Final Exam 2010

Notes:

For full credit you must *justify what you are doing*. **Show all work.** Pulling an obscure formula out of a hat may lose you points; you must at least say where it came from (e.g., [Jackson page 813]), or probably you should write how to show that from standard ways.

Credit may be reduced if you do something a very awkward or long-winded way, even if you get the right result in the end.

I will readily give hints, but will record what I tell you and allow for that in grading.

Short Answer Questions (4 points each)

1. In a ferroelectric material, an isolated solid sphere carries a uniform polarization field in the $\hat{z}$-direction. There is no external electric field. Sketch the position and sign of the bound charge, and also the electric field ($\mathbf{E}$) lines in inside and outside of the sphere.

2. Outline an explanation of the difference in behavior of paramagnetic, diamagnetic, and ferromagnetic materials in terms of the origins of each effect.

3. Use the appropriate Maxwell’s equations to derive the boundary conditions at interfaces for the $\mathbf{B}$ and $\mathbf{H}$ fields that apply with no free currents present.

4. Why the electric field ($\mathbf{E}$) is very small in a deep conical hole? (You can assume the standard solutions for the Laplace’s equation.)

5. What do you understand by “covariance”? Also write the Maxwell’s equations in covariant form for both the fields ($\mathbf{E}$ and $\mathbf{B}$) and the potentials ($\phi$ and $\mathbf{A}$).
6. [Jackson 3.12].

An infinite, thin, plane sheet of conducting material has a circular hole of radius $a$ cut in it. A thin, flat disc of the same material and slightly smaller radius lies in the plane, filling the hole, but separated from the sheet by a very narrow insulating ring. The disc is maintained at a fixed potential $V$, while the infinite sheet is kept at zero potential.

a. Using appropriate cylindrical coordinates, find an integral expression involving Bessel functions for the potential at any point above the plane.

b. Show that the potential at a perpendicular distance $z$ above the center of the disc is

$$
\Phi_0(z) = V \left(1 - \frac{z}{\sqrt{a^2 + z^2}}\right).
$$

c. Show that the potential at a perpendicular distance $z$ above the edge of the disc is

$$
\Phi_a(z) = \frac{V}{2} \left(1 - \frac{K(k)}{\pi a} \right)
$$

where $k = 2a/\sqrt{a^2 + 4a^2}$, and $K(k)$ is the complete elliptic integral of the first kind.

Formulas:

$$
\int_{\infty}^{-\infty} \frac{du}{(1+u^2)^{3/2}} = 2,
\int_{-\infty}^{\infty} \frac{du}{1+u^2} = \pi.
$$


7. Consider the volume in the half space $x \geq 0$, with $\Phi(x=0, y, z) = V$ on the surface $S$ defined by $x = 0$. Suppose there is a charge $q$ at $(a, 0, 0)$.

a. Find the Dirichlet Green function $G_D(x, x')$ for $x > 0$.

b. Find the potential $\Phi(x, y, z)$ at any point for $x > 0$, by using the $G_D$ and the “magic” rule.

c. Find the charge density $\sigma(y, z)$ on the surface.

Formulas: $\int_{-\infty}^{\infty} \frac{du}{(1+u^2)^{3/2}} = 2$, $\int_{-\infty}^{\infty} \frac{du}{1+u^2} = \pi$.

8. A conducting spherical shell of radius $R$ is cut into three segments extending, respectively, from $\theta = 0$ to $\theta = 60^\circ$, from $\theta = 60^\circ$ to $\theta = 120^\circ$, and from $\theta = 120^\circ$ to $\theta = 180^\circ$. They are insulated from one another. The uppermost and lowermost segments are grounded ($\Phi = 0$) and the central segment is held at potential $V$.

a. Find the first four terms in a multipole expansion ($q_{lm}$, $0 \geq l \geq 3$) of the potential for $r > R$.

b. What is the total charge on the sphere?

c. What is the quadrupole moment of the charge distribution on the sphere?

d. What is the potential at the center of the sphere?