Test 2

1. A distribution of charge has a charge density

$$\rho(\boldsymbol{x}) = \frac{3Q}{8\pi} e^{-r} \cos^2\theta.$$

Find the potential energy

$$W = \frac{1}{8\pi\varepsilon_0} \int\!\!\int \frac{\rho(\boldsymbol{x})\rho(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}\, d^3x\, d^3x'$$

You can use the "2 electron atoms" from the class.

2. Consider a thin slab of ferromagnetic material M with a regular polygonal disk and thickness t. A polygon has n sides and the distance L from the center to the perpendicular sides. Assume $t \ll L$. The slab carries an uniform magnetization $\boldsymbol{M}$, oriented normal to the disk.

   a. Find all bounded currents.

   b. Find an expression for the magnitude and direction of the magnetic field $\boldsymbol{B}$ in the center *to leading order in the small parameter* t/L. It may be useful to use

$$\int \frac{dx}{(1+x^2)^{3/2}} = \frac{x}{(1+x^2)^{1/2}}$$

   and

$$\frac{\tan\theta}{(1+\tan^2\theta)^{1/2}} = \sin\theta.$$

   c. Relate $n \to \infty$ to a circular disk.

   d. Find the expression for the $\boldsymbol{H}$ field along the axis at the center $(0,0,0)$ and at just outside of the disk $(0,0,t^+/2)$, *to first order in* t/L.

3. A ring of radius R lies in the xy-plane with its center at the origin, and carries uniform charge per unit length $\lambda = q/2\pi R$.

   a. Compute the dipole moment of this charge distribution.

   b. Compute all independent components of the quadrupole moment tensor,

$$Q_{ij} \equiv \int (3x_i x_j - r^2\delta_{ij})\rho(\boldsymbol{x})\, d^3x.$$

   [Hint: make full use of symmetry before doing any integrals.]

   c. Show that for $r \gg R$ the potential can written in spherical coordinates as

$$\Phi(r,\theta) \approx \frac{q}{4\pi\varepsilon_0}\left(\frac{A}{r} + \frac{B}{r^3}P_2(\cos\theta) + \dots\right),$$

   and explicitly determine the A and B coefficients.