Three questions

1. What is a quark-gluon plasma?
2. What is a perfect liquid?
3. Why is the QGP a nearly PL?
Hot, hotter, hottest …

Genre:
Comedy / Crime / Romance / Thriller

Eating Takoyaki (squid balls) fresh from the grill in Osaka/Japan

Nucleons + mesons
Quark-gluon plasma
Melting nuclear matter
Heating Things Up

- What heat does to matter:
  - Increases disorder (entropy)
  - Speeds up reactions
  - Helps overcome potential barriers

- States / phases of matter:
  - **Solid** [long-range correlations, shear elasticity]
  - **Liquid** [short-range correlations]
  - **Gas** [few correlations]
  - **Plasma** [charged constituents] (solid / liquid / gaseous)
QCD (Nuclear) Matter

- Matter governed by the laws of Quantum Chromodynamics can also take on different states:
  - Solid, e.g. crust of neutron stars
  - Liquid, e.g. all large nuclei
  - Gas, e.g. nucleonic or hadronic gas ($T \approx 7$ MeV)
  - Plasma - the QGP ($T > T_c \approx 150 – 200$ MeV)

- The QGP itself may exist in different phases:
  - Gaseous plasma ($T \gg T_c$)
  - Liquid plasma ($T, \mu$ near $T_c, \mu_c$ ?)
  - Solid, color superconducting plasma ($\mu \gg \mu_c$)
QCD phase diagram

- Critical end point
- Hadronic matter
- Quark-Gluon
- Plasma
- 1st order line
- Chiral symmetry broken
- Chiral symmetry restored
- Color superconductor

- RHIC
- Nuclei
- Neutron stars
- $\mu_B$
QCD equation of state

Degrees of freedom:

\[ v = \left[ (2 \times 8) + \frac{7}{4} \times (2 \times 3 \times N_f) \right] \times \left( 1 - O(g^2) \right) \]

\[ \frac{\pi^2}{30} v = 16.0 \]

Lattice QCD

Indication of weak coupling?
Color screening

\[ -\nabla^2 \phi^a = g \rho_G^a(\phi^b) + g \rho_Q^a(\phi^b) \]

Induced color density \( \rho^a = -\mu^2 \phi^a \)

with \( \mu^2_G = (gT)^2, \quad \mu^2_Q = \frac{N_F}{6}(gT)^2 \)

Static color charge (heavy quark) generates screened potential

\[ \phi^a = t^a \frac{\alpha_s}{r} e^{-\mu r} \]
Plasma two-stream instability
Turbulent color fields

M. Strickland, hep-ph/0511212
Quark masses

Heat “melts” the quark condensate: QCD mass disappears above $T_c$.

(Partial) chiral symmetry restoration
Lattice - susceptibilities

\[ \chi_{XY} = \frac{\partial^2 \ln Z(T, \mu_i)}{\partial \mu_x \partial \mu_y} \ln Z(T, \mu_i) = \langle XY \rangle - \langle X \rangle \langle Y \rangle \]

\[ \langle XS' \rangle \approx \sum_i x_i s_i n_i \]

\[ C_{XS} = -3 \frac{\langle XS \rangle - \langle X \rangle \langle S \rangle}{\langle S^2 \rangle - \langle S \rangle^2} \]
The historical path to the QGP

Arrow of time

quarks gain QCD mass and become confined
The practical path to the QGP...

...is hexagonal and 3.8 km long

Relativistic Heavy Ion Collider
Phenomenology provides the connection
Space-time picture

Bjorken formula

\[
\begin{align*}
    s(\tau) & \sim \frac{dN(\tau) / dy}{dV(\tau) / dy} \\
    & \leq \frac{(dN / dy)_{\text{final}}}{\pi R^2 \tau}
\end{align*}
\]

- \( s(\tau_0 = 1 \text{ fm/c}) \approx 33/\text{fm}^3 \)
- or \( T(\tau_{eq}) \approx 300 \text{ MeV} \)

in Au+Au (200 GeV)
Some important results from RHIC:

- Chemical equilibration (incl. s-quarks!)
  - $u, d, s$-quarks become light and unconfined
- Elliptic flow
  - rapid thermalization, low viscosity
- Collective flow pattern related to valence quarks
- Jet quenching
  - parton energy loss, high color opacity
- Strong energy loss of $c$ and $b$ quarks (why?)
- Charmonium suppression is not increased compared with lower (CERN-SPS) energies
Chemical equilibrium

\[ \frac{Y_1}{Y_2} \approx \exp \left( -\frac{(m_1 - m_2)}{T_{ch}} \right) \]

Increase in flavor correlation length \( r_c \gg 1 \text{ fm} \) + gluon thermalization

Sudden hadronization?
Jet quenching in Au+Au

Suppression of hadrons

No suppression for photons

Yield in A+A

\[
R_{AA}(p_T) = \frac{d^2N_{AA}/dp_Tdy}{T_{AA}(d^2\sigma_{NN}/dp_Tdy)}
\]

Area density of p+p coll’s in A+A

Cross section in p+p coll’s

Without nuclear effects:

\[
R_{AA} = 1.
\]
Radiative energy loss: 

\[ \Delta E \sim \rho L^2 \left\langle k_T^2 \right\rangle \]

Scattering centers = color charges

\[ \hat{q} = \rho \int q^2 dq^2 \frac{d\sigma}{dq^2} = \rho \sigma \left\langle k_T^2 \right\rangle = \int dx^- \left\langle F_i^+(x^-) F^{+i}(0) \right\rangle \]

Density of scattering centers

Range of color force

Scattering power of the QCD medium:
q-hat at RHIC

Eskola et al.

**RHIC**

\[ \sqrt{s_{NN}} = 200 \text{ GeV}, \left( k^+ + k^\pi^- \right)/2 \]

\[ A_{eff} = 181 \text{ (0-5\%)} \]

\[ L_{coll} = 5.0 \text{ fm} \]

**RHIC data**

- sQGP?
- QGP
- Pion gas
- Cold nuclear matter

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Where does the “lost” energy go?

Lost energy of away-side jet is redistributed to angles away from 180° and low transverse momenta $p_T < 2 \text{ GeV/c}$ (Mach cone?).
Collision Geometry: Elliptic Flow

Elliptic flow ($v_2$):
- Gradients of almond-shape surface will lead to preferential expansion in the reaction plane
- Anisotropy of emission is quantified by $2^{nd}$ Fourier coefficient of angular distribution: $v_2$
  - prediction of fluid dynamics

Bulk evolution described by relativistic fluid dynamics,
- assumes that the medium is in local thermal equilibrium,
- but no details of how equilibrium was reached.
- **Input:** $\varepsilon(x,\tau_i)$, $P(\varepsilon)$, ($\eta$, etc.).
Elliptic flow: early creation

Time evolution of the energy density:

Flow anisotropy must generated at the earliest stages of the expansion, and matter needs to thermalize very rapidly, before 1 fm/c.
$v_2(p_T)$ vs. hydrodynamics

Failure of ideal hydrodynamics tells us how hadrons form.

Mass splitting characteristic property of hydrodynamics.
Quark number scaling of $v_2$

In the recombination regime, meson and baryon $v_2$ can be obtained from the quark $v_2$:

$$v_2^B(p_t) = 3v_2^q\left(\frac{p_t}{3}\right)$$
Question #2…

What is a “Perfect Liquid”? 
An ideal gas is one that has sufficiently strong interactions to reach thermal equilibrium (on a reasonable time scale), but sufficiently weak interactions so that their effect on $P(n,T)$ can be neglected.

This ideal can be approached arbitrarily by diluting the gas and waiting very patiently until equilibrium is achieved.

A perfect liquid is one that obeys the Euler equations, i.e. a fluid that has zero viscosity and infinite thermal conductivity.

Observation of dissipative effects requires velocity or temperature gradients.

Velocity gradients at RHIC are sizable: Knudsen # $Kn = \frac{\lambda_f}{R} \sim 0.1$
What is viscosity?

Shear and bulk viscosity are defined as coefficients in the expansion of the stress tensor in gradients of the velocity field:

\[ T_{ik} = \varepsilon u_i u_k + P (\delta_{ik} + u_i u_k) - \eta \left( \nabla_i u_k + \nabla_k u_i - \frac{2}{3} \delta_{ik} \nabla \cdot u \right) + \zeta \delta_{ik} \nabla \cdot u \]

Microscopically, \( \eta \) is given by the rate of momentum transport:

\[ \eta \approx \frac{1}{3} n \bar{p} \lambda_f = \frac{\bar{p}}{3 \sigma_{tr}} \]

Unitarity limit on cross sections suggests that \( \eta \) has a lower bound:

\[ \sigma_{tr} \leq \frac{4\pi}{\bar{p}^2} \quad \Rightarrow \quad \eta \geq \frac{\bar{p}^3}{12\pi} \quad \text{QGP} \rightarrow 0.1 \text{s} \]
Lower bound on $\eta/s$

A heuristic argument for $(\eta/s)_{\text{min}}$ is obtained using $s \sim 4n$:

$$\eta \approx \frac{1}{3} n \left( \bar{p} \bar{V} \right) \left( \frac{\lambda_f}{\bar{V}} \right) \approx \frac{1}{12} s \left( \frac{\varepsilon}{n} \right) \tau_f$$

But the uncertainty relation dictates that $\tau_f (\varepsilon/n)^{1/3} \geq \hbar$, and thus:

$$\eta \geq \frac{\hbar}{12} s \approx \frac{\hbar}{4\pi} s$$

(It is unclear whether this relation holds in the nonrelativistic domain, where $s/n$ can be much larger than 4. But is obeyed by all known substances.)

For $N=4$ SUSY SU($N_c$) Yang-Mills the bound is saturated at strong coupling:

$$\eta = \frac{s}{4\pi} \left[ 1 + \frac{135 \zeta(3)}{(8g^2 N_c)^{3/2}} + \cdots \right]$$
String theory weighs in

General argument [Kovtun, Son & Starinets, PRL 94 (2005) 111601] based on duality between thermal QFT and string theory on curved background with D-dimensional black-brane metric, e.g.:

$$ds^2 = \frac{r^2}{R^2} \left[ -\left( 1 - \frac{r_0^4}{r^4} \right) dt^2 + \sum_{i=1}^{3} dx_i^2 \right] + \frac{R^2}{r^2} \left( 1 - \frac{r_0^4}{r^4} \right)^{-1} dr^2$$

Kubo formula for shear viscosity:

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt \, d^3x \, e^{i\omega t} \left\langle \left[ T_{xy}(t, \vec{x}), T_{xy}(0,0) \right] \right\rangle$$

Dominated by absorption of (thermal) gravitons by the black hole:

$$\sigma_{\text{abs}}(\omega) = \frac{8\pi G}{\omega} \int dt \, d^3x \, e^{i\omega t} \left\langle \left[ T_{xy}(t, \vec{x}), T_{xy}(0,0) \right] \right\rangle \to a \quad \text{(horizon area)}$$

Therefore: $$\eta = \frac{\sigma_{\text{abs}}(0)}{16\pi G} = \frac{a}{16\pi G} = \frac{s}{4\pi}$$ because $$s = \frac{a}{4G}$$
Viscosity of materials

Temperature dependence of the shear viscosity of typical fluids:

Minimum of $\frac{\eta}{s}$ for all common materials is far above $\frac{1}{4\pi}$.
Elliptic flow “measures” $\eta_{\text{QGP}}$

Relativistic viscous hydrodynamics:

$$\partial_\mu T^{\mu\nu} = 0 \quad \text{with}$$

$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - Pg^{\mu\nu} + \eta (\partial^\mu u^\nu + \partial^\nu u^\mu - \text{trace})$$

$$\eta \approx \frac{1}{3} n p \lambda_f = \frac{\bar{p}}{3\sigma_{\text{tr}}}$$

Boost invariant hydrodynamics with $T_0 \tau_0 \sim 1$ requires $\eta/s \leq 0.1$

Small shear viscosity implies:

The QGP is an almost perfect liquid
Question #3: Why…

…is the QGP a nearly perfect liquid, i.e. why is its ratio $\eta/s$ closer to the lower bound than for any other known substance?

Perturbation theory suggests $\eta/s \sim (5 - 10)/4\pi$.

Are parton cross sections nonperturbatively large? If so, why?

Or is there a non-collisional origin of viscosity?
Viscosity of plasmas is known (since 1970s) to be often dominated by momentum transport in “turbulent” electromagnetic fields, which arise due to instabilities of collective modes.

The Quark-Gluon Plasma is a plasma, after all!
Expansion → Anisotropy

Perturbed equilibrium distribution:
\[
f(p) = f_0(p) \left[ 1 + f_1(p) \left( 1 \pm f_0(p) \right) \right]
\]
\[
f_0(p) = \exp\left[ -u_\mu p^\mu / T \right]
\]

For shear flow of ultrarelat. fluid:
\[
f_1(p) = -\frac{5\eta / s}{2ET^2} \left( p^i p^j - \frac{1}{3} \delta_{ij} \right) \Delta_{ij}(u)
\]
\[
\Delta_{ij}(u) = \nabla_i u_j + \nabla_j u_i - \frac{2}{3} \delta_{ij} \nabla \cdot u
\]

Momentum space anisotropy of expanding fluid is a measure of the ratio: shear viscosity / entropy density.

Anisotropic momentum distributions always generate instabilities of soft color field modes.
Instability $\rightarrow$ color turbulence

Wavelength and growth rate of unstable modes can be calculated perturbatively:

$$k_z \sim gQ_s, \quad \Gamma \sim gQ_s < k_z$$

Exponential growth saturates when $B^2 > g^2 T^4$.

Mrowczynski
Strickland et al.
Arnold et al.
QGP viscosity – anomalous

Classical expression for shear viscosity:

\[ \eta \approx \frac{1}{3} n \bar{p} \lambda_f \]

Momentum change in one coherent domain:

\[ \Delta p \approx g Q^a B^a r_m \]

Anomalous (collisionless) mean free path:

\[ \lambda_f^{(A)} \approx r_m \left\langle \frac{\bar{p}^2}{(\Delta p)^2} \right\rangle \approx \frac{\bar{p}^2}{g^2 Q^2 \left\langle B^2 \right\rangle r_m} \]

Anomalous viscosity due to random color fields:

\[ \eta_A \approx \frac{n \bar{p}^3}{3 g^2 Q^2 \left\langle B^2 \right\rangle r_m} \approx \frac{9}{4} s T^3 \left\langle B^2 \right\rangle r_m \]
Shear viscosity

Take moments of

\[
\left[ \frac{\partial}{\partial t} + \frac{p}{E_p} \cdot \nabla_r - \nabla_p \cdot D(p) \cdot \nabla_p \right] f(r, p, t) = C[f] \quad \text{with } p_z^2
\]

\[
D_{ij}(p) = \int_{-\infty}^{t'} \langle F_i^a(\vec{r}(t'), t') U_{ab}(\vec{r}, r) F_j^b(r, t) \rangle
\]

\[
\vec{F}^a = g \left( \vec{E}^a + \vec{\nabla} \times \vec{B}^a \right) = \text{color force}
\]

\[
\frac{1}{\eta} = O(1) \frac{N_c}{N_c^2 - 1} \left\langle F^2 \right\rangle \tau_m + O(10^{-2}) \frac{g^4 \ln g^{-1}}{T^3} \equiv \frac{1}{\eta_A} + \frac{1}{\eta_C}
\]

\[
\int dt' \left\langle F_i^{+}(t') F^{+i}(t) \right\rangle = \left\langle F^2 \right\rangle \tau_m \equiv \hat{q}
\]

\[
= \text{jet quenching parameter !!!}
\]

M. Asakawa, S.A. Bass, B.M.,

PRL 96:252301, 2006

hep-ph/0608270
Connecting jets with the medium

Hard partons probe the medium via the density of colored scattering centers:

\[ \hat{q} = \rho \int q^2 dq^2 \left( \frac{d\sigma}{dq^2} \right) \sim \int dx^- \left\langle F_+^- (x^-) F_+^+ (0) \right\rangle \]

If kinetic theory applies, thermal gluons are quasi-particles that experience the same medium. Then the shear viscosity is:

\[ \eta \approx \frac{1}{3} \rho \left\langle p \lambda_r (p) \right\rangle = \frac{1}{3} \left\langle \frac{p}{\sigma_r (p)} \right\rangle \]

In QCD, small angle scattering dominates:

\[ \sigma_r (p) \approx \frac{2 \hat{q}}{\langle p \rangle^2 \rho} \]

With \( \langle p \rangle \sim 3T \) and \( s \approx 3.6 \rho \) (for gluons) one finds:

\[ \frac{\eta}{s} \approx 1.25 \frac{T^3}{\hat{q}} \quad \text{(A. Majumder, B.M., X.N. Wang, hep-ph/0703082)} \]

From RHIC data: \( T_0 \approx 335 \text{ MeV}, \hat{q}_0 \approx 2.8 \text{ GeV}^2/\text{fm} \rightarrow (\eta / s)_0 \approx 0.10 \)
Manifestations?

- Possible effects on QGP probes:
  - Longitudinal broadening of jet cones (observed – “ridge”)
  - Anomalous diffusion of charm and bottom quarks (observed)
  - Synchrotron-style radiation of soft, nonthermal photons?
  - Field induced quarkonium dissociation?
  - No unstable modes for quarks: quasi-particle picture of QGP is compatible with low viscosity
Summary

The RHIC program has shown that

- equilibrated matter is rapidly formed in heavy ion collisions;
- wide variety of probes available at collider energies;
- systematic study of matter properties is possible.

QGP appears to be a strongly coupled, maybe turbulent color liquid with novel and unanticipated transport properties.

Experimental surprises have become a gold mine for theorists:

- extreme opaqueness of matter to colored probes;
- large enhancement of baryon production;
- collective flow phenomena indicating strong coupling;
- connection to string theory and AdS/CFT duality.

Exciting times for QGP physics at RHIC-2 and LHC lie ahead!
THE END