Experimental control of a chaotic point process using interspike intervals

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A physical point process generated by passing a continuous, deterministic, chaotic signal through an integrate-and-fire device is controlled using proportional feedback incorporating only the time intervals between events. This system is unique in that the mean time between events can be adjusted independent of the dynamics of the underlying chaotic system. It is found that the range of feedback parameters giving rise to control as a function of the mean firing time exhibits surprisingly complex structure, and control is not possible when the mean interspike interval is comparable to or larger than the underlying system memory time. [S1063-651X(98)00708-7]

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Many systems evolve such that long periods of inactivity are punctuated by brief, nearly identical bursts of activity. Typical examples of such systems include certain laser instabilities [1] or a spontaneously firing collection of neurons [2]. Such ''point processes'' may be characterized by the sequence of time intervals between events (interspike intervals, or ISI's) rather than a dynamical variable sampled at regular time intervals. In some instances, the ISI's fluctuate in a deterministically chaotic manner. For example, Witkowski *et al.* [3] have suggested that the interbeat intervals recorded from a fibrillating heart are chaotic. Since the occurrence of chaos often degrades the performance of devices or indicates disease, it is valuable from a clinical as well as a fundamental standpoint to investigate the implementation of chaos control [4] of point process generated by various mechanisms.

Recently, Carroll [5] studied experimentally a system that naturally produces pointlike events: a network of four coupled electronic circuits whose individual dynamics are governed by equations similar to the FitzHugh-Nagumo model of a neuron. He demonstrated that the dynamics of the network can be controlled using proportional feedback incorporating the ISI's where the mean ISI time (denoted by T^*) is set approximately by the inverse of the decay rate of a ''slow'' variable. In a separate investigation, Ding and Yang [6] demonstrated theoretically control of a point process generated by passing a continuous chaotic signal through a threshold-crossing device using a similar feedback protocol. In this case, T^* is set approximately by the characteristic time scale of the chaotic fluctuations of the underlying dynamical system.

In this article, we investigate experimentally the control of a point process generated by passing a continuous signal $s(t)=\hat{\mathbf{n}}^{\mathbf{T}}\mathbf{y}(t)$ from a chaotic electronic circuit through an integrate-and-fire device as shown schematically in Fig. 1(a), where $\mathbf{y}$ is the state vector of the circuit and $\hat{\mathbf{n}}$ is the measurement direction in phase space. The device generates a chaotic sequence of spikes when the value of an integral reaches a threshold Θ, determined recursively from

$$\int_{t_n}^{t_{n+1}}[s(t)+\phi]dt=\Theta, \tag{1}$$

where the time interval between spikes is given by $T_{n+1}=t_{n+1}-t_n$. The offset ϕ ensures that the argument of the integral is positive definite when the dynamical system is in the neighborhood of the desired stabilized state. The interspike intervals T_n constitute a point process, derived from the underlying system $s(t)$.

Our goal is to convert the chaotic sequence of ISI's to a periodic sequence by applying small perturbations to an accessible variable or parameter of the underlying system using a proportional feedback algorithm. We note that there exist two types of signals $s(t)$ giving rise to a periodic sequence of ISI's. One is a constant signal (denoted by s^*) giving rise to a period-1 sequence where $T^*=\Theta/(s^*+\phi)$. For this signal, the sequence of ISI's remains periodic even when the parameters of the underlying dynamical system or the integrate-and-fire device change slightly (e.g., from parameter drift). The other type of signal is a periodic wave form, generating a periodic sequence of ISI's only when Θ and ϕ are tuned precisely. Extremely small parameter changes will render this sequence quasiperiodic. Therefore, only the continuous signal, corresponding to a period-1 periodic sequence of events, can be observed (and hence stabilized) in an experimental setting.

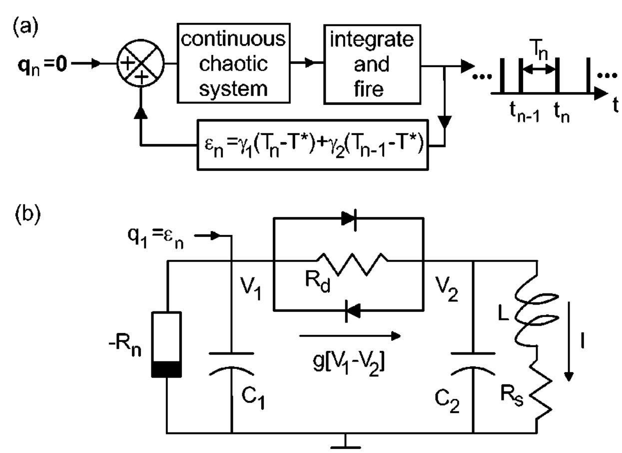

FIG. 1. (a) Scheme for controlling a chaotic series of interspike intervals using negative feedback where the intervals are generated by passing a signal from a continuous chaotic system through an integrate-and-fire device. (b) Chaotic electronic circuit.

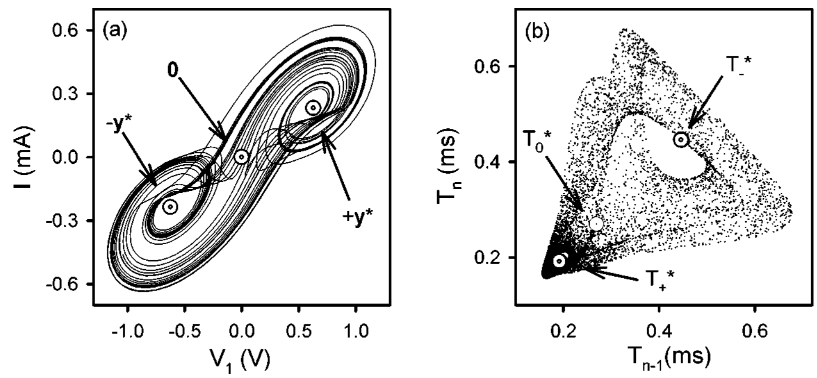

FIG. 2. (a) Projection in phase space of the chaotic attractor of the electronic circuit shown in Fig. 1(b). (b) Reconstruction of the chaotic attractor using the interspike intervals.

Our experimental system differs from those of Carroll [5] and Ding and Yang [6] in that T^* can be adjusted arbitrarily independent of the time scales characterizing the underlying chaotic system. We note that Sauer [7] has demonstrated that an ISI sequence such as that generated from Eq. (1) contains all information necessary to reconstruct the topology of the underlying chaotic system for an arbitrary setting of T^*. This suggests that control might be possible for some range of experimental parameters. On the other hand, Racicot and Longtin [8] recently observed that nonlinear forecastability of the system dynamics is lost when T^* is comparable to or larger than the characteristic memory time of the underlying system, where the memory time is given approximately by the inverse of the largest positive Lyapunov exponent. We provide experimental evidence that this loss of forecastability dramatically limits the ability to control the chaotic dynamics. In addition, we show that the range of feedback parameters for which control is effective as a function of T^* exhibits nontrivial structure.

The underlying chaotic system is an electronic circuit consisting of a negative resistor R_n and passive linear and nonlinear components connected as shown schematically in Fig. 1(b). The dynamics of this system are well described [9] by the set of dimensionless equations

$$dV_1/dt = V_1/R_n - g[V_1 - V_2] + q_1, \tag{2a}$$

$$dV_2/dt = C_1(g[V_1 - V_2] - I)/C_2 + q_2, \tag{2b}$$

$$dI/dt = V_2 - R_m I + q_3, \tag{2c}$$

where V_1 and V_2 are the voltage drops across capacitors $C_1 = 45$ nF and $C_2 = 45$ nF, respectively, and I is the current flowing through the inductor $L = 252$ mH. In Eq. (2) and in the following, all voltages are normalized to the diode voltage $V_d = 0.8$ V, all currents to $I_d = (V_d/R) = 0.34$ mA for $R = \sqrt{L/C_1} = 2.37$ kΩ, all resistances to R, and time to $\tau = \sqrt{LC_1} = 1.06$ ms. The current flowing through the parallel combination of the resistor and diodes (type 1N914B) is denoted by $g[V] = (V/R_d) + I_r[\exp(\alpha V) - \exp(-\alpha V)]$, where $R_d = 3.4$, $I_r = 1.64 \times 10^{-5}$ is the reverse current of the diodes, and $\alpha = 11.6$. The other circuit parameters are $R_m = R_L + R_s = 0.195$, where $R_L = 0.041$ is the dc resistance of the inductor, and $R_s = 0.154$ is a resistor placed in series with the inductor. The elements of the vector $\mathbf{q}_n = (q_1, q_2, q_3)^{\mathbf{T}}$ are the closed-loop feedback signals that attempt to stabilize the system about its fixed points. We denote the circuit state vector in phase space by $\mathbf{y} = (V_1, V_2, I)^{\mathbf{T}}$.

The circuit displays ''double scroll'' behavior as shown in Fig. 2(a) for $R_n = 1.07$ (kept fixed throughout this study) and $\mathbf{q}_n = \mathbf{0}$, where the unstable steady states at the center of the ''scrolls'' with coordinates $\pm \mathbf{y}^*$ and $\mathbf{0}$ are indicated. Our task of stabilizing a periodic sequence of ISI's corresponds to stabilizing the dynamics of the underlying system about one of these fixed points. The most unstable eigenvalues characterizing the fixed points $\pm \mathbf{y}^*$ are given by $\lambda_u^* = 0.95 \pm i5.88$ as determined experimentally by observing the dynamics of the system in a neighborhood of the fixed points. For future reference, the ''memory time'' corresponding to these states is given approximately by $\tau_m = 1/\mathrm{Re}(\lambda_u^*) = 1.05$.

Figure 2(b) shows an experimental reconstruction of the attractor from the ISI's with $\hat{\mathbf{n}} = (1,0,0)^{\mathbf{T}}$, $\phi = 1.84$, and $\Theta = 0.56$, where the three possible ISI's corresponding to the periodic sequences $T_\pm^*$ and T_0^* (corresponding to $\pm \mathbf{y}^*$ and $\mathbf{0}$ of the underlying system) are indicated. The ISI's are determined by measuring $V_1(t)$ with a high-impedance voltage follower, summing this voltage with an adjustable offset voltage ϕ, and feeding the combined signal to an analog electronic integrator whose value is monitored by a Schmidt trigger that fires when the threshold Θ is crossed, and then resets. An analog time-to-voltage converter is initialized by the firing of the Schmidt trigger. The value of this converter is sampled and held at a value proportional to T_n when the Schmidt trigger fires at the next threshold-crossing event, and is then reset. This process is repeated to determine the next ISI while the previous value T_{n-1} is transferred to auxiliary sample-and-hold device. It is seen that the attractor undergoes some deformation during the reconstruction, but general topological features appear to be preserved, consistent with the work of Sauer [7].

We control the sequence of ISI's by perturbing an accessible system parameter using a standard closed-loop feedback protocol given by

$$\varepsilon_n = \gamma_1(T_n - T^*) + \gamma_2(T_{n-1} - T^*), \tag{3}$$

where γ_j ($j = 1,2$) are gain parameters. Note that the perturbations vanish when the system is stabilized about the desired state $T_n = T_{n-1} = T^*$. For simplicity, we consider only adjustments to the current injected into the V_1 node of the

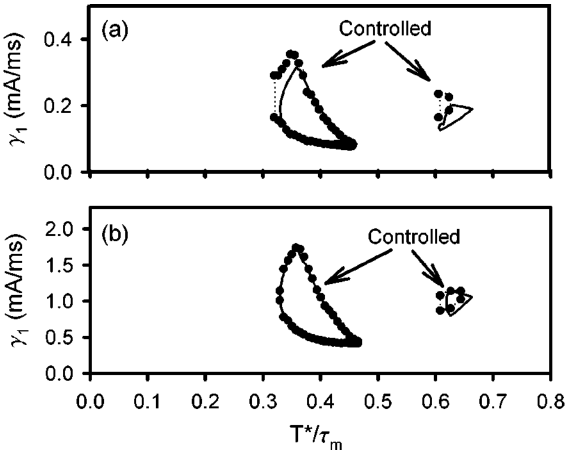

FIG. 3. Experimentally observed (filled circles) and theoretically predicted (solid lines) domains of control for (a) $\phi=-0.61$, and (b) $\phi=0.24$. The memory time of the unstable fixed point is given by $\tau_m=1.05$ (corresponding to 1.11 ms in physical units).

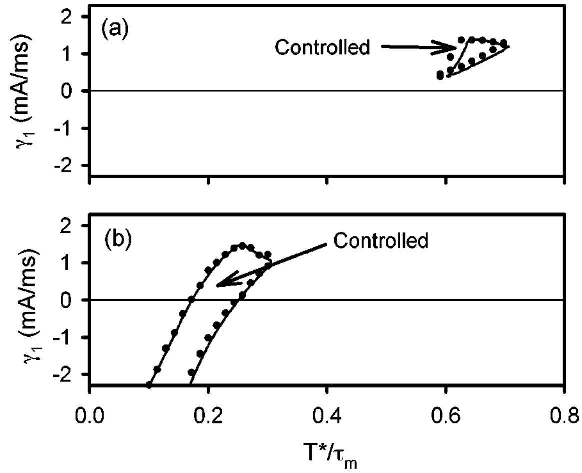

FIG. 4. Experimentally observed (filled circles) and theoretically predicted (solid lines) domains of control for (a) $\gamma_2=-0.26$ mA/ms, and (b) $\gamma_2=1.32$ mA/ms for $\phi=0.24$. Compare these figures to Fig. 3(b) with $\gamma_2=0$ and the same value of ϕ.

underlying electronic circuit so that $\mathbf{q}_n=(\varepsilon_n,0,0)^{\mathbf{T}}$. We note that a protocol similar to that desribed by Eq. (3) has been used to control the dynamics of biological systems such as the heart [4] and brain [10].

Our primary goal is to determine the range of values of T^* for which control is possible when the parameters of the underlying dynamical system remain fixed. This range can be visualized quickly by plotting the domain of values of the feedback parameter γ_1 that successfully stabilize the desired periodic sequence as a function of T^* (the ''domain of control'') for various values of the remaining feedback and integrate-and-fire parameters. For brevity, we only consider stabilization of the state $\mathbf{y}^*$ with $\hat{\mathbf{n}}=(1,0,0)^{\mathbf{T}}$ (corresponding to $s^*=0.79$). We have verified that our main conclusions described below are not affected by this choice.

Figure 3 shows the measured domain of control (solid dots) for two values of the offset ϕ with $\gamma_2=0$, where Θ is adjusted to obtain the desired value of T^*. It is seen that the shape of the domains stretches in the vertical dimension such that the gain required to maintain control increases with increasing ϕ. In addition, the domains are quite complex in that there exist multiple, isolated domains, although these may be simply connected when considering the full space of parameters spanned by γ_1, γ_2, ϕ, and Θ [11]. Of considerable interest is the observation that control is not possible beyond $T^*\sim0.64\tau_m$ and that this maximum value is rather insensitive to the value of the offset ϕ. Similar behavior is observed for other values of ϕ.

Figure 4 shows the effects of adding information from the previous ISI into the control protocol; the solid dots indicate the measured domain of control for various values of the gain parameter γ_2 with $\phi=0.24$. Inclusion of this information shifts the domain of control along both the T^* and γ_1 axes, in some cases limiting the range to very small values of T^*. However, in no case explored in the experiment have we observed control beyond $T^*\sim0.7\tau_m$ (only a slight increase beyond what is observed with $\gamma_2=0$).

It is possible to motivate theoretically some of our observations by considering the case of a 1D unstable system whose dynamics in the presence of closed-loop feedback is governed by

$$ds/dt=\lambda s+\varepsilon_n, \tag{4}$$

where the eigenvalue $\lambda=1/\tau_m$ is real and positive and ε_n is the feedback signal at firing time t_n. The goal of the feedback perturbations, given by Eq. (3), is to stabilize the system to its fixed point $s^*=0$ using only the ISI's generated by the protocol given in Eq. (1).

The stability analysis is simplified considerably by developing a map-based description of the system in the presence of feedback, which allows us to predict its evolution from firing time t_n to the next firing time t_{n+1}. Such a mapping can be obtained by integrating Eq. (1) over a single ISI, inserting this result into the integrate-and-fire protocol (1), and by considering only small deviations $\delta T_n=T_n-T^*$

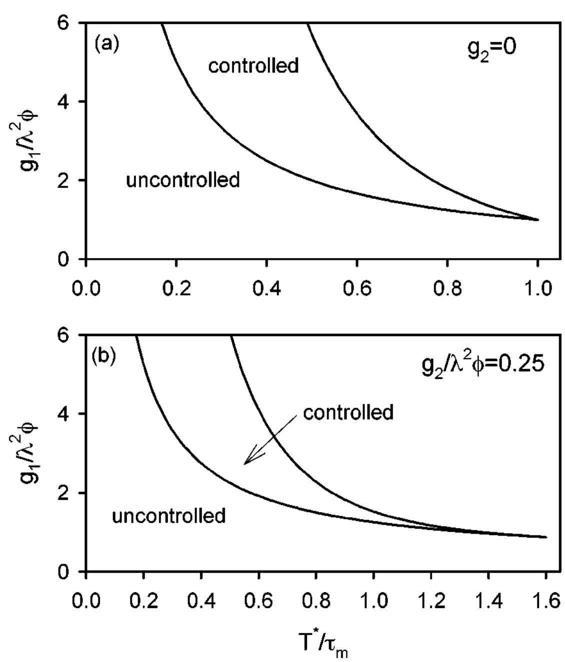

FIG. 5. Theoretically predicted domain of control from the simple 1D model for (a) $\gamma_2=0$, and (b) $\gamma_2/\lambda^2\phi=0.25$.

about the desired ISI. We find that the dynamics of the system is described approximately by the mapping

$$\begin{bmatrix} s_{n+1}/\phi \\ \delta T_{n+1}\lambda \\ \varepsilon_{n+1}/\lambda\phi \end{bmatrix} = \begin{bmatrix} \exp(\lambda T^*) & 0 & -c \\ c & 0 & c+\lambda T^* \\ cG_1 & G_2 & (c+\lambda T^*)G_1 \end{bmatrix} \times \begin{bmatrix} s_n/\phi \\ \delta T_n\lambda \\ \varepsilon_n/\lambda\phi \end{bmatrix}, \tag{5}$$

where $s_n = s(t_n)$, $G_j = \gamma_j/\lambda^2\phi$, and $c = 1-\exp(\lambda T^*)$. Equation (5) is valid under conditions when the system is in a neighborhood of the fixed point such that $s_n \ll \phi$, $\varepsilon_n \ll \lambda\phi$, and $\delta T_n \ll T^*$.

The periodic sequence of ISI's, and hence the fixed point of the underlying system, is stabilized by the feedback if and only if all the eigenvalues of the matrix in Eq. (5) have magnitude less than one. The domain (region) of control can be determined by direct computation of the eigenvalues or application of the Schur-Cohn stability criterion. It is interesting to note that the offset ϕ does not play a role in the stability of the system other than rescaling the gain parameters, consistent with our experimental observations.

Figure 5 shows the theoretically predicted domains of control for two values of the feedback gain parameters γ_2. For $\gamma_2 = 0$, we find that control is not possible beyond precisely $T^* = \tau_m$; for $\gamma_2 \neq 0$, control is not possible beyond $T^* \simeq 1.6\tau_m$, which occurs when $\gamma_2 \simeq 0.25$. This demonstrates that the mean firing time must be less than or on the order of the memory time of the system, consistent with our experimental observations in the higher-dimensional system. It is seen, however, that the domains shown in Fig. 5 exhibit structure that is simpler than that observed in the experiment. Indeed, the simple theoretical analysis gives only a qualitative guide to what we find experimentally.

Our analysis may be extended to higher dimensions to describe completely the dynamics of the circuit with closed-loop feedback in the neighborhood of the fixed point. In analogy to Eq. (4), consider an M-dimensional system whose dynamics is governed by

$$d\mathbf{y}/dt = \mathbf{F}(\mathbf{y}) + \mathbf{q}_n, \tag{6}$$

where $\mathbf{F}$ is the nonlinear flow, $\mathbf{q}_n = \hat{\mathbf{q}}\varepsilon_n$, and $\hat{\mathbf{q}}$ is the feedback direction. The goal of the feedback is to stabilize a periodic sequence of ISI's, and hence an unstable fixed point $\mathbf{y}^* = \mathbf{F}(\mathbf{y}^*)$ of the underlying continuous system. As before, the stability analysis is simplified by developing a mapping of the dynamics. Following a procedure similar to that described above and considering only small deviations $\mathbf{x}_n = \mathbf{y}_n - \mathbf{y}^*$ about the fixed point, we find that the dynamics is described approximately by the mapping

$$\begin{bmatrix} \mathbf{x}_{n+1}/\phi \\ \delta T_{n+1}\lambda^* \\ \varepsilon_{n+1}/\lambda^*\phi \end{bmatrix} = \begin{bmatrix} \exp(\mathbf{A}T^*) & \mathbf{0} & -(\mathbf{I}-e^{\mathbf{A}T_{n+1}})\mathbf{A}^{-1}\hat{\mathbf{q}} \\ \hat{\mathbf{n}}^{\mathbf{T}}\mathbf{A}^{-1}\lambda^*\mathbf{C} & 0 & \hat{\mathbf{n}}^{\mathbf{T}}(\mathbf{C}+\mathbf{A}^{-1}T^*)(\lambda^*\mathbf{A}^{-1})^2\hat{\mathbf{q}} \\ \hat{\mathbf{n}}^{\mathbf{T}}\lambda^*\mathbf{A}^{-1}\mathbf{C}G_1 & G_2 & \hat{\mathbf{n}}^{\mathbf{T}}(\mathbf{C}+\mathbf{A}^{-1}T^*)(\lambda^*\mathbf{A}^{-1})^2\hat{\mathbf{q}}G_1 \end{bmatrix} \begin{bmatrix} \mathbf{x}_n/\phi \\ \delta T_n\lambda^* \\ \varepsilon_n/\lambda^*\phi \end{bmatrix}, \tag{7}$$

where $\mathbf{A} \equiv \partial\mathbf{F}/\partial\mathbf{y}$ is the Jacobian of the nonlinear flow evaluated at the fixed point, $\mathbf{C} = \mathbf{I} - \exp(\mathbf{A}T^*)$, $\mathbf{I}$ is the $M \times M$ identity matrix, and λ^* is the real part of the largest eigenvalue of $\mathbf{A}$. For our experimental system, the Jacobian is given by

$$\mathbf{A} \equiv \begin{bmatrix} 1/R_n - g'[V_{1n}^* - V_{2n}^*] & g'[V_{1n}^* - V_{2n}^*] & 0 \\ (C_1/C_2)g'[V_{1n}^* - V_{2n}^*] & -(C_1/C_2)g'[V_{1n}^* - V_{2n}^*] & -(C_1/C_2) \\ 0 & 1 & -R_m \end{bmatrix}, \tag{8}$$

where $g' = \partial g/\partial V_1 = -\partial g/\partial V_2$, and $\hat{\mathbf{q}} = (1,0,0)^{\mathbf{T}}$.

As for the one-dimensional system, we determine the range of parameter values for which the system is stable, that is, when the eigenvalues of the matrix in Eq. (7) have magnitude less than one. The domains of control determined using this procedure are shown as solid lines in Figs. 3 and 4. We see that the experimental measurements agree well with the theoretical predictions. Both theory and experiment display a surprising level of complexity in the structure of the control domains, which, our results suggest, may not necessarily be simply connected for some parameter values.

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