The Chiral Magnetic Effect

A Status Report

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Selected Publications

- The Chiral Magnetic Effect
  - D. Kharzeev, L. McLerran, H. Warringa, NPA 803 ('08) 227
  - K. Fukushima, D. Kharzeev, H. Warringa, PRD 78 ('08) 074033
  - K. Fukushima, H. Warringa, PRD 80 ('09) 034028
  - K. Fukushima, D. Kharzeev, H. Warringa, NPA 836 ('10) 311
  - K. Fukushima, D. Kharzeev, H. Warringa, PRL 104 ('10) 212001
  - D. Kharzeev, Ann. Phys. 325 ('10) 205 [Review]
  - M. Asakawa, A. Majumder, B. Müller, PRC 81 ('10) 064912
  - S. Nam, PRD 82 ('10) 045017
  - V. Orlovsky, V. Shevchenko, arXiv:1008.4977
  - B. Müller, A. Schäfer, Phys. Rev. C82 ('10) 057902

- RHIC Data
  - B. Abelev et al [STAR Coll.], PRL 103 ('09) 251601
  - B. Abelev et al [STAR Coll.], PRC 81 ('10) 054908
Selected Publications II

- Theoretical Alternatives
  - A. Bzdak, V. Koch, J. Liao, PRC 81 ('10) 031901; arXiv:1008.4919
  - F. Wang, PRC 81 ('10) 064902
  - J. Liao, V. Koch, A. Bzdak, arXiv:1005.5380

- Lattice QCD studies
  - P. Buividovich et al, PRD 80 ('09) 054507
  - M. D’Elia, S. Mukherjee, F. Sanfilippo, PRD 82 ('10) 051501
Overview

- Theoretical foundations
- The chiral magnetic effect
- From CME to charge fluctuations
- Results for CME
- The STAR data
- Alternative explanations
- Challenges
Nonabelian gauge fields are characterized by a topological current (Chern-Simons current):

\[ K^\mu = \frac{g^2}{32\pi^2} \varepsilon^{\mu\nu\alpha\beta} A^a_\nu \left( F^a_{\alpha\beta} - \frac{g}{3} f_{abc} A^b_\alpha A^c_\beta \right) \]

which gives rise to a discrete “charge” (winding number)

\[ \nu = \int d^3x \ K^0 \in \mathbb{Z} \quad K^0 = \frac{1}{16\pi^2} \left( g^2 \mathbf{A}^a \cdot \mathbf{B}^a - \frac{1}{3} g^3 f_{abc} \mathbf{A}^a \cdot (\mathbf{A}^b \times \mathbf{A}^c) \right) \]

when the gauge field configuration is a pure gauge (= vacuum). Transitions between vacua with different values of the winding number require gauge fields with nonzero energy:
The real vacuum state can be any superposition of the vacuum states with definite winding number $\nu$; for example a coherent state:

$$|\theta\rangle = \sum_{\nu \in \mathbb{Z}} e^{i\nu \theta} |0_{\nu}\rangle$$

If such a state is realized in nature, the QCD Lagrangian effectively acquires a term:

$$\Delta \mathcal{L}_\theta = \frac{\theta}{32\pi^2} F_{\mu\nu}^a \ast F^{a\mu\nu} = \frac{\theta}{8\pi^2} \mathbf{E}^a \cdot \mathbf{B}^a$$

that violates T, CP, and P invariance. The presence of such a term in the real world can be probed by searching for an electric dipole moment of the neutron (which violates these symmetries). The current exp. limit on a neutron edm implies: $\theta < 0.7 \times 10^{-11}$.

Thus T, CP, and P are very well respected by QCD in our normal vacuum.

Morley & Schmidt (1985) and Kharzeev, Pisarski & Tytgat (1998) speculated that a vacuum state with $\theta \neq 0$ could be excited in relativistic heavy ion collisions.
The $\eta$ - $\eta'$ problem

There are nine “ground state” pseudoscalar mesons ($\pi$, $\eta$, $K$, $\eta'$) which all should be much lighter than other hadrons with similar quark content, because they are the Goldstone bosons associated with spontaneous chiral symmetry breaking.

True for ($\pi$, $K$), but not for $\eta$ (548 MeV) and $\eta'$ (958 MeV). Why not?

The reason is coupling to the ($gg$) channel, allowed only for flavor singlet state $\eta_0$.

$$
\eta = \eta_8 \cos \theta - \eta_0 \sin \theta \\
\eta' = \eta_8 \sin \theta + \eta_0 \cos \theta
$$

**Problem:** Perturbative coupling is too small to explain the large $\eta'$ mass!

**Solution:** Vacuum fluctuations of the gluon field are strongly enhanced by fluctuations between vacuum fields with different winding numbers.
Instantons

Gauge field energy as function of winding number $\nu$.

Field configurations interpolating between different integer values of $\nu$ are not pure gauges and do not solve the YM equations in real time, but they solve the YM equations in imaginary time: “instantons”!

Veneziano-Witten formula

$$\chi_{\text{top}} = \int d^3 x \langle \nu(x) \nu(0) \rangle \approx \frac{F^2 m_{\eta}^2}{2 N_f}$$

$\eta'$ mass “measures” the winding number fluctuations in the QCD vacuum: $\chi_{\text{top}} = 1/fm^4$. 
$T \neq 0$: Sphalerons

Sphaleron transition rate is determined by classical gauge field dynamics at thermal equilibrium: naïvely rate $\sim (\alpha_s T)^4$, but diffusion at saddle point adds another factor $\alpha_s \ln(1/\alpha_s)$. Numerical estimate: $\frac{1}{2} T^4 \sim 2.5/fm^4$ @ $T = 300$ MeV.


Electroweak sphalerons are thought to play an important role in the cosmological baryon asymmetry, but they (and their QCD counterparts) have never been observed in experiment.
Gluon $\mathbf{E}^a \cdot \mathbf{B}^a$ interacts with $\mathbf{E} \cdot \mathbf{B}$ of the electromagnetic field via a quark loop:

$$\mathcal{L}_{\text{QED}} = \kappa \alpha \alpha_s (\mathbf{E}^a \cdot \mathbf{B}^a) (\mathbf{E} \cdot \mathbf{B})$$

Effective interaction from $\eta, \eta'$ exchange:

$$\kappa \approx \frac{1.46}{\pi^2 f_\eta^2 m_{\eta'}}$$

Pseudoscalar mesons interact with electromagnetic $\mathbf{E} \cdot \mathbf{B}$ via a triangular quark loop:

$$\mathcal{L}'_{\text{QED}} = \sum_{i=\pi^0, \eta, \eta'} \frac{\alpha}{\pi f_i} \phi_i \mathbf{E} \cdot \mathbf{B}$$

$f_i$ is the nonperturbative "meson decay constant"
Maxwell’s equations dictate that the effective interaction \( \mathcal{L}_{\text{QED}} = \mathcal{P} E \cdot B \) gives rise to an anomalous current \( j_{\text{an}} = (\partial_t \mathcal{P}) B = \sigma_\chi B \) (D’Hoker & Goldstone - 1985).

If \( (\partial_t \mathcal{P}) \) has a nonzero expectation value, this implies a new kind of conductivity \( \sigma_\chi \) (the chiral conductivity), which “violates” parity, because \( B \) is parity-even and \( j \) is parity-odd.

But even if \( (\partial_t \mathcal{P}) \) has a vanishing expectation value, nonvanishing fluctuations can exist:

\[
\langle j_i(x) j_k(x') \rangle = \langle \partial_t \mathcal{P}(x) \partial_t \mathcal{P}(x') \rangle B_i(x) B_k(x')
\]

Strong \( B \)-fields exist in non-central relativistic heavy ion collisions; maybe the fluctuations of the anomalous current can be observed!

(Kharzeev, McLerran, Waringa ‘07)
Chiral magnetic effect

Low-frequency limit of anomalous current $j_{an} = \sigma_\chi \mathbf{B}$ is determined by the axial anomaly (Fukushima, Kharzeev & Warringa '08, Kharzeev & Warringa '09). For a thermal quark-gluon plasma:

$$\sigma_\chi \sim - \sum_f \frac{3e_f^2 \alpha_s}{4\pi^3 T^2} \int dt \left( \bar{E}^a \cdot \bar{B}^a \right) = -\frac{4\alpha}{\pi T^2} (\Delta \nu)$$
Hadronic anomalous current

Vector meson dominance (VMD) relates the electromagnetic hadronic current to the neutral rho-meson field:

\[ j^\mu = -\frac{e}{g_{\rho\pi\pi}} \rho^\mu \]

Thus, to generate an “anomalous” current, the B-field needs to convert a pseudoscalar meson into a rho-meson (Asakawa, Majumder, BM ’10):

The relevant interactions are well known from radiative decays:

\[ \Gamma_{\rho^0 \rightarrow \pi^0 \gamma} = 3\alpha g_{\rho\pi\gamma}^2 \frac{p_{\text{cm}}^3}{m_{\rho}^2} \approx 90 \pm 12 \text{ keV} \]

\[ \Gamma_{\eta' \rightarrow \rho \gamma} = \alpha g_{\rho\eta'\gamma}^2 \frac{p_{\text{cm}}^3}{m_{\eta'}^2} \approx 60 \pm 5 \text{ keV} \]

Suppressed, if \( B \) is constant, but not if \( B \) is strongly time dependent.
STAR Data
The STAR data

Azimuthal two-particle correlation with respect to the reaction plane, discriminating between charges of equal or opposite sign:

\[
C^{(\pm,\pm)} = \left\langle \cos \left( \phi^{(\pm)}_\alpha + \phi^{(\pm)}_\beta - 2\Psi_{RP} \right) \right\rangle
\]

\[
\Delta Q \sim C^{(++)} + C^{(--)} - 2C^{(+-)}
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Larger correlation for more peripheral collisions

Larger B?
The STAR observable

\[ C^{(\pm,\pm)} = \left\langle \cos \left( \phi^{(\pm)}_\alpha + \phi^{(\pm)}_\beta - 2\Psi_{RP} \right) \right\rangle \quad \Rightarrow \quad \Delta Q \sim C^{(++)} + C^{(--)} - 2C^{(+-)} \]

- **CME**
  - \( \cos(\pi/2 + \pi/2) = -1 < 0 \)
  - \( \cos(3\pi/2 + 3\pi/2) = -1 < 0 \)
  - \( \cos(\pi/2 - \pi/2) = \cos(3\pi/2 - 3\pi/2) = +1 > 0 \)

- “Trouble Effect”
  - \( \cos(0 + \pi) = -1 < 0 \)
  - \( \cos(0 - \pi) = -1 < 0 \)
The STAR data

Both $\langle \cos(\phi_\alpha + \phi_\beta) \rangle$ and $\langle \cos(\phi_\alpha - \phi_\beta) \rangle$ are negative for like-sign charges!

This does not prove the absence of the CME, but it shows that either
- the CME is nearly canceled by some other effect, or
- the CME is much smaller than the effect causing the STAR data.
The momentum dependence of the effect seen by STAR can be explained by a modest momentum shift (~ 100 MeV/c) of the correlated pair spectrum.

Combining with elliptic flow $v_2$, many effects can contribute to the STAR signal: Coulomb interaction, transverse momentum conservation, etc.
Other backgrounds

Local charge conservation + $v_2$:

$$\gamma_{\alpha \beta} \equiv \langle \cos(\phi_\alpha + \phi_\beta) \rangle = \langle \cos \phi_\alpha \cos \phi_\beta \rangle - \langle \sin \phi_\alpha \sin \phi_\beta \rangle$$

\[ \gamma_p = \gamma_{++} - \frac{1}{2}(\gamma_{++} + \gamma_{--}) \]

Enhanced by elliptic flow

Cluster decays (F. Wang ’10)

Initial-state energy density fluctuations (Petersen, Renk, Bass ’10)

...combined with $v_2$ give correct magnitude of angular correlation, but require assumptions about charge dependence.
CME quantified
CME mechanisms (I)

- **CGC mechanism**: Two gluons from the initial nuclei fuse in the pseudoscalar channel and generate an anomalous current in the strong magnetic field;

- **Glasma mechanism**: Gluons in the “glasma” generate an anomalous current in the strong magnetic field via a winding number fluctuation;

- **QGP mechanism**: Gluons in the equilibrated quark-gluon plasma generate an anomalous current in the strong magnetic field via a winding number fluctuation (“sphaleron”);

- **Corona mechanism**: A neutral pion in the hadronic corona generates an anomalous current by converting into a rho-meson in the strong magnetic field;

- **Hadronic gas mechanism**: A neutral pion in the final hadronic gas phase generates an anomalous current by converting into a rho-meson in the strong magnetic field.
Time scales

Time scale for winding number transitions: \( \tau_{\text{sph}} \approx 1/T \approx 0.5 - 1 \text{ fm/c} \approx \tau_{\text{inst}} \).

Time scale for other chirality changing processes (quark mass, instantons):
\( \tau_{\chi,\text{diff}} \approx 1/m_q \approx (1 - \text{many}) \text{ fm/c} \).

Life-time of the strong magnetic field: \( \tau_B \approx 2R/\gamma \approx 0.1 - 0.2 \text{ fm/c} \).

Intrinsic time scale of hadronic anomalous current interactions:
\( \tau_{\text{had}} \approx 1/m_{\eta'} \approx 0.2 \text{ fm/c} \).

\[ B_{\text{int}} = \int_{-\infty}^{\infty} dt B(0,x_{\perp},t) \]

\[ B_{\text{int}} \approx 2.3Z\alpha b / R^2 \]

\[ eB(\text{MeV}) \]

\[ eB(\text{MeV}^2) \]

\[ \tau(\text{fm}) \]
From $J$ to $\Delta Q$

The anomalous current is not directly observable. What is observed is the final charged particle distribution and its asymmetry with respect to the reaction plane:

$$\Delta Q = \int d^3p \int_{\Sigma_f} \frac{d\sigma_{\mu} p^\mu}{E} \sum_i e_i f_i(x, p) \text{sgn}(p_z) \theta(\sigma_{\mu} p^\mu)$$

Transformation of spatial charge asymmetry into a momentum space asymmetry requires either collective flow or opacity during freeze-out (or both):

Charge asymmetry is created early; it must be transported to the freeze-out surface. Locally separated charged be depleted by diffusion processes.

Altogether, it is a complex transport problem, but one describable with current technology (viscous hydro + diffusion & hadron cascade).
From $J$ to $\Delta Q$ (II)

Two ways to proceed:

1. For isochronous freeze-out, a position-momentum correlation requires collective flow.

$$\Delta Q = \int d^3x \int d^3p \sum_i e_i f_i(x, p; \tau_f) \text{sgn}(p_z)$$

where

$$f_i(x, p; \tau_f) = \exp \left[ -u_\mu(x) p^\mu / T_f + e_i \mu_Q(x) / T_f \right]$$

For weak flow, expand $f(x,p)$ in first order in $\mathbf{v}$. For charged pions only one finds the simple result:

$$\Delta Q \approx \frac{3}{2} \int d^3 x \, \rho(x, \tau_f) \, v_z(x)$$

then use the continuity equation $\partial_t \rho = -\nabla \cdot \mathbf{j}_\text{an}$ to obtain:

$$\langle (\Delta Q)^2 \rangle \approx \frac{9}{4} \int_0^{\tau_f} dt \, dt' \int d^3 x \int d^3 x'$$

$$\times \nabla v_z(x, \tau_f) \cdot \langle \mathbf{j}_\text{an}(x, t) \mathbf{j}_\text{an}(x', t') \rangle \cdot \nabla' v_z(x', \tau_f).$$
2. In the **geometric approximation**, one simply assumes that all charges in the upper hemisphere are emitted upwards, and vice versa:

\[
\Delta Q = \int_{z>0} d^3 x \, \rho(x) = \int d^4 x \, \delta(z) j_z(x)
\]

Predictions using the two approaches differ by a factor \(\sim 30\) (#1 is smaller)!

The weak flow approximation differs from the geometric approximation by a factor

\[
\Theta = \frac{\langle (\Delta Q)^2 \rangle_{\text{flow}}}{\langle (\Delta Q)^2 \rangle_{\text{geo}}} \approx v_f^2 \xi_j / R
\]

where \(\xi_j\) is the anomalous current correlation length.

For \(\xi_j \approx 1\) fm, \(v_f \approx 0.5\) and \(R \approx 7\) fm, one has \(\Theta \approx 0.035 \Rightarrow \text{large uncertainty.} \)
From $J$ to $\Delta Q$ (IV)

A more complete treatment of final state effects needs to include advective effects and charge diffusion in the continuity equation:

$$\frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla \rho - D_{\text{ch}} \nabla^2 \rho = -\nabla \cdot \mathbf{j}_{\text{an}}.$$

This requires the numerical calculation of the charge asymmetry transport in the framework of relativistic hydrodynamics (or other transport models).
Results
This mechanism can be enhanced if the $\eta'$ mass is lowered by medium interactions.

Because CGC color fields are nearly transverse, the fields in $E^a \cdot B^a$ must come from different nuclei. The gluon matrix element can be expressed in terms of the nuclear gluon distribution:

$$\langle j^3(x)j^3(x') \rangle \approx e^2 C \langle (E^a \cdot B^a)(x)(E^b \cdot B^b)(x') \rangle$$

with

$$C = \frac{g_{\rho \eta'}^2}{g_{\rho \pi \pi}^2} \frac{3(Z\alpha)^2 \alpha_s^2 \cos^2 \theta}{(2\pi f_\eta)^2 m_{\eta'}^2 m_\rho^2} \frac{b^2 \gamma^2}{R^6}$$

$$\langle (E^a \cdot B^a)(E^b \cdot B^b) \rangle \propto [x_0 G(x_0)]^2 T_{AA}(\mathbf{x}_\perp; b)$$

This mechanism can be enhanced if the $\eta'$ mass is lowered by medium interactions.
CGC mechanism (II)

The nuclear gluon density can be related to the CGC saturation scale $Q_s$ by:

$$A [\xi_0 G(\xi_0)] = \frac{(N_c^2 - 1) R^2 Q_s^2}{8\pi^2 \alpha_s}$$

To calculate the up-down charge asymmetry fluctuations, we calculate the current through the reaction plane and assume that all charges above are emitted upwards and all charges below are emitted downwards (an overestimate!). After a lengthy calculation involving various additional “reasonable” approximations one finds:

$$\langle (\Delta N_{\text{ch}})^2 \rangle = \begin{cases} 
C \frac{9(N_c^2 - 1) v_f^2}{(8\pi)^3 \alpha_s^2} Q_s^2 f(b) \approx 5 \times 10^{-5} v_f^2 \frac{b^2}{R^2} f(b) & \text{(weak flow)} \\
C \frac{3(N_c^2 - 1)}{32\pi^4 \alpha_s^2} Q_s^3 R f(b) \approx 1.7 \times 10^{-3} \frac{b^2}{R^2} f(b) & \text{(geometric)}
\end{cases}$$

with $f(b) \approx 1 - (b^2/R^2)(1 - b/4R)^2$
We now consider the hadronic corona which never forms a QGP.

Assuming a thermal pion gas with $T = 150$ MeV and integrating over the entire B-field history, one finds in the geometric approximation:

$$
\langle (\Delta N_{ch})^2 \rangle \approx \frac{\pi Z \alpha g_{\rho \pi \gamma} g_{\rho \pi \pi}}{768 e^2} \left( \frac{T}{m_\rho} \right)^6 \frac{b^2}{R^2} f(b) \approx 3 \times 10^{-5} \frac{b^2}{R^2} f(b)
$$

Both results are far too small to explain the STAR data, because this would require:

$$
\frac{\langle (\Delta N_{ch})^2 \rangle}{(N_{ch})^2} \approx 10^{-3} \frac{b^2}{R^2}
$$
Effective action

CME: \( \vec{j} = \sigma \chi \vec{B} \) with \( \sigma \chi = \sum_f \frac{3e^2 g^2}{16\pi^4 T^2} \int dt (\vec{E}^a \cdot \vec{B}^a) \)

Compare with Ohm’s law: \( \vec{j} = \sigma \vec{E} \)

\[
L_{\text{eff}} = -\int \langle \vec{j} \rangle \cdot d\vec{A} = \frac{1}{2} \sigma \int dt' \vec{E}(t')^2 \quad \Leftrightarrow \quad \tilde{L}_{\text{eff}} = \frac{i\sigma}{2\omega} |\vec{E}(\omega)|^2
\]

\[
L^{(\text{CME})}_{\text{eff}} = -\sum_f \frac{e^2 g^2}{8\pi^4 T^2} \int_0^{\tau_B} dt' (\vec{E} \cdot \vec{B}) \int_0^{\tau_{\text{sph}}} dt'' (\vec{E}^a \cdot \vec{B}^a)
\]

Minimize action: \( L = \frac{1}{2} f(B) E^2 + L^{(\text{CME})}_{\text{eff}} \quad \Rightarrow \quad E_{\text{min}} \)

Asymmetry: \( \frac{dN^{(+)}}{dN^{(-)}} \approx 2e \tau_B v E_{\text{min}} / T_f \quad \Rightarrow \quad \Delta^\pm = \frac{dN^{(+)}}{dN^{(+)}} - \frac{dN^{(-)}}{dN^{(-)}} \approx \frac{e \tau_B v E_{\text{min}}}{T_f \sqrt{N_{\text{domains}}}} \)

Final result: \( \Delta^\pm \approx 3.5 \times 10^{-7} \frac{b}{R} \) (compare with \( \Delta_{\text{exp}}^\pm \leq 6 \times 10^{-4} \))
“To Do” list

- Other experimental observables need to be studied, e.g.
  - in-plane (left/right) vs. out-of-plane (up/down) charge correlations;
  - new observables, such as E-by-E charge dipole.

- Beam energy dependence?
  - maximal $B$ is proportional to energy, but time-integrated $B$ is constant;

- System size dependence?
  - Central U+U can have $v_2 > 0$, but $B = 0$.

- If CME is much smaller than STAR effect, how small an effect could be seen with the “best” observable?

- Theoretical studies:
  - Realistic calculations of all mechanisms and backgrounds, or at least improved estimates, are needed.
  - Lattice QCD studies of QCD matter in the presence of strong $B$ field; problem: only static $B$ fields can be studied.
The **Chiral Magnetic Effect** occurs in both, the partonic and the hadronic phases of QCD. It exists as a consequence of well known (but not fully tested) properties of QCD; the question is whether its effect in heavy ion collisions is large enough to be observable.

The phenomenology of the STAR observation points to a different cause of the effect, related to the effect of elliptic flow on charged particle correlations.

“Local parity violation” is a misnomer (in my humble opinion):

![Analogy: Particle in oscillator potential.](image)

The measured position fluctuates event by event, but a measured value $x \neq 0$, or $\langle x^2 \rangle \neq 0$ does not indicate parity violation, as long as $\langle x \rangle = 0$. 
The END