Effective Field Theories and Multiple Scales in Quantum Physics

Thomas Mehen, Duke U.

CTMS Workshop
Theory and Applications of Multiscale Modelling
9/11/09
Units in Nuclear/Particle Physics

- **Special Relativity**
  \[ E = mc^2 \quad c = 3.00 \times 10^{-8} \text{ m/s} \]

- **Quantum Mechanics**
  \[ L = n\hbar \quad \hbar = 1.05 \times 10^{-34} \text{ Js} \]

- **‘natural units’**
  \[ \hbar = c = 1 \]
  all dimensionful quantities expressed in terms of energy

  - Energy \( \sim \) Momentum \( \times c \sim \) Mass \( \times c^2 \)
  - Length \( \sim \) \( \frac{\hbar c}{\text{Energy}} \)
  - Time \( \sim \) \( \frac{\hbar}{\text{Energy}} \)

- Large Mass/Energy \( \leftrightarrow \) Small Length/Time
  \[ \hbar c = 197.3 \text{ MeV} \times 10^{-15} \text{ m} = 197.3 \text{ MeV fm} \]

- **Electron mass**
  \[ m_e = 0.511 \text{ MeV} / c^2 = 9.11 \times 10^{-31} \text{ kg} \]

- **Proton mass**
  \[ m_p = 938 \text{ MeV} / c^2 = 1.67 \times 10^{-27} \text{ kg} \]

- **Proton radius**
  \[ 0.88 \text{ fm} \]
The Many Mass Scales of Elementary Particle Physics

fermions (matter)

<table>
<thead>
<tr>
<th>Leptons</th>
<th>spin = 1/2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flavor</td>
<td>Mass GeV/c²</td>
</tr>
<tr>
<td>νₑ</td>
<td>&lt;1 × 10⁻⁸</td>
</tr>
<tr>
<td>e</td>
<td>0.000511</td>
</tr>
<tr>
<td>νₘ</td>
<td>&lt;0.0002</td>
</tr>
<tr>
<td>μ</td>
<td>0.106</td>
</tr>
<tr>
<td>νₜ</td>
<td>&lt;0.02</td>
</tr>
<tr>
<td>τ</td>
<td>1.7771</td>
</tr>
</tbody>
</table>

Quarks (spin = 1/2)

<table>
<thead>
<tr>
<th>Flavor</th>
<th>Approx. Mass GeV/c²</th>
<th>Electric charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>u up</td>
<td>0.003</td>
<td>2/3</td>
</tr>
<tr>
<td>d down</td>
<td>0.006</td>
<td>-1/3</td>
</tr>
<tr>
<td>c charm</td>
<td>1.3</td>
<td>2/3</td>
</tr>
<tr>
<td>s strange</td>
<td>0.1</td>
<td>-1/3</td>
</tr>
<tr>
<td>t top</td>
<td>175</td>
<td>2/3</td>
</tr>
<tr>
<td>b bottom</td>
<td>4.3</td>
<td>-1/3</td>
</tr>
</tbody>
</table>

Unified Electroweak (spin = 1)

<table>
<thead>
<tr>
<th>Name</th>
<th>Mass GeV/c²</th>
<th>Electric charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>γ</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>W⁻</td>
<td>80.4</td>
<td>-1</td>
</tr>
<tr>
<td>W⁺</td>
<td>80.4</td>
<td>+1</td>
</tr>
<tr>
<td>Z⁰</td>
<td>91.187</td>
<td>0</td>
</tr>
</tbody>
</table>

Strong (color) (spin = 1)

<table>
<thead>
<tr>
<th>Name</th>
<th>Mass GeV/c²</th>
<th>Electric charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>g</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

hadrons (quark composites)

Baryons qqq and Antibaryons q̅q̅q
Baryons are fermionic hadrons. There are about 120 types of baryons.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Quark content</th>
<th>Electric charge</th>
<th>Mass GeV/c²</th>
<th>Spin</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>proton</td>
<td>uud</td>
<td>1</td>
<td>0.938</td>
<td>1/2</td>
</tr>
<tr>
<td>p̄</td>
<td>anti-proton</td>
<td>uudd</td>
<td>-1</td>
<td>0.938</td>
<td>1/2</td>
</tr>
<tr>
<td>n</td>
<td>neutron</td>
<td>udd</td>
<td>0</td>
<td>0.940</td>
<td>1/2</td>
</tr>
</tbody>
</table>

Mesons q̅q
Mesons are bosonic hadrons. There are about 140 types of mesons.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Quark content</th>
<th>Electric charge</th>
<th>Mass GeV/c²</th>
<th>Spin</th>
</tr>
</thead>
<tbody>
<tr>
<td>π⁺</td>
<td>pion</td>
<td>uū</td>
<td>+1</td>
<td>0.140</td>
<td>0</td>
</tr>
<tr>
<td>K⁻</td>
<td>kaon</td>
<td>sū</td>
<td>-1</td>
<td>0.494</td>
<td>0</td>
</tr>
<tr>
<td>ρ⁺</td>
<td>rho</td>
<td>uū</td>
<td>+1</td>
<td>0.770</td>
<td>1</td>
</tr>
</tbody>
</table>
Nuclear Physics

proton mass: \( m_p = 938.3 \text{ MeV} \)

neutron mass: \( m_n = 939.6 \text{ MeV} \)

\[ m_p - m_n = 1.3 \text{ MeV} \]

pion mass: \( m_{\pi} \approx 137 \text{ MeV} \rightarrow \text{range of nuclear force} \sim 1.4 \text{ fm} \)

binding energy per nucleon (lead): \( \sim 9 \text{ MeV} \)

deuteron (np) binding energy: \( 2.2 \text{ MeV} \)

Atomic Physics \((H_2)\)

\( m_e = 0.511 \text{ MeV} \)

fine structure constant: \( \alpha = \frac{1}{137} \)

\[ a_0 = \frac{1}{m_e \alpha} = 0.5 \times 10^{-10} \text{ m} \]

binding energy: \( \frac{1}{2} m_e \alpha^2 = 13.6 \text{ eV} \)

Quantum Physcis provides numerous problems with widely separated scales
Quantum Field Theory: Language of Particle Interactions

action: \( S[\phi] \)  
local field: \( \phi(x, t) \)  
classical e.o.m: \( \frac{\delta S}{\delta \phi} = 0 \)

\[
S[\phi] = \int d^4x \mathcal{L}[\phi] 
\]

\[
S[\phi] = \int d^4x \frac{1}{2} (\partial_\mu \phi \partial^\mu \phi - m^2 \phi^2) + g\phi^3 + \lambda\phi^4 + \lambda'\phi^5 + \ldots 
\]

kinetic (quadratic) terms  
interaction terms

Quantization:

kinetic terms: non-interacting particles w/ relativistic dispersion relation

\[
E = \sqrt{p^2 c^2 + m^2 c^4} 
\]

(local) interactions: 3 or more particles meeting at a point in space/time

absorption/emission of quanta, collisions

coupling constants: \( g, \lambda, \lambda', \ldots \)
Quantum Electrodynamics (QED)

Feynman Diagrams: visual recipe for calculating transition amplitude

- Photons
- Electrons/positrons
- Free particle propagation
- Interactions

\( \propto \sqrt{\alpha} \)
\( \alpha = \frac{1}{137} \)

Perturbation Theory for \( e^- e^- \rightarrow e^- e^- \)

\[ O(\alpha) \]
\[ O(\alpha^2) \]

Basic Idea

Draw all diagrams to up to \( O(\alpha^n) \), \( n \) determined by accuracy required
QFT is a machine that computes physics given $S[\phi]$

How do you figure out $S[\phi] = \int d^4x \mathcal{L}[\phi]$?

- Experiments and a lot of guesswork!
- Symmetry
- Dimensional Analysis - Renormalization Theory
Symmetry in QM

- Rotational Invariance of $H_2$

\[ H = \frac{\vec{p}^2}{2m} - \frac{e^2}{r} \]

- Degenerate (same energy) Multiplets

S-waves

P-waves - multiplet of 3

- Constrains Dynamics - Wigner-Eckart Theorem

\[ \langle J, m | \vec{r} \cdot \vec{E} | J', m' \rangle \propto C_{J,J';m,m'} \langle J || \vec{r} \cdot \vec{E} || J' \rangle \]
Symmetries in QFT

- **Global symmetry, e.g. isospin**

  Heisenberg noticed nuclear force (almost) invariant under $n \leftrightarrow p$

  \[
  N = \begin{pmatrix} n \\ p \end{pmatrix} \quad \vec{\pi} = \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}
  \]

  $N \rightarrow UN \quad \vec{\pi} \rightarrow R \vec{\pi} \quad U, R$ are $SU(2)$ matrices

- **Lorentz Invariance - Special Relativity**

  form of kinetic terms for spin-0, 1/2, 1 particles, $S[\phi]$ is L.I. invariant

- **Gauge (local) symmetry**

  (QED) $\psi \rightarrow e^{ie\Lambda(x)}\psi \quad A_\mu \rightarrow A_\mu + \partial_\mu \Lambda(x)$

  electron

  photon

  (SM) gauge symmetry $\leftrightarrow$ forces $\leftrightarrow$ gauge bosons
Symmetry is not enough!

\[ \mathcal{L}_{\text{QED}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} (i \partial - m_e) \psi - e \bar{\psi} A \psi \]

\[ + b_1 \bar{\psi} \sigma_{\mu\nu} F^{\mu\nu} \psi + c_1 (F_{\mu\nu} F^{\mu\nu})^2 + c_2 (\epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta})^2 + \ldots \]

Blue terms allowed by symmetry, but not required to due accurate atomic physics calculations, e.g. Lamb Shift (2S-2P splitting), \( a_e, a_\mu, \ldots \)

Why?

Some dimensional analysis: \( (\hbar = c = 1) \) \[ \ldots = \text{mass dimension} \]

\[ [S[\phi]] = 0 \rightarrow [\mathcal{L}] = 4 \]

\[ [A_\mu] = 1 \quad [\partial_\mu] = 1 \quad [F_{\mu\nu}] = 2 \quad [\psi] = 3/2 \]

\[ [m_e] = 1 \quad [e] = 0 \quad [b_1] = -1 \quad [c_1] = [c_2] = -4 \]
Question: How do very heavy particles affect low energy physics at low energies?

### β decay

\[ n \rightarrow p + e^- + \bar{\nu}_e \]

(\text{(udd) (uud)})

\[ m_p - m_n = 1.3 \text{ MeV} \]

\[ m_W = 80.4 \text{ GeV} \]

\[ E_W \neq \sqrt{p^2 + M_W^2} \]

Virtual particle: how far can it go?

Interaction looks local! Replace W boson with effective interaction

**Fermi Theory:**

\[ \mathcal{L}_{eff} = G_F \psi_p^\dagger \psi_n \psi_\nu^\dagger \psi_e \]

\[ G_F \sim \frac{1}{M_W^2} \]
All QFT’s are low energy Effective Field Theories:

- $10^{19}$: Planck scale
- $10^{15}$: Unification scale
- $10^2$: Top quark mass
- $1$: Hadronic processes
- $10^{-8}$: Atomic processes

- String theory?
- Grand unified theory?
- Supersymmetry?
- Electroweak
- QCD
- QED

{Standard Model}
all terms in the Lagrangian allowed by symmetry are expected: “That which is not forbidden is compulsory”, Gell-Mann

but not all terms are equally important (and thankfully? most are not)

\[
\mathcal{L}_i = \int d^4 x \ g_i \mathcal{O}_i \quad [\mathcal{O}_i] = d_i \quad [g_i] = 4 - d_i
\]

\( \mathcal{E} \) : energy scale of physics we’re interested in
\( \Lambda \) : physical cutoff (e.g. lightest particle that’s being ignored)

\[
g_i \sim \frac{1}{\Lambda^{d_i - 4}} \quad \int d^4 x \ \mathcal{O}_i \sim \mathcal{E}^{d_i - 4} \quad \mathcal{L}_i \sim \left( \frac{\mathcal{E}}{\Lambda} \right)^{d_i - 4}
\]

but not all terms are equally important (and thankfully? most are not)

relevant \( d_i - 4 < 0 \)

marginal \( d_i - 4 = 0 \)

irrelevant \( d_i - 4 > 0 \)
Path Integral formulation of QFT

\[ \int D\phi_L D\phi_H e^{iS(\phi_L,\phi_H)} = \int D\phi_L e^{iS(\phi_L).} \]

\[ e^{iS(\phi_L)} = \int D\phi_H e^{iS(\phi_L,\phi_H)}. \]

Wilson: ‘integrating out’ high energy modes

Renormalization Group Equations

\[ \Lambda \frac{d}{d\Lambda} g_i = \gamma_{ij}(\{g\}) g_j \]

quantum effects can modify scaling expectations from naive dimensional analysis
Running Coupling Constants

\[ q = p - p': \text{momentum transfer} \]

\[ \propto \frac{\alpha}{q^2} \]

\[ \sim \alpha \log\left(\frac{|q|^2}{m_e^2}\right) \quad |q| \gg m_e \]

Vacuum Polarization screens bare charge

\[ \alpha(m_e) = 1/137 \]

atomic physics

\[ \alpha(90 \text{ GeV}) = 1/128 \]

LEP: \( e^+ e^- \rightarrow Z_0 \rightarrow X \)

Heisenberg: \( q \leftarrow \hbar/r \)
Quantum Chromodynamics (QCD)

QED
- Electrons/positrons: (+,-) charges
- Photon: Neutral

QCD
- Quarks: Three colors (r,g,b)
- Gluons: 8 colors (e.g., r g)

Key Difference: Gluons colored and self-interact
Asymptotic Freedom

\[ \alpha_s(Q) \]

\[ Q(\text{GeV}) \]

- \[ \alpha(Q \to \infty) \to 0 \]
- \text{IR - Strong Coupling}

\[ \Lambda_{\text{QCD}} \sim 500 \text{ MeV} \]

(Gross, Politzer, Wilczek; Nobel 2004)

\[ \text{Screening} \]

\[ \text{Anti-Screening} \]

\[ \text{perturbation theory in UV} \]

- Confinement – Colorless Hadrons
- Chiral Symmetry Breaking
Confinement

- Physical Strongly Interacting Particles (hadrons) are Colorless
  - Mesons - $q \bar{q}$
  - Baryons - $qqq$

- Potential Energy between quark and antiquark

$$V_{q\bar{q}}(r) \approx -\frac{4}{3} \frac{\alpha_s(1/r)}{r} + \sigma r$$

force between quark and antiquark separated by $\sim 1$ fm equivalent to 14 tons!

try to pull a quark-antiquark, create more mesons

$\sim 1$ fm

$\bar{q} q$
Hydrogen atom

\[ |H\rangle \sim |p^+e^-\rangle + \alpha |p^+e^−γ\rangle + \alpha^2 |p^+e^-e^+e^-\rangle \]

Hydrogen a two-body system up to small corrections

Meson (\bar{q}q + ...) 

\[ \alpha_s \sim 1 \]

many-body system: many Fock states contribute
Effective Field Theories

- isolate relevant degrees of freedom

- small expansion parameter (ratio of scales):

- Lagrangian is most general consistent w/ symmetries of underlying theory, i.e. QCD

- Power Counting
  must be able to count powers of expansion parameter in Lagrangian and in Feynman diagrams so we can systematically compute to a given order

- Physics at high scales (> 1 GeV) determines coefficients in Lagrangian
determine by matching (e.g. Fermi theory), or experimentally, or numerical simulations
RG Improved Perturbation Theory for Strong Interactions
Weak decays of heavy quarks

\[ b \rightarrow c \, \bar{u} \, d \]

like Fermi theory for $\beta$ decay

large strong interaction corrections

\[ 1 + c_1 \alpha_s(m_b) \log \left( \frac{m_W^2}{m_b^2} \right) + c_2 \alpha_s(m_b)^2 \log^2 \left( \frac{m_W^2}{m_b^2} \right) + \ldots \]

\[ \sim 0.2 \quad \sim 5 \]

\[ m_W = 80.4 \text{ GeV} \quad m_b \approx 4 - 5 \text{ GeV} \]

RGE for Fermi theory can be used to resum series to all-orders
easier to do with EFT than full Theory
Chiral Theories of Low Energy QCD
Chiral Symmetry

- chirality = helicity (for a massless particle)

\[ \vec{s} \cdot \vec{p} = -1/2 \quad \quad \vec{s} \quad \quad \vec{s} \cdot \vec{p} = +1/2 \]

- \[ q = q_L + q_R \quad SU_L(3) \times SU_R(3) \]

\[ q_L \rightarrow U_L q_L \quad q_R \rightarrow U_R q_R \]

- Symmetry of \( \mathcal{L}_{QCD} \), but not the vacuum

\[ \langle 0 | \bar{q}_L q_R + \bar{q}_R q_L | 0 \rangle \neq 0 \]

Spontaneous Chiral Symmetry Breaking
Vacuum: \( \langle \bar{q}_L q_R \rangle \leftrightarrow \uparrow \)

Excited State: \( \langle \bar{q}_L U_L^\dagger(x) U_R(x) q_R \rangle \leftrightarrow \uparrow \)

\( \lambda = \frac{2\pi}{k} \)

recover vacuum as \( \lambda \to \infty \ (k \to 0) \quad E(k) \propto k \)

relativity: \( E(k) = \sqrt{k^2 + m^2} \)

Spontaneous Symmetry Breaking \( \rightarrow \) Massless Particle

Goldstone Boson
real world \( m_u, m_d, m_s \neq 0 \) pseudo-Goldstone Bosons

\[
m^2_{PGB} \propto m_q \langle 0 | \bar{q}q | 0 \rangle
\]

Light hadrons of QCD: pions, kaons, and eta’s

\[
m_\pi \sim 140 \text{ MeV} \quad m_K \sim 496 \text{ MeV} \quad m_\eta = 547 \text{ MeV}
\]

Chiral Symmetry Breaking determines the low lying degrees of freedom of QCD, constrains their self-interactions and interactions with other hadrons
Interactions vanish as $k \to 0$ derivatively coupled

weakly interacting at low energies:

Chiral Perturbation Theory (ChiPT)

$$\mathcal{L} = \frac{f_{\pi}^2}{8} \text{Tr} \partial_\mu \Sigma \partial^\mu \Sigma^\dagger + \frac{f_{\pi}^2 B_0}{4} \text{Tr}(m_q \Sigma + m_q \Sigma^\dagger) + \ldots$$

perturbative expansion in $\frac{Q^2}{\Lambda_\chi^2}$

$$\Lambda_\chi = 4\pi f_{\pi} \sim 1 \text{ GeV}$$
Coupling to nucleons

\[ a^{I \pi \nu}_{\pi N} = C_I \frac{m_\pi}{f_\pi^2} \] (within 15\%-25%)

Long range nuclear spin-tensor force

\[ \frac{g_A^2}{2f_\pi^2} \frac{\vec{q} \cdot \vec{\sigma}}{\vec{q}^2 + m_\pi^2} \]
Large Scattering Lengths
Quantum Mechanics of shallow bound states

\[ V(r) - E_B \]

\[ \psi(r) \propto \frac{e^{-r/a}}{r} \quad \sigma = \frac{4\pi}{p^2 + 1/a^2} \quad E_B = \frac{1}{2\mu a^2} \]

universal properties

\( a \gg R \)

\( R \) - range of force \quad \( a \) - scattering length

since physics observables can all be computed in terms of a single parameter, simple EFT with single relevant operator can reproduce this physics

higher order corrections: expansion in \( R/a \)

2- and 3-body scattering easily calculated within this EFT

originally developed for few-body nuclear physics

deuteron - weakly bound neutron and proton \( E_B = 2.2 \text{ MeV} \)
neutron-deuteron scattering at low energies

NLO EFT: 5 parameters, 1-D coupled integral equations

Potential model: 20+ parameters in 2+3 body potentials
Faddeev equations (9-dimensional Schroedinger Eq.)

By focusing on relevant degrees of freedom
EFT reduces parameters, computational complexity

(N. Hammer, T.M. PLB516:353(2001))
Current Topical Applications

- **Cold Trapped Atoms**
  experimental atomic physicist can tune scattering length at will using Feshbach Resonances (Prof. John Thomas, Duke U.)

- **Non-Relativistic Conformal Symmetry as** \( a \rightarrow \infty \)
  

- **X(3872) (discovered in 2003)**
  shallow bound state of D(c\(\bar{u}\)) mesons \( E_B < 0.7 \text{ MeV} \)

**XEFT:**
Conclusions

- Focusing on relevant degrees of freedom, interactions at a given scale useful for systematic analysis of physics problems

- Effective interactions - ‘coarse graining’ - can simplify computational problems

  Examples: resumming strong interaction logarithms neutron-deuteron scattering at low energies