Assignment 1 (due Friday 1/22/10)

1. a. A vector field $\mathbf{E}$ satisfies $\nabla \times \mathbf{E} = 0$. Prove that for any domain $V$ bounded by a surface $S$

$$\int_V (\nabla \cdot \mathbf{E}) d^3x = \oint_S (\hat{n} \cdot \mathbf{E}) d\mathbf{a} - \frac{1}{2} \oint_S |\mathbf{E}|^2 \hat{n} d\mathbf{a}.$$

You may assume any of the vector formulas and theorems given on Jackson’s endplates, but make it clear when you are doing so. *Hint:* Generalize the divergence theorem to a tensor ($T_{ij}$), and use $T_{ij} = E_i E_j$.

b. Similarly, another vector field $\mathbf{B}$ satisfies $\nabla \cdot \mathbf{B} = 0$. Prove

$$\int_V (\nabla \times \mathbf{B}) \times \mathbf{B} d^3x = \oint_S (\hat{n} \cdot \mathbf{B}) \mathbf{B} d\mathbf{a} - \frac{1}{2} \oint_S |\mathbf{B}|^2 \hat{n} d\mathbf{a}.$$

2. Find the electric field a distance $z$ from the center of a spherical surface of radius $R$, which carries a uniform charge density $\sigma$. Treat the case $z < R$ as well as $z > R$. Express your answers in terms of the total charge $q$ on the sphere.

3. Two spheres, each of radius $R$ and carrying uniform charge densities $+\rho$ and $-\rho$, respectively, are placed so that they partially overlap. Call the vector from the positive center to the negative center $\mathbf{d}$. Show that the field in the region of overlap is constant, and find its value.

4. Jackson’s problem 5.1. (Hard)

5. Jackson’s problem 5.3. (Easy)


7. Jackson’s problem 5.5.