Test 2

1. [Jackson 3.5b]
A hollow sphere of inner radius $a$ has the potential specified on its surface to be $\Phi = V(\theta, \phi)$. Show that the solution for the potential inside the sphere:

$$\Phi(x) = \sum_{lm} \left(\frac{r}{a}\right)^l Y_{lm}(\theta, \phi) \int V(\theta', \phi') Y^*_l(\theta', \phi') d\Omega'$$

You cannot assume from Jackson sections 3.9 and 3.10, but you can assume:

a. The image Green’s function for a sphere [Jackson eq. 2.16]:

$$G(x, x') = \frac{1}{|x - x'|} - \frac{a}{x'|x - a^2 x'/x'|^2}$$

b. The spherical expansion for an unit potential [Jackson eq. 3.70]:

$$\frac{1}{|x - x'|} = \sum_{lm} \frac{4\pi}{2l + 1} \frac{r_{<}^l}{r_{>}^l+1} Y_{lm}(\theta, \phi) Y^*_l(\theta', \phi')$$

c. The magic rule [Jackson eq. 1.42]:

$$\Phi(x) = \frac{1}{4\pi \varepsilon_0} \int_V G(x', x) \rho(x') d^3x' - \frac{1}{4\pi} \oint_S \left( \Phi(x') \frac{\partial G}{\partial n'}(x', x) - G(x', x) \frac{\partial \Phi}{\partial n'}(x') \right) da'$$

2. A very long dielectric cylinder of radius $R$ and dielectric constant $\kappa = \varepsilon/\varepsilon_0$ is placed in a field $E_0$ perpendicular to its axis. What is the electric field $E$ inside the dielectric cylinder?