Final Exam 2009 (24hr takehome)

Notes:

You are on your honor to abide by the rules in the handout About the Final. Violations will be treated extremely seriously: see http://www.phy.duke.edu/graduate/resources/integrity.pdf.

For full credit you must justify what you are doing. Show all work. Pulling an obscure formula out of a hat may lose you points; you must at least say where it came from (e.g., [Jackson page 813]), or probably you should write how to show that from standard ways.

Credit may be reduced if you do something a very awkward or long-winded way, even if you get the right result at the end.

I will readily give hints, but will record what I tell you and allow for that in grading.

1. A grounded conductor has the shape of an infinite plane with a hemispherical bulge of radius $a$. A charge $q$ is placed above the center of the bulge, a distance $b$ from the plane (or $b-a$ from the top of the bulge).
   a. What is the force on the charge?
   b. What is the potential $\Phi(r, \theta)$?
   c. What fraction of the total induced charge is induced on the bulge?

2. a. Jackson 3.22.
   b. Using this Green’s function, find the potential $\Phi(\rho, \phi)$ if the potential is $V$ on the circular surface (i.e. $\Phi(a, \phi) = V$) and the potential is 0 on the straight surfaces (i.e. $\Phi(\rho, 0) = \Phi(\rho, \beta) = 0$).

3. Elliptic cylindrical coordinates $(u, v, z)$ are related to rectangular coordinates $(x, y, z)$ by

   \[
   x = c \cosh u \cos v \\
   y = c \sinh u \sin v \\
   z = z
   \]

   where $c$ is a positive constant, with $0 \leq u < \infty$; $0 \leq v < 2\pi$; $-\infty < z < \infty$. The constant-$u$ curves are confocal ellipses and the constant-$v$ curves are confocal hyperbolas. If $z$ is uniform, the Laplace equation $\nabla^2 \psi(u, v) = 0$ is just

   \[
   \frac{\partial^2 \psi}{\partial u^2} + \frac{\partial^2 \psi}{\partial v^2} = 0.
   \]

   An infinitely long elliptical cylinder in a homogeneous medium of relative permeability $\kappa \equiv \mu/\mu_0$, is introduced into a homogeneous magnetic field $H_0$ making angle $\gamma$ with the major axis. Use the elliptic cylindrical coordinates with $u = u_0$ at the surface.

   Show that the magnetic scalar potential $\Phi_M$ is

   \[
   \Phi_M(u, v) = H_0(x \cos \gamma + y \sin \gamma) + H_0(1 - \kappa) \frac{ab(a+b)}{\sqrt{a^2 - b^2}} \left( \frac{\cos \gamma \cos v}{a + \kappa b} + \frac{\sin \gamma \sin v}{b + \kappa a} \right) e^{-u} \quad u > u_0
   \]

   \[
   \Phi_M(u, v) = H_0(a + b) \left( \frac{\cos \gamma}{a + \kappa b} x + \frac{\sin \gamma}{b + \kappa a} y \right) + C^{st} \quad u < u_0
   \]

   where the ellipse has semiaxes $a$ and $b$. 