Final Exam 2009 (24hr takehome)

Notes:

You are on your honor to abide by the rules in the handout *About the Final*. Violations will be treated extremely seriously: see `http://www.phy.duke.edu/graduate/resources/integrity.pdf`.

For full credit you must *justify what you are doing.* **Show all work.** Pulling an obscure formula out of a hat may lose you points; you must at least say where it came from (e.g., [Jackson page 813]), or probably you should write how to show that from standard ways.

Credit may be reduced if you do something a very awkward or long-winded way, even if you get the right result in the end.

I will readily give hints, but will record what I tell you and allow for that in grading.

1. A grounded conductor has the shape of an infinite plane with a hemispherical bulge of radius a. A charge q is placed above the center of the bulge, a distance b from the plane (or $b-a$ from the top of the bulge).
   a. What is the force on the charge?
   b. What is the potential $\Phi(r,\theta)$?
   c. What fraction of the total induced charge is induced on the bulge?

2. a. Jackson 3.22.
   b. Using this Green's function, find the potential $\Phi(\rho,\phi)$ if the potential is V on the circular surface (*i.e.* $\Phi(a,\phi)=V$) and the potential is 0 on the straight surfaces (*i.e.* $\Phi(\rho,0)=\Phi(\rho,\beta)=0$).

3. Elliptic cylindrical coordinates (u,v,z) are related to rectangular coordinates (x,y,z) by

$$\begin{aligned} x &= c\cosh u\cos v \\ y &= c\sinh u\sin v \\ z &= z \end{aligned}$$

where c is a positive constanti, with $0\le u<\infty$; $0\le v<2\pi$; $-\infty<z<\infty$. The constant-u curves are confocal ellipses and the constant-v curves are confocal hyperbolas. If z is uniform, the Laplace equation $\nabla^2\psi(u,v)=0$ is just

$$\frac{\partial^2\psi}{\partial u^2}+\frac{\partial^2\psi}{\partial v^2}=0.$$

An infinitely long elliptical cylinder in a homogeneous medium of relative permeability $\kappa\equiv\mu/\mu_0$, is introduced into a homogeneous magnetic field H_0 making angle γ with the major axis. Use the elliptic cylindrical coordinates with $u=u_0$ at the surface.

Show that the magnetic scalar potential Φ_M is

$$\Phi_M(u,v)=H_0(x\cos\gamma+y\sin\gamma)+H_0(1-\kappa)\frac{ab(a+b)}{\sqrt{a^2-b^2}}\left(\frac{\cos\gamma\cos v}{a+\kappa b}+\frac{\sin\gamma\sin v}{b+\kappa a}\right)e^{-u}\qquad u>u_0$$

$$\Phi_M(u,v)=H_0(a+b)\left(\frac{\cos\gamma}{a+\kappa b}x+\frac{\sin\gamma}{b+\kappa a}y\right)+C^{\text{st}}\qquad u<u_0$$

where the ellipse has semiaxes a and b.