Time-delay signatures in broadband chaos generated by optoelectronic oscillators

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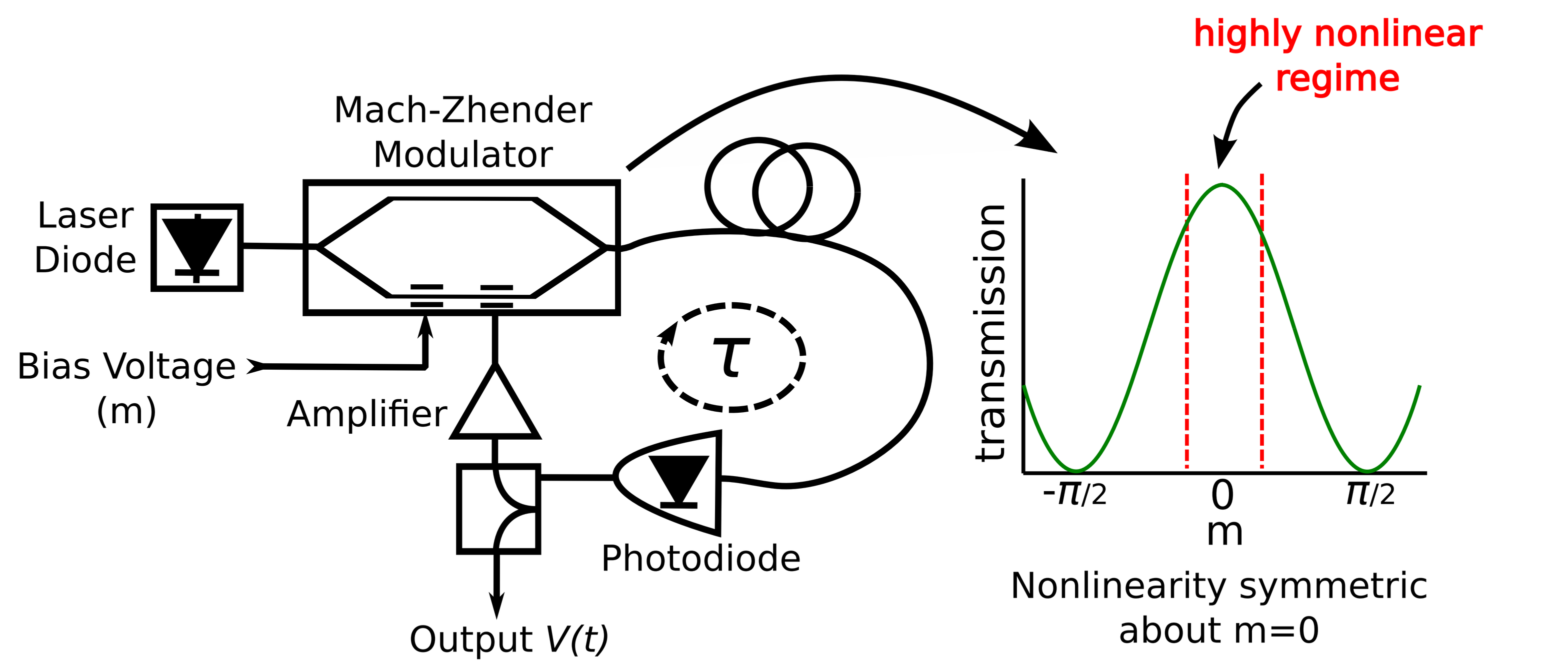
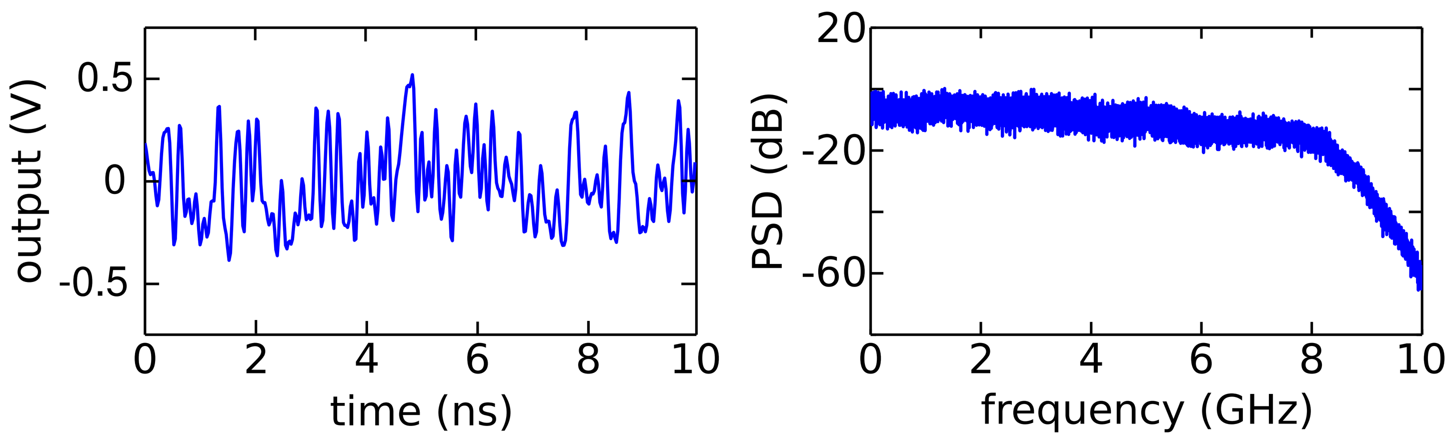
Revolutionary Research . . . Relevant Results

Introduction

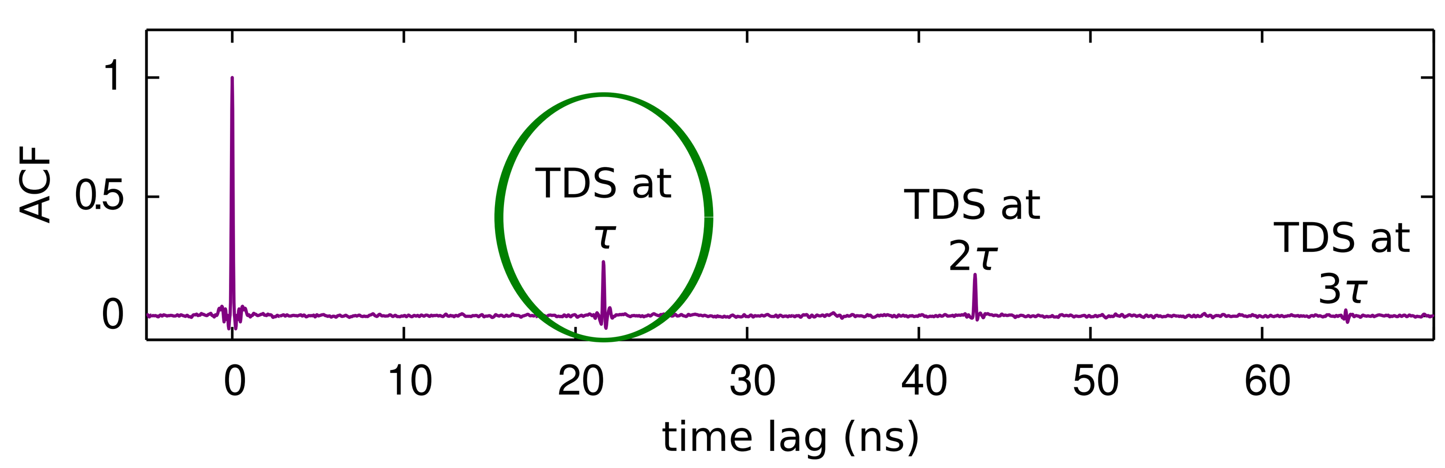
Optoelectronic oscillators (OEOs) use nonlinear time-delayed feedback to produce high-speed and broadband chaos when operated in the highly nonlinear regime [1,2]. By this, we mean that the linear contribution to the nonlinearity is small or vanishes completely. While the nonlinearity itself is symmetric, we observe an interesting dynamical symmetry breaking in a single oscillator and a partial restoration of the symmetry when two nonidentical oscillators are coupled.



Typical experimental chaotic time series and power spectrum for m=0



Despite the complexity of the dynamics, time-delay signatures (TDS) can easily be identified with simple methods, such as the normalized autocorrelation function (ACF).



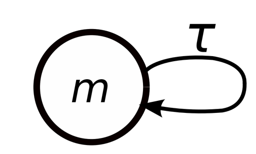
We find that:

☆ the operating point of the nonlinearity (m) strongly influences the peak size of the TDS of a single OEO
  • gives rise to an asymmetric relationship

☆ in a network of two mutually delay-coupled OEOs both the operating point (m) and coupling strength (c) determine the size of the TDS
  • partially restores the symmetry

☆ the size of the TDS is useful for sensing changes in the coupling strength in a small network of OEOs

Single OEO



Delay-differential equations used to model dynamics:

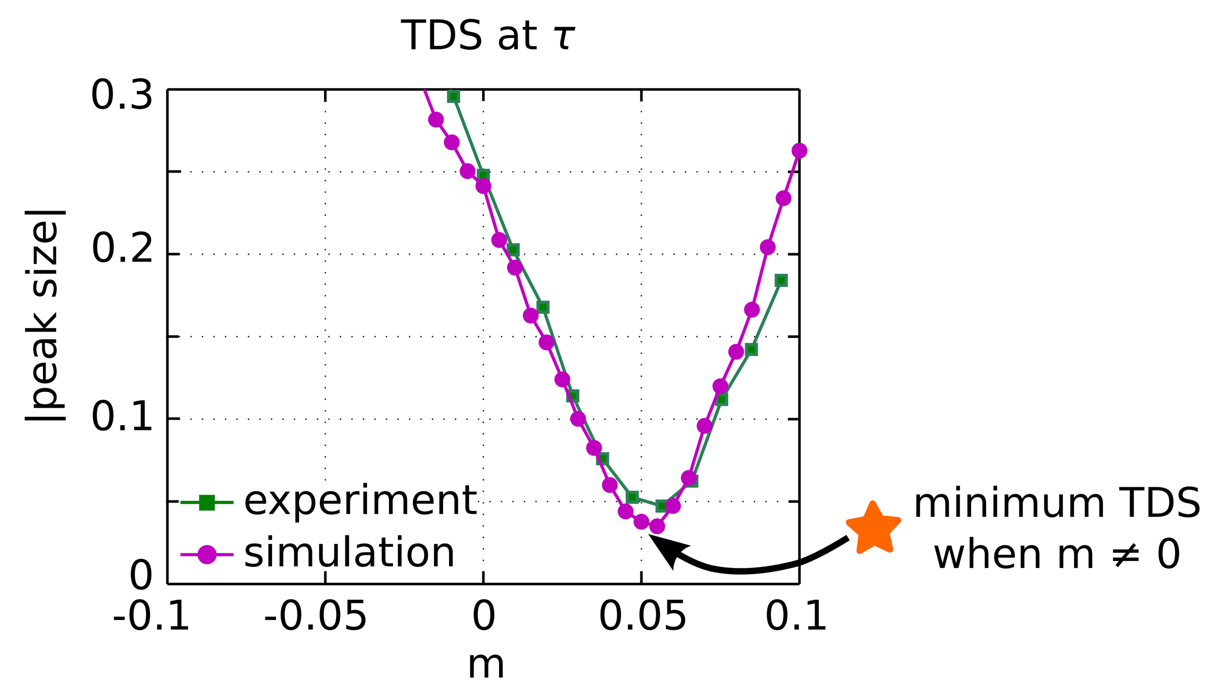
$$\dot{V}(t) = \Delta\{-V(t) - U(t) + F[V(t-\tau)]\}$$

$$\dot{U}(t) = \Delta\epsilon V(t)$$

Nonlinear feedback term:

$$F[V] = (\gamma g/d)\{\cos^2[m + d\tanh(V/g)] - \cos^2(m)\}$$

Properties of TDS in a single OEO:
- simple, asymmetric relationship with operating point (m)
  ⇒ symmetry breaking
- can be minimized, but not completely eliminated



A plot of the (simulated) trajectories for different values of m in phase space relative to the periodic nullclines [3] helps explain the value of m where this minimization occurs.

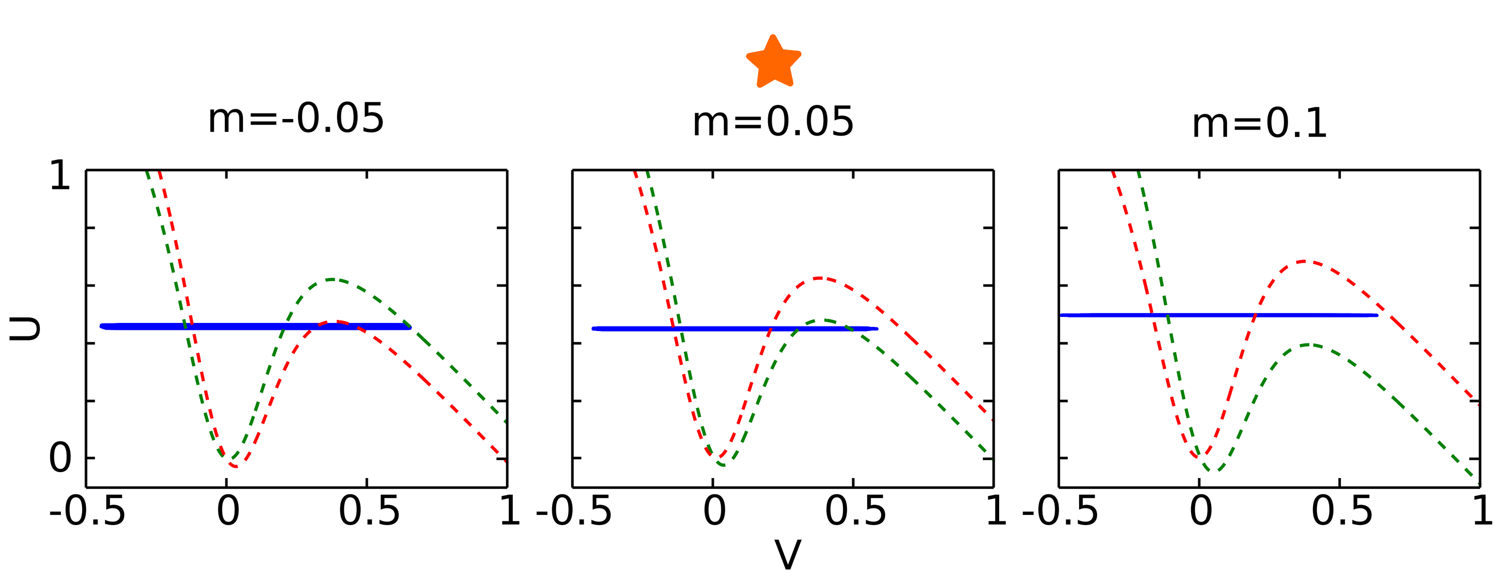
To find nullcline:

$$\dot{V} = 0 \quad \Rightarrow \quad U(t) = -V(t) + F[V(t-\tau)]$$

two choices: $V(t-\tau) = +V(t)$ (period = τ)
$V(t-\tau) = -V(t)$ (period = 2τ)

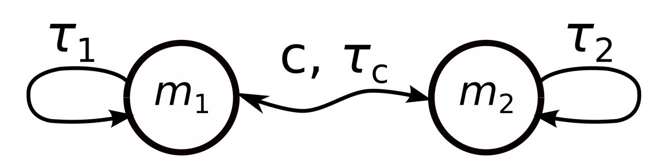
Equations for periodic nullclines:

$$U_+ = -V + F[+V] \qquad U_- = -V + F[-V]$$



The TDS is a minimum when the maximum of V is in between the two periodic nullclines.

Two Coupled OEOs



Coupled delay-differential equations:

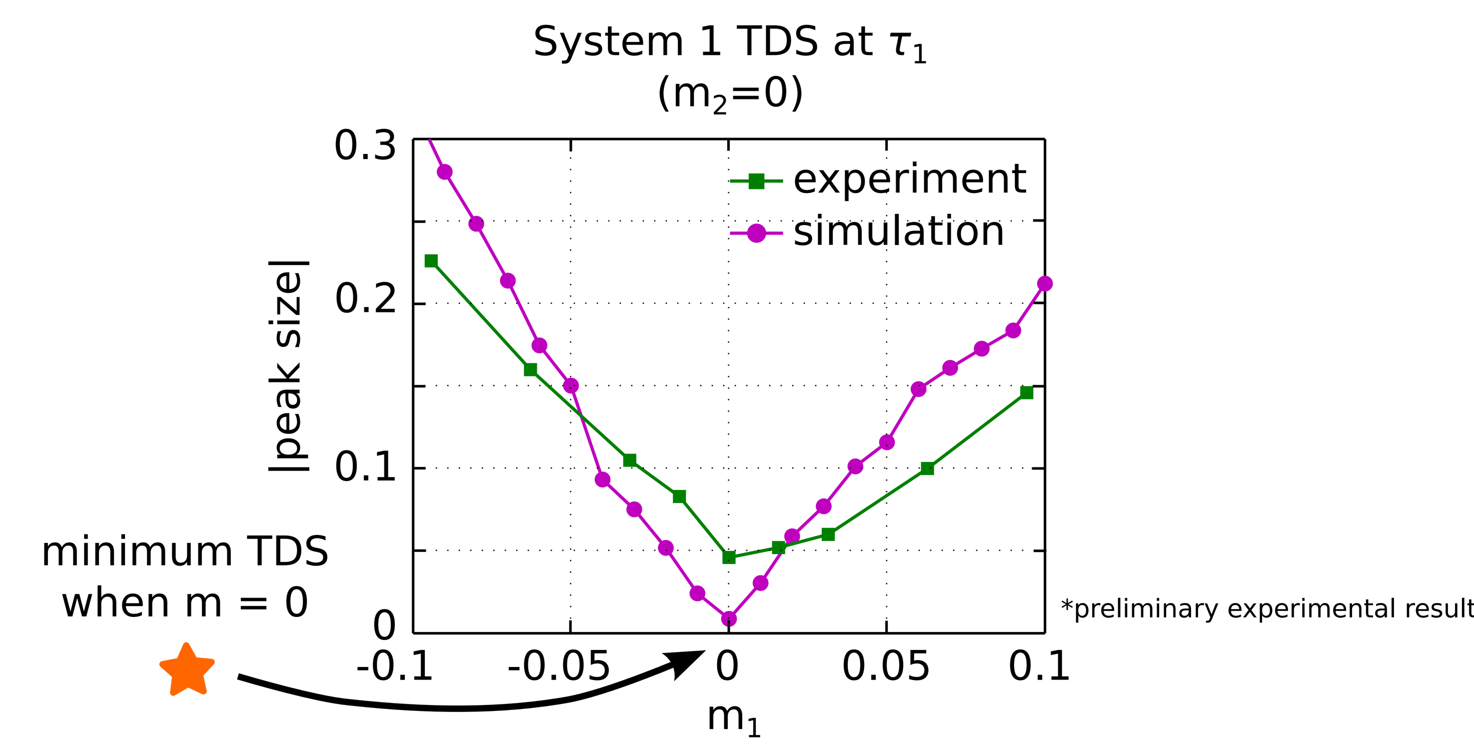
$$\dot{V}_1(t) = \Delta\{-V_1(t) - U_1(t) + F[V_1(t-\tau_1) + cV_2(t-\tau_c)]\}$$

$$\dot{U}_1(t) = \Delta\epsilon V_1(t)$$

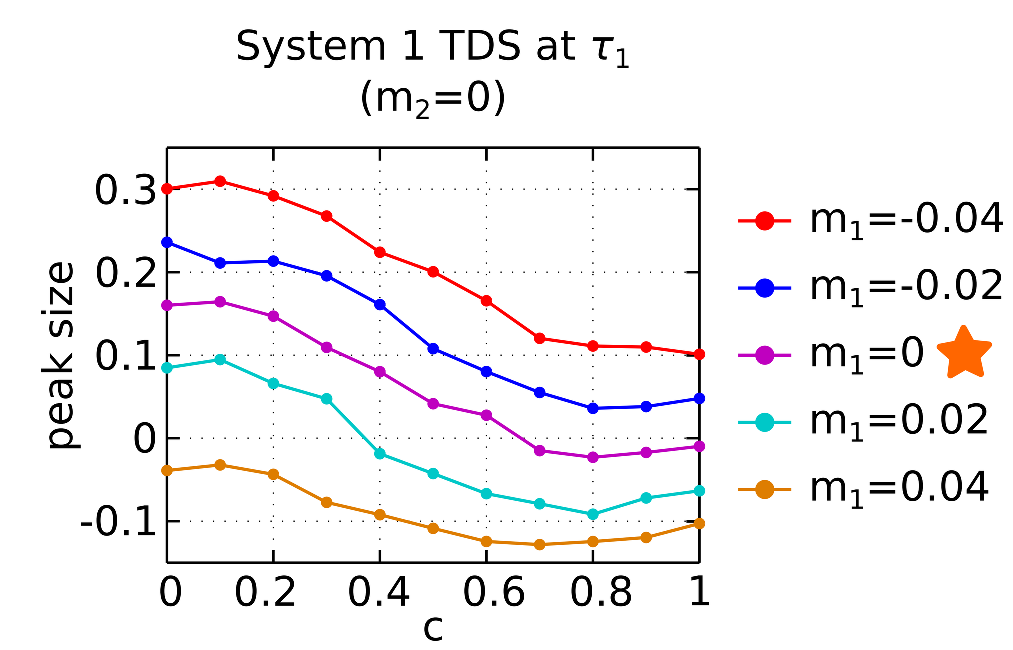
$$\dot{V}_2(t) = \Delta\{-V_2(t) - U_2(t) + F[V_2(t-\tau_2) + cV_1(t-\tau_c)]\}$$

$$\dot{U}_2(t) = \Delta\epsilon V_2(t)$$

For $\tau_1 \neq \tau_2 \neq \tau_C$ and c=1:
- dynamics are unsynchronized
- coupling nonidentical OEOs restores symmetry in the location of the minimum TDS



Simulations show that the size of the TDS also depends on the coupling strength.



By monitoring the dynamics and parameters of only one node, the coupling strength between the nodes can be infered from the TDS.

Summary:
- Symmetric nonlinearity gives rise to asymmetric TDS with respect to m in single OEO
- Bidirectional coupling of two nonidentical OEOs partially restores the symmetry in TDS
- Knowledge of how TDS behave allows us to extract network parameters from limited measurements

References
[1] Callan *et al.*, *Phys. Rev. Lett.* **104**, 1113901 (2010)
[2] Larger & Dudley, *Nature* **465**, 41 (2010)
[3] Rosin *et al.*, *Europhys. Lett.* **96**, 34001 (2011)

The authors gratefully acknowledge the support of the ONR MURI "Exploiting Nonlinear Dynamics for Novel Sensor Networks."

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