Controlling lasers by use of extended time-delay autosynchronization

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A method is described for suppressing chaotic instabilities in lasers by use of a specific form of controlling-chaos feedback. The technique is easy to implement and requires only application of small perturbations to an accessible system parameter or variable. © 1998 Optical Society of America

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The performance of optical devices incorporating lasers can be degraded when the intensity or the frequency of the beam generated by the laser fluctuates in an erratic manner as a result of temporal instabilities or chaos. Chaos is a particularly insidious source of erratic dynamics since it cannot be suppressed by use of better shielding from the environment. One solution to this problem is redesign the entire system to avoid chaos, but this may prove impractical or costly. Another solution is to suppress the instabilities by application of small perturbations to an accessible system variable or parameter, such as the pump rate, in a form prescribed by chaos-control theory. This technique takes advantage of the unstable states embedded in the attractor and is often easy to implement, and the control parameters can be determined straightforwardly from experimental observations without a detailed model of the system. Such methods were used successfully to control the dynamics of lasers both in theoretical investigations and in experiments.

Recently, Chang et al. described a continuous-feedback scheme for efficiently stabilizing unstable steady states (USS’s) of nonlinear dynamic systems based on a specific form of a controlling-chaos scheme known as extended time-delay autorschization. Stabilizing the USS’s (the cw state) of lasers is often desirable for applications because of the high degree of coherence of such states. The control protocol attempts to stabilize an USS by making small adjustments $\epsilon(t)$ to an accessible system parameter $q$ (nominal value $\bar{q}$) when the system is in a neighborhood of the state. The protocol uses a feedback loop in which the error signal is determined through the relation

$$\frac{d\epsilon}{dt} = -\omega_0\epsilon + \gamma d\xi/dt,$$

where $\xi(t) = \mathbf{n} \cdot \mathbf{z}(t)$ is the dynamical state of the system, $\mathbf{z}(t)$ is the system state vector, $\mathbf{n}$ is the measurement direction in phase space, $\gamma$ is the feedback gain parameter, and $\omega_0$ is the feedback cutoff frequency. Note that the form of Eq. (1) is identical to a single-pole high-pass filter, which is familiar from elementary electronics. The scheme is easy to implement, does not require knowledge of the coordinate $\mathbf{z}$ in phase space of the desired USS, and is stable against the well-known high-frequency instabilities that plague derivative feedback. In addition, $\epsilon(t)$ vanishes when the system is in the USS, since $d\xi(t)/dt = 0$ in this case.

The primary purpose of this Letter is to point out that feedback of the form prescribed by Eq. (1) may be ideally suited for stabilizing the USS’s of lasers, especially when the lasers fluctuate on a fast time scale. For an idealized laser model, it is found theoretically that there exists a wide range of feedback parameters giving rise to stable behavior (known as the domain of control) and that the controller can automatically track slow variation or drift of the laser parameters. Although it is well known that this idealized laser model does not describe quantitatively the behavior of typical lasers, these observations highlight the potential of the feedback schemes. Detailed studies of specific laser systems must be undertaken to ascertain whether the controlling-chaos techniques presented here will be useful in real-world applications.

For simplicity, consider a single-mode, homogeneously broadened, resonant, two-level laser whose dynamics is governed by the following set of dimensionless equations:

$$dE/dt = -\kappa(E - v + E_{\text{inj}}),$$

$$dv/dt = -(v - Ew),$$

$$dw/dt = -\gamma_l(w + Ev - w_p),$$

where $E$ is the laser field strength inside the cavity, $v(w)$ is the atomic polarization (population inversion), $\kappa(\gamma_l)$ is the cavity (atomic-inversion) decay rate, $w_p$ is the inversion that is due to the pumping process in the absence of a field, and $E_{\text{inj}}$ accounts for the possibility of injecting a field into the cavity. In Eqs. (2), the field strength is normalized to its saturation value, the inversion to its value at the first laser threshold, the polarization to its value for a field at the saturation strength, and time to the inverse of the polarization decay rate. The state vector in phase space is denoted by $\mathbf{z} = [E, v, w]^T$.

For low pump rates ($w_p < 1$) the laser is below threshold and resides on the stable steady state $\mathbf{z}^0 = [0, 0, w^0_p]^T$. At $w_p = 1$, the state $\mathbf{z}^0$ becomes unstable through a pitchfork bifurcation (the first laser...
For a bad cavity law detector, where \( T_m \) beam generated by the laser and sensed by a square-law detector, the largest eigenvalue of the desired USS is comparable with the inverse of the real part of the latency must be taken into account when it is adjustment of the pump rate (control-loop latency); the time between the sensing of the state of the laser and the pump rate in the presence of feedback straightforwardly by linearizing the linear dynamics of the laser can be determined directly from experimental measurements. It is found that stabilization of the USS’s is not overly sensitive to the parameter \( \omega_0 \) so long as \( \omega_0 \) is much smaller than the characteristic frequency of the chaotic fluctuations.

In Fig. 2, the precise domain (between the solid curves) for \( \kappa = 4, \gamma_i = 0.5, \omega_0 = 0.1, \) and \( E_{inj} = 0 \) is shown in comparison with the approximate domain (between the dashed curves). It can be seen that control is possible for all \( w_p \) and that the results are in close agreement.

This analysis is applicable only for the case in which the trajectory of the chaotic system is in a neighborhood of the USS, whereas the chaotic trajectory never visits this neighborhood, as seen in Fig. 1. Direct numerical integration of Eqs. (2) and (3) indicates that the states are globally stable in the presence of feedback. However, the size of the transient control perturbations can often attain unphysical values. A method for circumventing this problem is suggested by Fig. 2(b).

A careful inspection of the figure reveals that the domain of control encompasses a horizontal band in the approximate range \( 0.9 > \gamma_i T_m > 0.3 \), implying that the states can be controlled for all \( w_p \) without adjustment of \( \gamma_i \). Thus the control loop can automatically track slow changes or drift in the pump rate so long as the loop can adiabatically follow these changes (i.e., slow in comparison with the response time of the loop, \( \sim \omega_0^{-1} \)). The procedure for stabilizing the USS by use of only small perturbations is to turn on control when the pump rate is low so that the USS’s \( z_{ss}^\pm \) are stable in...
the absence of control, set $\gamma$, in the range 0.3–0.9, and then slowly adjust $w_p$ to the desired value.

In the second feedback scheme, called coherent control, the control perturbation is an optical field $E_{\text{inj}}$ injected into the laser whose nominal value is zero. Filtering a fraction of the optical field emitted by the laser with a short, high-finesse Fabry–Perot interferometer generates coherent control all optically, as shown schematically in Fig. 3(a). When the interferometer is adjusted so that one of its longitudinal modes coincides with the optical carrier frequency of the laser and its free spectral range is much larger than the spectral content of the chaotic fluctuations, the injected field is governed approximately by

$$dE_{\text{inj}}/dt = -\omega_0 E_{\text{inj}} + \gamma_c \sqrt{T_m} dE/dt,$$

where an optical attenuator or amplifier in the beam path adjusts the feedback parameter $\gamma_c$ and the cavity length and the finesse adjust the cutoff frequency $\omega_0$. Note that $E_{\text{inj}}$ vanishes when control is successful, which distinguishes the controlling-chaos technique from other methods of frequency locking and narrowing of lasers by use of a field reflected from a Fabry–Perot interferometer.8

The stability of the USS's in the presence of coherent feedback can be determined with the methods outlined in the above discussion on incoherent control. The exact domain of control is shown in Fig. 3(b) for $\kappa = 4$, $\gamma = 0.5$, $\omega_0 = 0.1$, and $\epsilon = 0$. For a narrow filter, the domain for either state $z^+$ or $z^-$ is given approximately by

$$\gamma_c > \frac{1}{2\kappa} \left( \kappa(2\gamma + 1)/(\gamma + 1) + \gamma + 1 \right),$$

$$\{4\gamma \omega_p [\kappa/(\gamma + 1) - 1] + (\gamma + 1)^2 - \kappa(6\gamma^2 + 4\gamma - \kappa - 2)/(\gamma + 1)^{3/2} \}. \quad (6)$$

The approximate result is indistinguishable from the exact result shown in Fig. 3. As with incoherent control, it can be seen that stabilization of the USS is effective for arbitrarily large pump rates. Also, the procedure for stabilization of a USS by use of only small perturbations is to turn on control when the pump rate is low so that the USS's $z^+$ are stable in the absence of control, set $\gamma_c$ to a large enough value (determined from Fig. 3(b)], and then slowly adjust $w_p$ to the desired value.

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