A TECHNIQUE FOR DIRECTLY MEASURING THE GRAVITATIONAL ACCELERATION OF ANTIHYDROGEN

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ABSTRACT

The gravitational force on antimatter has never been directly measured. A method is suggested for measuring the acceleration of antimatter ($\overline{g}$) by measuring the deflection of a beam of neutral antihydrogen atoms in the Earth's gravitational field. While a simple position measurement of the beam could be used, a more efficient measurement can be made using a transmission interferometer. A 1% measurement of $\overline{g}$ should be possible from a beam of about 100,000 atoms, with the ultimate accuracy being determined largely by the number of antihydrogen atoms that can be produced. A method is suggested for producing an antihydrogen beam appropriate for this experiment.

1. Introduction

1.1. Motivation

There has never been a direct measurement of the gravitational acceleration of antimatter. Even the Witteborn and Fairbanks[1] experiment only measured the gravitational force on electrons and not positrons. So while most physicists expect that antimatter will fall towards the earth with the same acceleration as matter, we do not *know* that it will. There are even some recent experimental results that suggest that the dark matter problem is really a problem with gravity,[2] and if this is true, then the gravitational acceleration of antimatter will not be the same as matter. Furthermore, it is likely that a theory could be found that would be able to explain any experimental result. It is simply a question of how many cherished assumptions would need to be thrown out in the process. Of course, even a result exactly as expected would be important, and would provide an important constraint on theoretical attempts to unify gravity with the rest of the Standard Model.

1.2. History

The ability to make a direct measurement of the gravitational acceleration of antimatter hinges on the availability of slow-moving antimatter. Until recently, what little antimatter has been available on this planet has been relativistic. In fact, it was the lack of an appropriate source of positrons that prevented Witteborn and Fairbanks[1] from measuring the gravitational force on positrons. However, the development of electromagnetic trapping techniques has made it possible to hold and cool particles (charged or neutral)[3], and the construction of the Low Energy Antiproton Ring (LEAR) at CERN has made it possible to trap substantial numbers of antiprotons[4]. This has led to an experiment which will attempt to measure the gravitational acceleration of antiprotons[5]. However, such a measurement is extremely difficult[6], since electromagnetic forces on the charged antiproton can easily overpower the force of gravity on the particle. Nevertheless, by using H^- ions as a control, the experimenters hope to eventually make a 1% measurement of the gravitational acceleration of antiprotons in the earth's gravitational field ($\overline{g}$).

While a measurement of the gravitational acceleration of antiprotons may be possible, a measurement made with neutral antimatter would be free of the systematic problems which plague a measurement made with charged particles. A number of ideas have been presented for ways to measure the earth's gravitational force on antihydrogen[7,8,9]. However, most of these methods require the ability to capture a substantial number of antihydrogen atoms in traps. Since producing antihydrogen and then capturing it in traps is almost certainly much more difficult than producing antihydrogen without catching it, a neutral antimatter gravity experiment that does not rely on trapping neutral atoms should be easier to perform. Some designs for this kind of experiment are discussed below.

2. Measuring the Deflection of a Beam

The experiment suggested here is to measure the gravitational deflection of a "beam" of neutral antihydrogen atoms. The word "beam" is in quotes because the intensity is unimportant; it is not necessary to ever have more than one $\overline{H}$ at a time. However, the atoms would be produced with a velocity component in a particular direction which is significantly larger than the thermal velocity of the $\overline{H}$, thus producing a "beam". Since the antihydrogen beam is crucial to this experiment, and since it is unlikely that sufficient quantities of antihydrogen could be produced to make a beam by collimating an isotropic source, we will briefly discuss some methods which might be used to produce the $\overline{H}$ beam at the end of this paper.

Perhaps the most obvious method to measure $\overline{g}$ using the antihydrogen beam would be to collimate the beam horizontally and measure its position after it had propagated a sufficient distance in a drift tube. A schematic of such an experiment is shown in Figure 1. Two methods are shown for measuring the position of the beam: a complex detector for measuring the precise position of each antihydrogen, and a much simpler method which only measures the asymmetry. In the first method, the position of each antihydrogen atom is measured at the end of a drift tube, either by measuring the trajectories of the charged particles produced by the $\overline{p}p$ annihilation when the $\overline{H}$ strikes the target, or by measuring the impact position directly, perhaps by detecting the nuclear recoil from the annihilation.

While a complicated detector would be able to give an accurate measurement of the impact position of each antihydrogen atom, its many electronic channels are unnecessary when the measurement could be made with only two channels. This could be done by covering half of the drift tube (either the upper half or the lower half) with a target, and measuring the up/down asymmetry of the beam by counting the annihilations at this target and the annihilations at a target covering the entire drift tube further downstream. Note that the targets need not be active. External counters could be used to detect the charged particles coming from the annihilations.

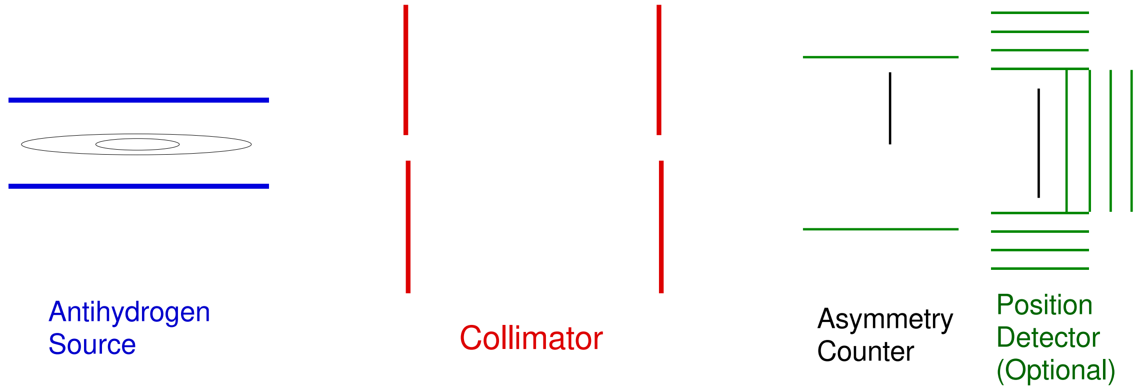

Fig. 1. Schematic of an experiment to measure $\overline{g}$ by directly measuring the deflection of a neutral antihydrogen beam. Two methods are shown for measuring the beam's position: a high precision vertex detector; and a simple target covering half of the drift tube. Scintillation counters can be used to count the annihilations in this target and at the end of the drift tube. Not drawn to scale.

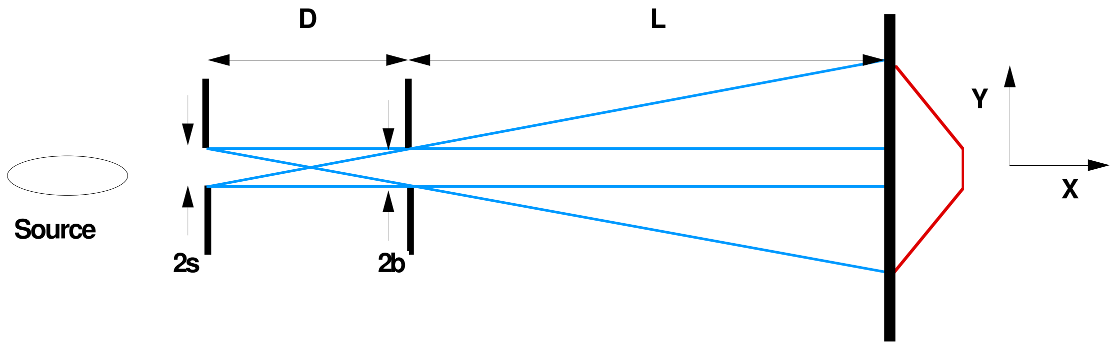

Fig. 2. Parameters characterizing a collimated beam. Not to scale.

The asymmetry α of a beam (for small deflections Δy) is given by the formula

$$\alpha \approx \frac{\Delta y}{b} \frac{D}{D+L}$$

where $2s$ and $2b$ are the slit widths for the first and second slits of the collimator, D is the separation of the two slits, and L is the drift distance to the asymmetry measurement. These dimensions are shown in Figure 2. The condition of small deflections is that $\Delta y < b + (b - s)\frac{L}{D}$. The acceleration of the beam between the collimator slits has been neglected in this approximation.

It is interesting to look at a concrete example to see how well this method might work. For the case where $b = s = 2$ mm, the slits are separated by 1 meter, and the asymmetry is measured 10 meters past the collimator ($D = 1$ m and $L = 10$ m), we find a beam of particles travelling at $v_x = 500$ m/s would drop about 2 mm if $\overline{g} = 10$ m/s^2, and the asymmetry would be almost 10%.

The accuracy to which $\overline{g}$ can be measured is given by:

$$\frac{\sigma(\overline{g})}{\overline{g}} = \sqrt{\left(\frac{\sigma(\alpha)}{\alpha}\right)^2 + \left(\frac{2\sigma(v_x)}{v_x}\right)^2}$$

where we have assumed that the dimensions of the apparatus are well known. The uncertainty on α is statistical:

$$\sigma(\alpha) = \sqrt{\frac{4ud}{N^3}}$$

where u and d are the number of particles that are detected in the upper and lower half, respectively, and $N = u + d$. The uncertainty in the velocity $\sigma(v_x)$ is difficult to estimate without knowing more details about how the antihydrogen is produced. A spread in the velocity is not a problem so long as the velocity distribution is known. However, for the sake of this analysis, we can characterize the uncertainty in the velocity with a temperature T_u:

$$\sigma(v_x) = \sqrt{\frac{kT_u}{m}}$$

where k is Boltzmann's constant and m is the antihydrogen mass. We can use this to calculate $\sigma(\overline{g})$ in the previous example: for $N = 1000$ and an uncertainty in v_x of $T_u = 1°$K, $\sigma(\overline{g})/\overline{g} = 0.5$, so $\overline{g}$ could be distinguished from $-g$. For $N = 10^4$ and $T_u = 0.1°$ K, the uncertainty in $\overline{g}$ would be about 16%.

The number N in the above examples are the total number of antihydrogen that make it through the collimator. Since the antihydrogen is produced with a finite transverse velocity distribution, only a fraction of the the total antihydrogen produced will pass through both slits. If the transverse velocity is characterized by a temperature of 1° K, then only 0.9% of a 500 m/s beam will pass through a $\pm$2mm slit at 1 meter, and only half of this will pass through a second slit 1 meter past the first. At $T = 0.1°$ K, the fraction is 1.4% through both slits. If we assume that an additional half of the beam would be lost to horizontal collimation, then a beam of about 1.5×10^6 antihydrogens is needed at 0.1° K to get 10^4 through the collimation, or 4.4×10^5 at 1° K to get 1000 through.

3. Measuring $\overline{g}$ with an Interferometer

While measuring the deflection of a collimated beam of antihydrogen atoms may be one of the most direct ways to measure the gravitational acceleration of antimatter, a more efficient measurement can be made using an interferometer. The concept is to set up an interference pattern with a pair of diffraction gratings, and to measure the phase position of the interference pattern with a third grating. The phase shift caused by gravitation can be measured either by rotating the apparatus with respect to gravity, by changing the length of the interferometer, or by comparing the phase shifts for beams of different velocities.

Perhaps the ideal interferometer for this experiment is a configuration that has been used for both neutron and atom interferometers[10,11]. This interferometer consists

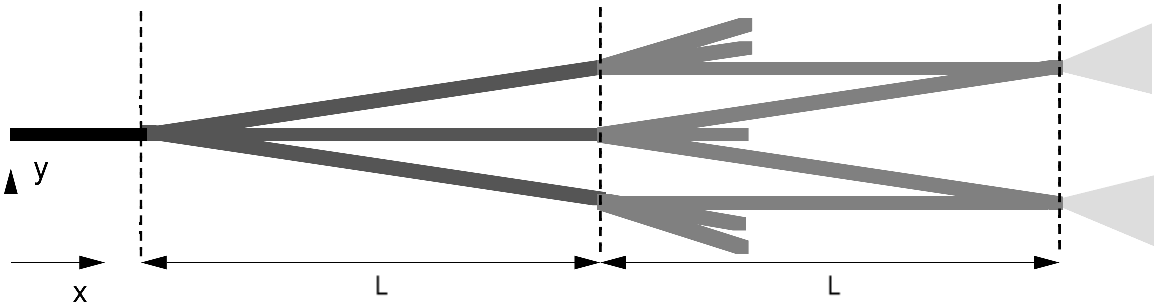

Figure 3: Three-grating interferometer for measuring $g_{antimatter}$. The three diffraction orders shown will contain most of the transmitted beam in roughly equal amounts. The orders which are drawn to the third grid cause an interference pattern with a frequency that matches the grid spacing. The diffraction orders which are not followed to the third grid do not contribute to this pattern, but rather cause a flat background.

of three equally-spaced transmission gratings, each with identical grid spacing. The first two gratings set up an interference pattern that is independent of both wavelength and the spacial coherence (incident wave direction) of the source[12]. This interference pattern has a spacial frequency which is equal to the grid spacing of the gratings, so the phase of the interference pattern can be analyzed with a third identical grating. The interference pattern is localized in X (the direction perpendicular to the grating planes), so while the distance between the first and second gratings is arbitrary, the distance from the second to the third grating must match the distance between the first and second gratings. A diagram of the interferometer is shown in Figure 3.

Not all of the diffraction orders from the first two gratings will contribute to the interference pattern. However, by using gratings with roughly 50% transmission (i.e. the slit width is half of the grating period), the even diffraction orders are suppressed, and most of the transmitted beam appears in the 0^{th} and $\pm 1^{st}$ order in roughly equal amounts. The orders that will interfere are shown in Figure 3. Ideally, approximately 4/9 of the beam transmitted through the second grating will contribute to the interference pattern.

The phase of the interference pattern can be measured by moving the third grating in the $\hat{y}$ direction. The transmission would be recorded as a function of the phase position of the grating: the transmission is highest when the interference peaks fall on the slits.

The interference pattern "falls" by the same amount that individual atoms fall while transversing the interferometer. Thus, for deflection D given by

$$D = \frac{1}{2}\overline{g}t^2 = 2\overline{g}\frac{L^2}{v^2}$$

where L is the separation between the gratings and v is the velocity of the antihydrogen, the phase shift $\Delta\phi$ is given by

$$\Delta\phi = D/d$$

where d is the line spacing (spatial frequency) of the grating. It is important to note that while the interference pattern is independent of velocity (wavelength), the deflection (or equivalently the phase shift) due to gravity is not. This means that a large velocity dispersion can wash out the interference pattern when the phase shift due to gravity becomes significant, so the beam used to make this measurement must either have a sufficiently small velocity dispersion, or else the velocity of each antihydrogen atom must be measured. We will return to this point when we discuss methods to produce the beam.

By solving the above formula for $\overline{g}$ we see that the uncertainty in the measurement of $\overline{g}$ is given by

$$\frac{\sigma(\overline{g})}{\overline{g}} = \sqrt{\left(\frac{\sigma(\Delta\phi)}{\Delta\phi}\right)^2 + \left(\frac{2\sigma(v)}{v}\right)^2 + \left(\frac{\sigma(d)}{d}\right)^2 + \left(\frac{2\sigma(L)}{L}\right)^2}.$$

If we assume that the distances L and d are well known, then the accuracy of the measurement is determined by how well the velocity is known, and how well the phase shift is measured. The latter term will be a function of the statistics.

3.1. Example Interferometer

An interferometer of the type described in the previous section has been constructed by a group at MIT[10] for use with a beam of sodium atoms. The parameters of the interferometer are $L = 0.65$ m, $d = 0.4\,\mu$m, and the slit width of each grating is half of the period. The sodium atoms were travelling at 1000 m/s, which corresponds to a wavelength of 16 pm. The interference signal had a contrast of 13%, which was about half of the calculated contrast (the difference was ascribed to a number of small effects such as grating imperfections). The authors were able to measure the phase of the interference pattern to 0.1 radian with 4000 atoms in the interference signal.

While this particular interferometer configuration would not be ideal for making the gravity measurement (for reasons that will be discussed below), it can be used as an "existence proof" to see what kind of accuracies should be possible with an interferometer. As an example, let us consider an antihydrogen beam with velocity $v = 10^4$ m/s, which corresponds to a wavelength of 40 pm and a deflection (for $\overline{g} = 9.8$ m/s^2) of 0.8 μm. An uncertainty in the phase measurement of 0.1 radian (for both a deflected and an undeflected beam) would lead to an uncertainty in the measurement of $\overline{g}$ of 1%. In addition to this, an uncertainty in the velocity corresponding to $T_u = 1°$K would lead to a total uncertainty of $\sigma(\overline{g})/\overline{g} \approx 2\%$.

The transmission through the third grating can be measured more accurately for antihydrogen than for sodium, since the antihydrogen which is not transmitted annihilates on the grating and can be recorded. Also, there should be little or no background to the annihilations. However, if we make the conservative assumption that it still takes 4000 atoms in the interference pattern to measure the phase to

0.1 radian, then the total number of antihydrogen atoms needed in the beam is $N_{\overline{H}} = 4000/\epsilon \times 2$. The factor of two comes from measuring the phase of both the deflected and the undeflected beam. The efficiency ϵ includes two factors of 0.5 for transmission through the first and second gratings, 4/9 for the diffraction orders which do not interfere, and a factor of precisely $1/\pi$ for reality, which comes from effects like the grating support structure, grating imperfections, and misalignments. These factors give $\epsilon = 3.5\%$, not including any beam that might be lost if collimation is necessary, for example if the gratings are too small to contain the entire beam. This means a beam of approximately 10^5 antihydrogen atoms with well known (or measurable) velocities could yield a 1% measurement of $\overline{g}$ using this interferometer.

As mentioned above, the interferometer used for the sodium beam is not optimal for the antihydrogen measurement. Because sodium has a much shorter wavelength than antihydrogen (at the same velocity), and because the researchers wanted to make a separated-beam interferometer, they were compelled to use a grating with as small a period as possible. Such a grating is very difficult to align: the period is too small to use lasers, and the alignment tolerances need to be less than the period over the active area of the grating to maintain phase coherence. In addition, the total size of the grating cannot be made very large, particularly since the grating's linearity must be better than the line spacing over the entire active area. A much better choice of the grating period for the gravity measurement would be at least an order of magnitude larger. This would greatly simplify the construction and alignment of the gratings. It would also allow a laser to be used for alignment and to give an instantaneous phase measurement. The interferometer could be lengthened to increase the gravitational deflection of the beam and compensate for the increase in d (and the corresponding decrease in $\Delta\phi$).

4. Making an Antihydrogen Beam

A number of authors have suggested methods for making antihydrogen[13]. The method which appears to be most suitable for making a (nonrelativistic) beam uses overlapping ion traps containing both antiprotons and positrons[14]. Antihydrogen is produced in the three-body process $\overline{p}+2e^+ \rightarrow \overline{H}+e^+$. The longitudinal motion of the antiprotons would be excited to make the antihydrogen form a beam. The calculated production rate for this process is quite high[15]:

$$\Gamma = 6 \times 10^{-13} \left(\frac{4.2}{T}\right)^{9/2} n_e^2 \; [s^{-1}]$$

per antiproton, where n_e is the positron density in number per cm^3. While this calculation includes the effect of the magnetic field on the motion of the positrons, there are a number of uncertainties resulting from other assumptions required for the calculation. For example, the effect of the magnetic field on the atomic states has been cited[13] as potentially affecting the calculated rates.

It is clear from discussions in sections 2 and 3 that the $\overline{H}$ beam either needs to have a narrow, well defined velocity distribution, or that the velocity of individual antihydrogen atoms needs to be measured. For the overlapping ion traps, it might be possible to get a narrow velocity distribution if the positrons occupy a fairly narrow region at the center of the harmonic well in which the antiprotons are oscillating. However, there could be problems from transverse heating of the antiprotons, and there would be some longitudinal cooling from collisions with the positrons. In addition, the continuous production of the antihydrogen would make it difficult to measure the velocity of individual atoms. Therefore, the more complicated ion trap shown in figure 4 might be better for making the antihydrogen beam.

The ion trap configuration shown in figure 4 could be used to make an $\overline{H}$ beam with a well-defined velocity and with a well-defined time structure. This would be done by alternately cooling the antiprotons, and then accelerating them with a small potential difference and passing them through a cold positron plasma to form antihydrogen. The trap and the interferometer could be inside of a completely closed system at cryogenic temperature, so that the plasmas could be held and cycled indefinitely.

The rate equation given at the beginning of this section can be used to estimate the number of antihydrogen atoms that would be produced per cycle. For example, about 600 antihydrogens would be produced by 10^6 antiprotons with $v = 1000$ m/s passing through a 1 cm long plasma of positrons with a density of 10^7 per cm^3. However, at much higher velocities, the assumptions that went into the calculation begin to break down. For example, the calculation assumes the ion (antiproton) is at rest, which is a good assumption so long as the $\overline{p}$ velocity is significantly below the positron's thermal velocity (8000 m/s at 4.2°K. Also, the $\overline{p}$ needs to remain in the plasma long enough for the atom to "evolve" through collisions to a low enough energy state to survive without being field-ionized as they exit the trap. At 4.2°K, this evolution time is 8×10^{-7} seconds, or about 0.8 mm at 10^3 m/s.

A major advantage of producing antihydrogen in this manner is that the $\overline{H}$ is formed at a specific time, so the velocity of the antihydrogen can be measured by recording the annihilation time. (The time in the cycle when the $\overline{H}$ are formed can be determined by measuring the difference in the average annihilation times at the different gratings.) Another advantage is that the acceleration voltage can be changed cycle-by-cycle. This would allow high velocity (small deflection) and low velocity (large deflection) data to be accumulated on alternate cycles, which would eliminate systematic errors in the measurement of $\Delta\phi$ resulting from drifts in the interferometer. Another advantage of the grating interferometer is that it does not care what atomic state the antihydrogen is in, so long as it is neutral. Most neutral atom traps, as well as an interferometer produced by light[16] (which incidentally has the potential to make very precise gravity measurements) generally require the atoms to be in a specific atomic state.

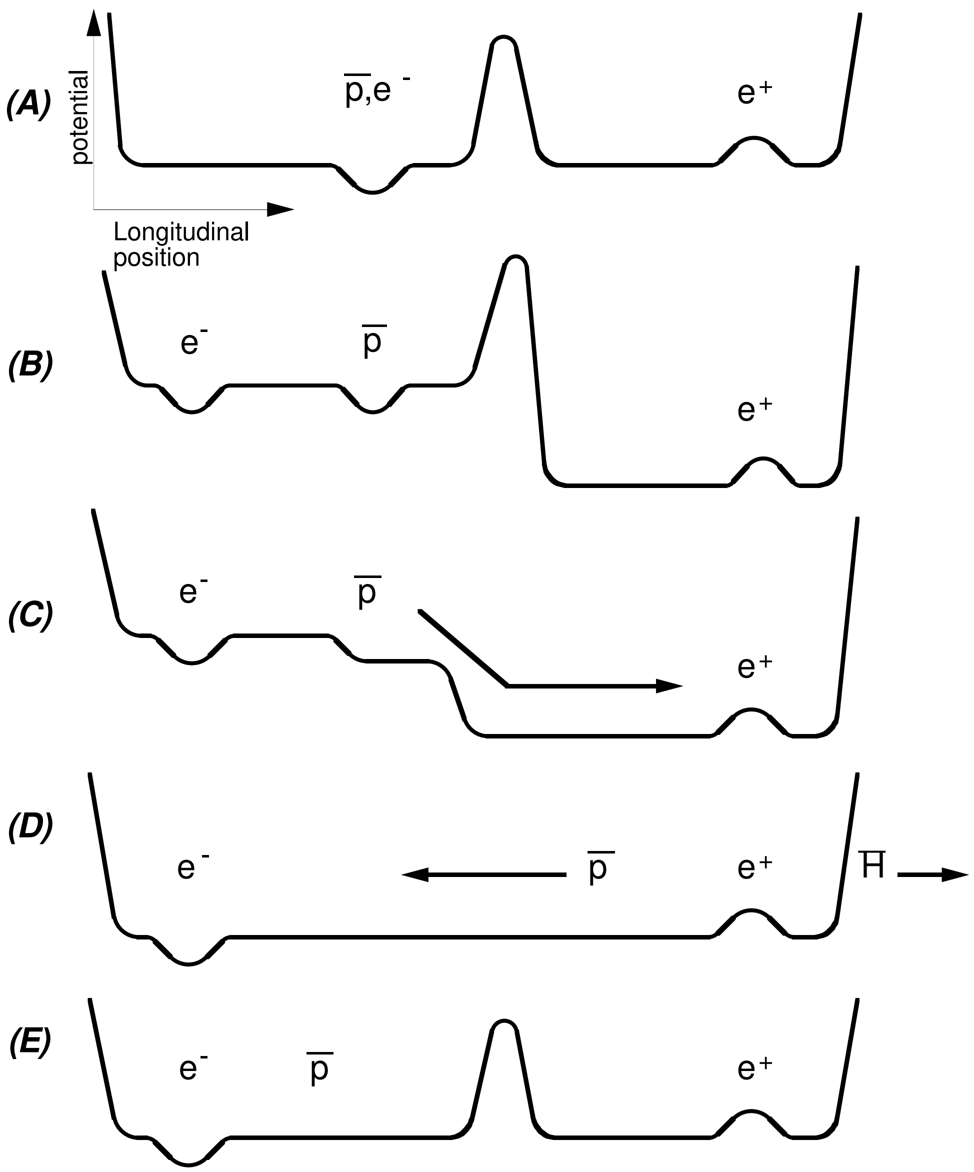

Figure 4: Ion trap cycle for making a well-controlled antihydrogen beam. In (A), the antiprotons are trapped and cooled with electrons on one side of the trap, while positrons are loaded into an inverted well on the other side of the trap. In (B), the electrons are ejected from the well containing the antiprotons and saved for the next cycle in an adjacent well. The potential of the antiproton well is raised relative to the side of the trap containing the positrons. In (C), the barrier separating the antiprotons and the positrons is lowered, and the antiprotons are accelerated by the potential difference. They pass through the positrons and form some antihydrogen with a well-defined velocity and at a well-defined time. In (D) and (E), the returning antiprotons are again trapped and cooled with the electrons, in preparation for another cycle.

5. Conclusions

A direct experimental measurement of the gravitational acceleration of antimatter has never been done, and the window of opportunity (the availability of low energy antiprotons) to make this fundamental measurement may be about to close. Several methods for making this measurement using a beam of neutral antihydrogen were discussed; the best of these uses a transmission-grating interferometer to measure the gravitationally-induced phase shift. This method should be capable of making a 1% measurement with a relatively small number (10^5) of antihydrogen atoms.

A method was suggested for making a beam of antihydrogen with a small velocity dispersion and with a definite time structure, which would allow the velocity of individual antihydrogen atoms to be measured. Of course, no one has accomplished making any non-relativistic antihydrogen yet, so the antihydrogen production is the most speculative aspect of this experiment.

The ultimate precision of this method for measuring $\overline{g}$ (assuming that a sufficiently large number of antihydrogen atoms can be produced) will be determined by how well the $\overline{H}$ velocity can be measured (a timing measurement), and by how well the dimensions of the interferometer are known. Even if the gravity measurement does not yield any surprises, the physics that can be done with an interferometer is very rich (there have been whole conferences devoted to neutron interferometry). If the calculated $\overline{H}$ production rate is correct and sufficient quantities of antihydrogen can be produced, then very precise limits on the difference between g and $\overline{g}$ could be set. This is an experiment that should be pursued while the opportunity exists.

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