}$ transition) is combined with the linear and orthogonally polarized information-carrying pulses. The coherence-preparation laser beam induces a Lorentzian-shaped amplifying resonance at frequency $\omega_R \approx \omega_d + \Delta_{hf}$ due to a stimulated hyperfine Raman scattering process [13], where $\Delta_{hf} = 2\pi(462 \text{ MHz})$ is the hyperfine splitting of the potassium $4S_{1/2}$ state. For future reference, we denote the linewidth of the resonance (half-width at half maximum) by $\gamma_R$.

Before attempting to measure $v_s$, we undertake a preliminary measurement using a Gaussian-like pulse with carrier frequency $\omega_0 \approx \omega_R$, as shown in Fig. 1(b) (filled circles). The pulse is turned on suddenly at a time $t = -1000$ ns (not shown) to a small amplitude that is well below the noise floor of our detection system. The pulse intensity then follows a Gaussian functional form ($1/e$ intensity half-width denoted by $\tau$), attaining its peak value at 0 ns, and is switched off suddenly at $t = 1000$ ns (also not shown). For a long pulse ($\gamma_R \tau \gg 1$), we find that the pulse distortion is reduced and the pulse delay time $t_{\text{del}}$ is maximized [14–17]. Unfortunately, such a long pulse also gives rise to a small value of the relative time delay $t_{\text{del}}/\tau$, a quantity we need to maximize in order to distinguish between the various velocities describing the propagating pulse. Therefore, we use a pulse width of the order of $\tau \sim 1/\gamma_R$, which we find maximizes $t_{\text{del}}/\tau$ but introduces a small amount of distortion [17].

Figure 1(b) shows the Gaussian-like pulse propagating through vacuum (filled circles) and through the slow-light medium (open circles). It is seen that the pulse is delayed by the slow-light effect and undergoes some broadening, as expected [14–17]. We do not observe any precursors related to the initial sudden turn on of the pulse at $t = -1000$ ns because their expected intensity is well below the noise floor of our detection system. A direct measurement of the waveforms reveals that the peak of the pulse propagating though the medium is delayed by 50.9 ns in comparison to the vacuum pulse. We find that the bandpass-coupled characteristic of the detector slightly shifts the peak of the pulses by an amount that is proportional to the pulse width. Correcting for this shift, we find that the peak of the pulse is delayed by $t_{\text{del}}^{\text{obs}} = 53$ ns. This experiment demonstrates that it is possible to delay a smooth pulse whose shape is chosen to suppress optical precursors; implications for information transfer and delay will be discussed below.

To obtain a quantitative understanding of our observations, we fit the waveforms to a model of the pulses and the slow-light medium. The vacuum waveform is modeled as a truncated Gaussian function passed through a bandpass filter to take into account the experimentally measured detector response. Figure 1(b) shows that the predicted vacuum waveform is in excellent agreement with the observed waveform using $\tau = 152.1$ ns. The slow-light waveform is fit by propagating the Gaussian pulse found above through a medium characterized by a linear susceptibility $\chi(\omega)$ and filtered by the detector response [18]. We find excellent agreement between the observed and predicted slow-light waveforms with a line-center intensity amplification coefficient $g_0 = 0.098 \text{ cm}^{-1}$ and $\gamma_R = 2\pi(1.41 \text{ MHz})$.

The narrow Raman resonance gives rise to laser beam amplification and large dispersion, as shown in Fig. 1(c). At the peak of the resonance, $n_g = 168$, resulting in $n_g/c = 5.95 \times 10^{-3}$. Such a slow group velocity will give a delay time of $t_{\text{del}} = L/v_g - L/c = 107.6$ ns for a very long pulse, which is considerably longer than $t_{\text{del}}^{\text{obs}}$ because the bandwidth of the Gaussian-like pulses used in our experiment is comparable to the bandwidth of the slow-light resonances, as discussed above and in Ref. [17]. Note that the wings of the slow-light feature are surrounded by broad regions of fast light, taking on its minimum value of $n_g \approx -18$ at $\Delta/2\pi \approx \pm 2.4$ MHz.

We now describe our measurement of the information velocity. In the experiment, we encode one bit of information on the Gaussian-shaped waveform using a fast optical telecommunication switch (OSW) driven by a step generator ($\sim 3$ ns rise time). A positive-going step is used to define one communication symbol (“1”), while a negative-going step is used to define another (“0”). The moment when the waveform goes high or low corresponds to a point of nonanalyticity [10–12]. We select these...
symbol waveforms because they allow us to smoothly turn on the pulse amplitude to a level above the noise floor of our detection electronics and to monitor the pulse delay.

Figure 2(a) shows the propagation of both symbols through the slow-light medium and through vacuum. Figure 2(b) shows an enlargement of the diagram in the vicinity of the transition between the symbols. From Fig. 2(a), it is seen that the slow-light medium delays the early part of the pulses during the smooth turn-on, identically to that observed for the full Gaussian-like pulses shown in Fig. 1(b). No information can yet be conveyed to a receiving party at the end of the communication channel because both symbols are the same for early times and hence they cannot be distinguished. From a simple visual inspection of the data, we see that the time when it is possible to first distinguish between the two symbols for the delayed pulses is nearly the same as the time when it is possible to first distinguish between the same two symbols propagating through vacuum. Hence, information arrives much faster than one might infer from \( v_g \), one primary conclusion of our experiment.

The arrival time of the information is determined by observing the pulses with a receiver that attempts to identify the incoming symbols as a function of time, with a certainty characterized by the bit error rate (BER). Before the arrival of this point of nonanalyticity at the detector, we expect no detected information, corresponding to BER = 1/2. Once the point of nonanalyticity (and the information it carries) propagates past the detector, the BER drops as the received information grows smoothly from zero. A symbol is considered detected when the BER falls below some threshold. Hence, the detection time of information is later than the time when information is first available at the detector by an amount we call the detection latency \( \Delta t \), even for pulses propagating through vacuum. Although it is possible to estimate \( \Delta t \) for a specific experimental apparatus, it cannot be measured directly because it requires measuring the point of nonanalyticity. In Fig. 2(b), we see that the manner in which the average symbol waveforms separate for the vacuum and delayed case are only slightly different so that the detection latency times (denoted by \( \Delta t_{\text{vac}} \) and \( \Delta t_{\text{del}} \), respectively) should be similar. Because our analysis makes no assumption about the sources of noise in the communication system, our general experimental approach and conclusions should hold even in the limit where quantum fluctuations are dominant [20].

To quantify our results, we determine the BER for the vacuum (Fig. 3, filled circles) and delayed (open circles) pulse pairs using an integrate-and-dump matched filter technique [8]. It is seen that the BER is high for final observation times in the range between 0 and 4 ns during which the pulse amplitudes are large but not yet distinct in the presence of noise [see Fig. 2(b)]. For longer observation times, the BER drops rapidly. In the BER range between 0.1 and 0.01, chosen to keep \( \Delta t_{\text{vac}} \) and \( \Delta t_{\text{del}} \) small, we determine the detection time for vacuum (delayed) pulse pairs \( \Delta t_{\text{vac}} \) (\( \Delta t_{\text{del}} \)) and the difference in detection times \( T_i = T_{\text{del}} - T_{\text{vac}} \). The time difference is approximately constant for BER values over this range; its average is equal to 1.02 ± 0.1 ns. Based only on a direct measurement of \( T_i \), one cannot determine whether an observed difference between the detection times is due to changes in the detection latencies or differences in the information velocities for vacuum (\( v_{\text{vac}} \)) and the slow-light medium (\( v_{\text{del}} \)) as can be seen by considering the relation among these quantities, which is given by

\[
T_i = (L/v_{\text{del}} - L/v_{\text{vac}}) + (\Delta t_{\text{del}} - \Delta t_{\text{vac}}). \tag{1}
\]

To gain some insight about the importance of detection latency, we analyze a mathematical model, based on Maxwell’s equations, that describes approximately our experiment. We find that the detection latency depends on the combined effects of the finite detector bandwidth and the symbol transition times, and noise in the measurement. Removing either nonideal effect would eliminate the detection latency, which, of course, is impossible in a real experiment. Consistent with previous research [10], this model predicts that \( v_{\text{del}} = v_{\text{vac}} = c \) in the absence of noise, and hence \( T_i \) is completely deter-
FIG. 3. Detecting the arrival of new information. Shown is the experimentally measured BER as a function of the upper limit of the integration time for the vacuum (filled circles) and delayed (open circles) pulses. Theoretical predictions based on a model of the experimental apparatus and slow-light medium are shown as lines.

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[18] The susceptibility is taken as \( \chi(\omega) = \chi_R(\omega) + i \chi_i(\omega) \), where \( \chi_R(\omega) = \beta g \epsilon R / (\omega - i \gamma_R) \) characterizes the Raman amplification resonance, and \( \chi_i(\omega) = i \pi \alpha_0 \epsilon R c T_s [ f_1 w(z_1) + f_2 w(z_2) ] / \omega_a \) characterizes the nearby Doppler-broadened absorption resonance arising from the hyperfine-split 4S(1/2) → 4P(1/2) transition. Here, \( g_0 \) is the line-center intensity amplification coefficient, \( \Delta = \omega - \omega_R \) is the detuning from the resonance, \( \alpha_0 = 1840 \text{ cm}^{-1} \), \( \gamma_a = 2\pi \epsilon (3 \text{ MHz}) \), \( T_s = 305 \text{ ps} \), \( f_1 = 1/8 \), \( f_2 = 5/24 \), \( z_i = T_s [ \Delta + 2 \pi \epsilon (1.68 \text{ GHz}) + i \gamma_a ] \), \( z_2 = T_s [ \Delta + 2 \pi \epsilon (2.14 \text{ GHz}) + i \gamma_a ] \), and \( w(z) \) is the complex error function; see Eq. 7.1.2 of Ref. [19].