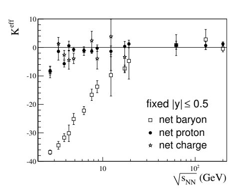
Fluid dynamical fluctuations of net-baryon density near the QCD critical point

Marlene Nahrgang Marcus Bluhm Steffen Bass Thomas Schäfer

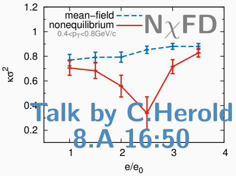
Quark Matter 2017, Chicago, USA February 8, 2017

SUBATECH, IMT Atlantique, Nantes, France

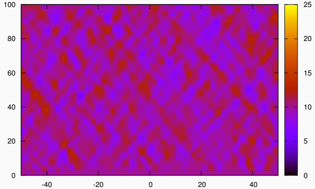
Goal of this talk



MN et al. EPJC72 (2012)



C.Herold, MN et al. PRC93 (2016)



MN et al., CPOD 2016

Present the first numerical implementation of diffusive dynamics of net-baryon density fluctuations at the QCD critical point:

- thoroughly tested against thermodynamic expectations
- includes effects of finite-resolution and -size and baryon conservation
- generation of non-Gaussian fluctuations from Gaussian white noise
- dynamics: nonequilibrium and memory effects!

MN, M. Bluhm, S. Bass, T. Schäfer, in preparation

Diffusive dynamics of net-baryon density

$$\partial_\mu N_B^\mu = 0 \quad \text{net-baryon number conservation}$$

The diffusive dynamics occur such as to minimize the free energy $\mathcal{F}$:

$$\partial_t n_B = \kappa \nabla^2 \left(\frac{\delta \mathcal{F}}{\delta n_B}[\delta n_B] \right)$$

Diffusive dynamics of net-baryon density

$$\partial_\mu N_B^\mu = 0 \quad \text{net-baryon number conservation}$$

The diffusive dynamics occur such as to minimize the free energy $\mathcal{F}$:

$$\partial_t n_B = \kappa \nabla^2 \left(\frac{\delta \mathcal{F}}{\delta n_B} [\delta n_B] \right) + \nabla \mathbf{J}$$

For the study of intrinsic fluctuations include a stochastic current:

$$J(t, x) = \sqrt{2T\kappa}\zeta(t, x)$$

Diffusive dynamics of net-baryon density

$$\partial_\mu N_B^\mu = 0 \quad \text{net-baryon number conservation}$$

The diffusive dynamics occur such as to minimize the free energy $\mathcal{F}$:

$$\partial_t n_B = \kappa \nabla^2 \left(\frac{\delta \mathcal{F}}{\delta n_B}[\delta n_B] \right) + \nabla \mathbf{J}$$

For the study of intrinsic fluctuations include a stochastic current:

$$J(t,x) = \sqrt{2T\kappa}\zeta(t,x)$$

$\zeta(t,x)$ is Gaussian: $\langle \xi^{(2n+1)} \rangle = 0$ and $\langle \xi^{(2n)} \rangle = \sum_{i \neq j = 0}^{2n} \langle \xi_i \xi_j \rangle^n$

$\zeta(t,x)$ is uncorrelated (white noise): $\langle \xi(x,t) \xi(0,0) \rangle = \delta(x)\delta(t)$

Diffusive dynamics of net-baryon density

$$\partial_\mu N_B^\mu = 0 \quad \text{net-baryon number conservation}$$

The diffusive dynamics occur such as to minimize the free energy $\mathcal{F}$:

$$\partial_t n_B = \kappa \nabla^2 \left(\frac{\delta \mathcal{F}}{\delta n_B}[\delta n_B] \right) + \nabla \mathbf{J}$$

For the study of intrinsic fluctuations include a stochastic current:

$$J(t,x) = \sqrt{2T\kappa}\,\zeta(t,x)$$

$\zeta(t,x)$ is Gaussian: $\langle \xi^{(2n+1)} \rangle = 0$ and $\langle \xi^{(2n)} \rangle = \sum_{i \neq j=0}^{2n} \langle \xi_i \xi_j \rangle^n$

$\zeta(t,x)$ is uncorrelated (white noise): $\langle \xi(x,t)\xi(0,0) \rangle = \delta(x)\delta(t)$

Fluctuation-dissipation theorem: $P_{\mathrm{eq}}[\delta n_B] = \frac{1}{\mathcal{Z}} \exp\left(\frac{-\mathcal{F}[\delta n_B]}{T} \right)$

Couplings motivated by 3-dimensional Ising model

$$\mathcal{F}[\delta n_B] = T\int \mathrm{d}^3 r\left(\frac{m^2}{2n_c^2}\delta n_B^2 + \frac{K}{2n_c^2}(\nabla\delta n_B)^2 + \frac{\lambda_3}{3n_c^3}\delta n_B^3 + \frac{\lambda_4}{4n_c^4}\delta n_B^4 + \frac{\lambda_6}{6n_c^6}\delta n_B^6\right)$$

The couplings depend on temperature via the correlation length $\xi(T)$:

$$m^2 = \frac{\tilde{m}^2}{\xi_0^3}\,, \quad \tilde{m} = \frac{1}{\xi/\xi_0}$$

$$K = \tilde{K}/\xi_0$$

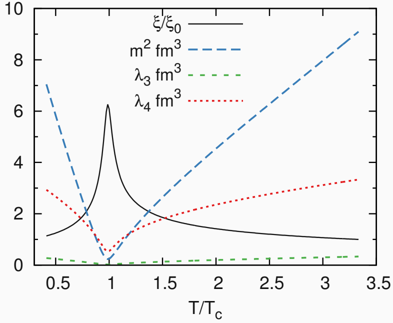
$$\lambda_3 = n_c\,\tilde{\lambda}_3\,(\xi/\xi_0)^{-3/2}$$

$$\lambda_4 = n_c\,\tilde{\lambda}_4\,(\xi/\xi_0)^{-1}$$

$$\lambda_6 = n_c\,\tilde{\lambda}_6$$

M. Tsypin PRL73 (1994); PRB55 (1997)

parameter choice:
$\xi_0 \sim 0.5$ fm, $n_c = 1/3$ fm^{-3}
$K = 1$ (surface tension)
$\tilde{\lambda}_3$, $\tilde{\lambda}_4$, $\tilde{\lambda}_6$ (universal but mapping to QCD)



in this Fig: $\tilde{\lambda}_3 = 1$, $\tilde{\lambda}_4 = 10$

Implementation and scenarios

The diffusion equation:

$$\partial_t n_B = \frac{D}{n_c}\left(m^2 - K\nabla^2\right)\nabla^2 n_B + D\nabla^2\left(\frac{\lambda_3}{n_c^2}\,\delta n_B^2 + \frac{\lambda_4}{n_c^3}\,\delta n_B^3 + \frac{\lambda_6}{n_c^5}\,\delta n_B^5\right) + \sqrt{2Dn_c}\nabla\zeta$$

- semi-implicit predictor-corrector scheme ($\langle n_B \rangle$ perfectly conserved)
- consider 3d system with propagation in one dimension
- diffusion constant $D = \kappa T/n_c = 1$ for equilibrium calculations, T-dependent for dynamical calculations
- equilibrium: let system equilibrate for long times
- dynamics: equilibrate at high temperature, then evolve

Implementation and scenarios

The diffusion equation:

$$\partial_t n_B = \frac{D}{n_c}\left(m^2 - K\nabla^2\right)\nabla^2 n_B \qquad -\vec{v}\cdot\nabla n_B$$
$$+D\nabla^2\left(\frac{\lambda_3}{n_c^2}\,\delta n_B^2 + \frac{\lambda_4}{n_c^3}\,\delta n_B^3 + \frac{\lambda_6}{n_c^5}\,\delta n_B^5\right) + \sqrt{2Dn_c}\nabla\zeta$$

- semi-implicit predictor-corrector scheme ($\langle n_B\rangle$ perfectly conserved)
- consider 3d system with propagation in one dimension
- diffusion constant $D = \kappa T/n_c = 1$ for equilibrium calculations, T-dependent for dynamical calculations
- equilibrium: let system equilibrate for long times
- dynamics: equilibrate at high temperature, then evolve
- (including advection does not change the results)

Implementation and scenarios

The diffusion equation:

Gauss+surface

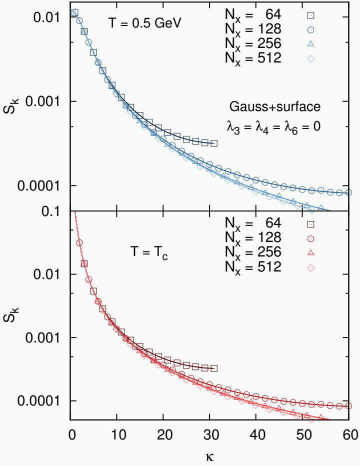
$$\partial_t n_B = \frac{D}{n_c}\left(m^2 - K\nabla^2\right)\nabla^2 n_B \qquad -\vec{v}\cdot\nabla n_B$$

Ginzburg-Landau

$$+D\nabla^2\left(\frac{\lambda_3}{n_c^2}\,\delta n_B^2 + \frac{\lambda_4}{n_c^3}\,\delta n_B^3 + \frac{\lambda_6}{n_c^5}\,\delta n_B^5\right) + \sqrt{2Dn_c}\nabla\zeta$$

- semi-implicit predictor-corrector scheme ($\langle n_B \rangle$ perfectly conserved)
- consider 3d system with propagation in one dimension
- diffusion constant $D = \kappa T/n_c = 1$ for equilibrium calculations, T-dependent for dynamical calculations
- equilibrium: let system equilibrate for long times
- dynamics: equilibrate at high temperature, then evolve
- (including advection does not change the results)

Gaussian fluctuations: static structure factor



Continuum structure factor ($\xi = \sqrt{K/m}$):

$$S(k) = \frac{n_c^2}{m^2}\frac{1}{1+\xi^2 k^2}$$

Discrete structure factor:

(depends on numerical scheme)

$$S_{\mathrm{dis}}(k) = \frac{n_c^2}{m^2}\frac{1}{1+\frac{2K}{m^2\Delta x^2}\left(1-\cos(k\Delta x)\right)}$$

$$\xrightarrow[\Delta x \to 0]{} S(k)$$

$\to$ perfectly reproduced!

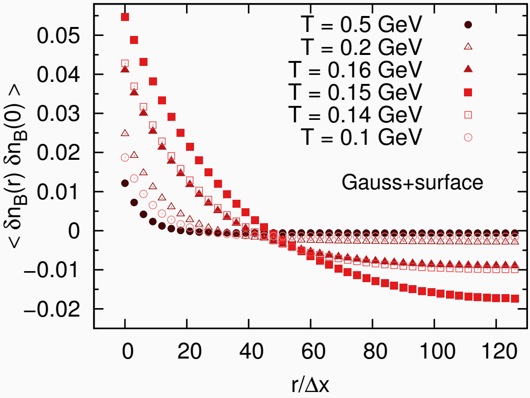
Finite surface tension suppresses high-frequency fluctuations!

(compared to purely Gauss)

Correlation function and - length

For $K = 0$ fluctuations are delta-correlated, finite surface tension leads to a finite correlation length with $\xi > \Delta x$.

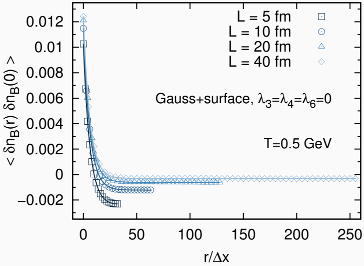
Thermodynamic correlation function: $\langle \delta n_B(r) \delta n_B(0) \rangle = \frac{n_c^2}{2m^2\xi} \exp\left(-\frac{|r|}{\xi}\right)$

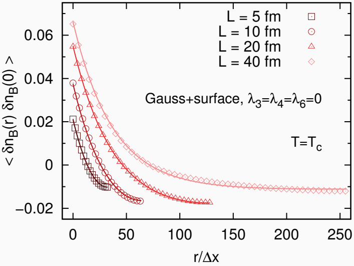


Broader spatial correlations for temperatures near $\mathbf{T_c = 0.15}$ GeV!

Correlation function and baryon conservation

Local fluctuations need to be balanced within L in order to conserve net-baryon density exactly.





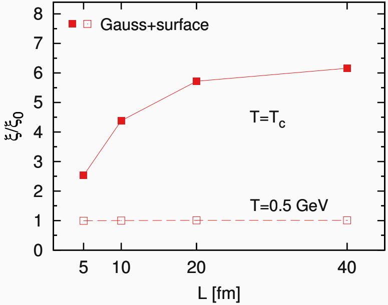
net-B conservation: $\int_L \mathrm{d}r \langle \delta n_B(r) \delta n_B(0) \rangle = 0$: $\rightarrow$ correction term < 0!

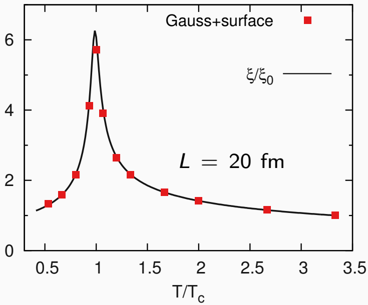
$\Rightarrow$ perfectly reproduced by the numerical result!

note: very large equilibration times needed for $L \rightarrow \infty$

Correlation length

For finite size L the correlation length is strongly limited $\xi < L$ due to net-baryon number conservation.

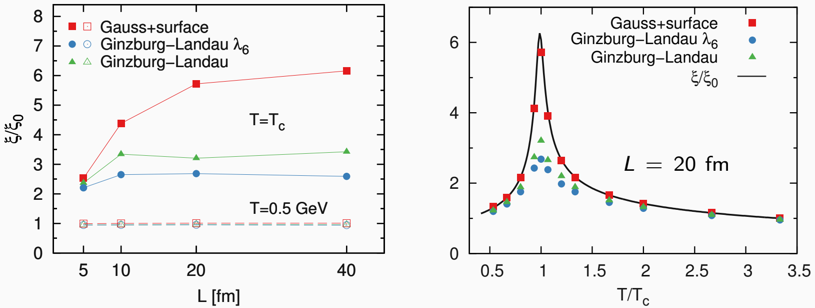




$\rightarrow$ very good realization of the input $\xi(T)$! (only achievable with $K \neq 0$!)

Correlation length

For finite size L the correlation length is strongly limited $\xi < L$ due to net-baryon number conservation.



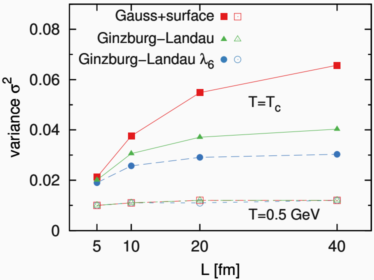
$\rightarrow$ very good realization of the input $\xi(T)$! (only achievable with $K \neq 0$!)

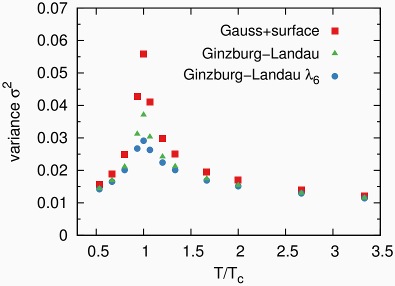
$\rightarrow$ reduction of the realized correlation length for GL.

Temperature dependence of Gaussian fluctuations

In finite systems: variance is reduced compared to TD expectation.

we choose $L = 20$ fm ($N_x = 256$) for the following studies



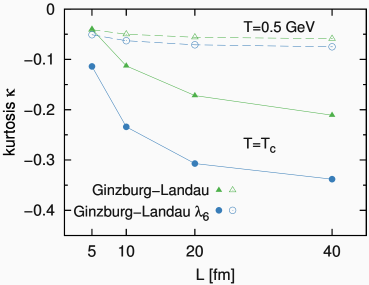


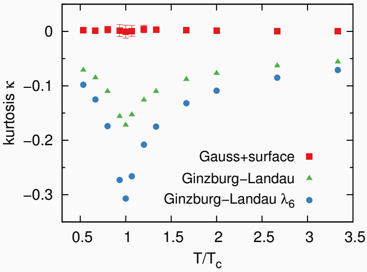
Nonlinear couplings in the Ginzburg-Landau model reduce the variance!

$\Rightarrow$ Could explain why no signal is observed in second-order moments!

Temperature dependence of non-Gaussian fluctuations

Kurtosis vanishes in the absence of nonlinear terms, for Gauss+surface.

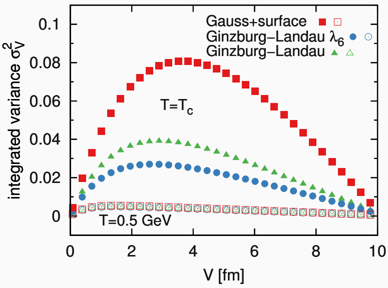


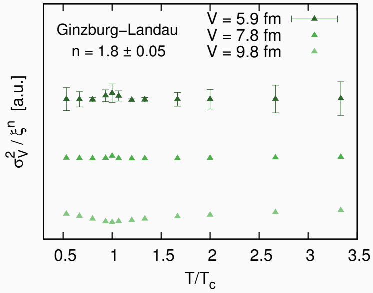


⇒ Negative kurtosis observed for finite nonlinear terms with a pronounced signal for T_c.

⇒ λ_6 term enhances magnitude of κ.

Local vs integrated variance





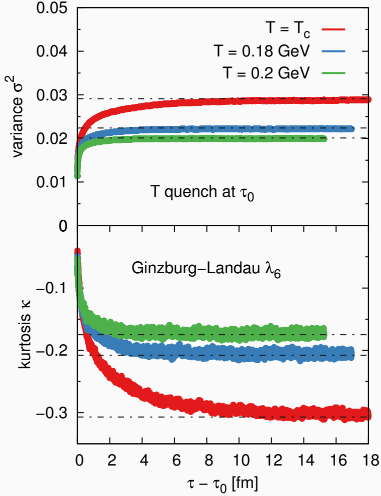
- In this talk: study of local variance (taken over $V = \Delta x$) with $\langle \delta n_B^2 \rangle \propto \xi$.
- Integrated variance: $\langle \delta n_B^2 \rangle_V = \frac{1}{V^2} \int \mathrm{d}x \int \mathrm{d}y \langle \delta n_B(x) \delta n_B(y) \rangle \propto \xi^2$

cf. M. Stephanov et al. PRL81 (1998); PRD60 (1999); PRL102 (2009)

- Including finite-size and net-baryon number conservation we obtain:

$$\langle \delta n_B^2 \rangle_V \sim \xi^n \text{ with } n \sim 1.80 \pm 0.05$$

Dynamics: temperature quench and equilibration



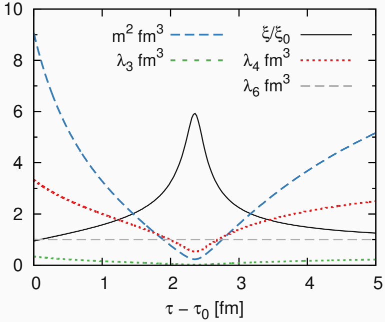
- temperature quench: at τ_0 temperature drops from 0.5 GeV to T
- λ_6 term stabilizes potential vs (spurious) dynamical effects
- fast initial relaxation
- variance approaches equilibrium faster than kurtosis
- long relaxation times near T_c

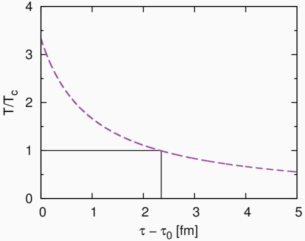
cf. B. Berdnikov, K. Rajagopal PRD61 (2000)

Dynamics: time-dependent couplings

time-dependent temperature

$$T(\tau) = T_0 \left(\frac{\tau_0}{\tau}\right)^{dc_s^2}$$

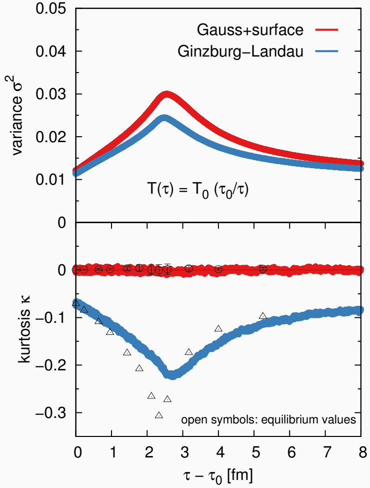




- choose $c_s^2 = 1/3$

  should be ($c_s^2 = c_s^2(T)$)
- initialize system at high temperature $T_0 = 0.5$ GeV
- T_c is reached at time 2.33 fm

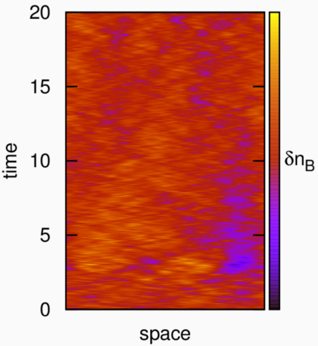
Dynamics: critical Gaussian and non-Gaussian fluctuations



- shift of extrema for variance and kurtosis (memory effects)
- |extremal value| in dynamical simulations below equilibrium values (nonequilibrium effects)
- no dynamical effects on non-Gaussianity in Gauss+surface model
- expected behavior with varying D and c_s^2 (expansion rate)

Conclusions

- successful verification of numerical implementation vs analytical results
- significant effect of net-baryon number conservation
- in presence of non-linear terms:
  - variance is reduced (consistent with experimental data)
  - generation of non-Gaussian fluctuations (from purely Gaussian white noise)
- dynamics: nonequilibrium and memory effects!



Thanks to: BEST COLLABORATION

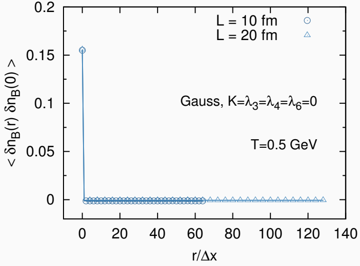
DAAD Deutscher Akademischer Austauschdienst German Academic Exchange Service

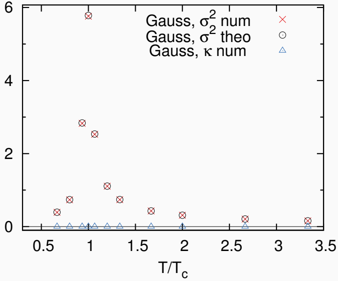
EU Horizon 2020, Marie Sklodowska-Curie No 665778 via National Science Centre, Poland, Polonez UMO-2016/21/P/ST2/04035

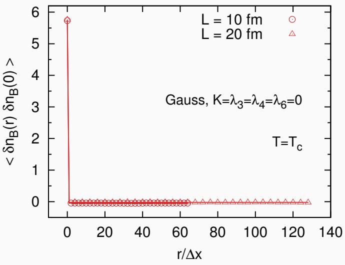
backup

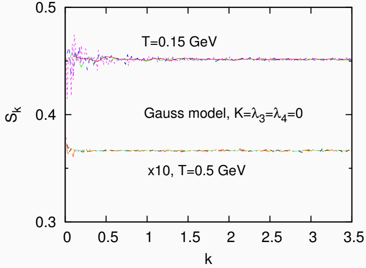
Gauss model: numerics vs analytical results

perfect reproduction of analytical results:

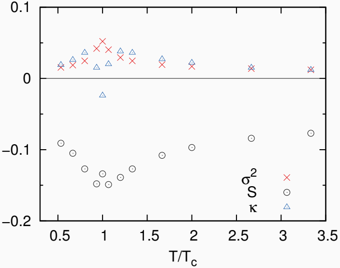




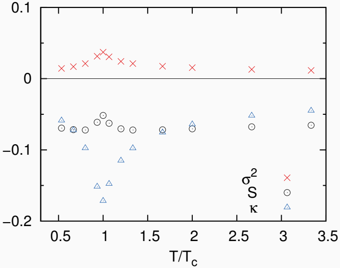




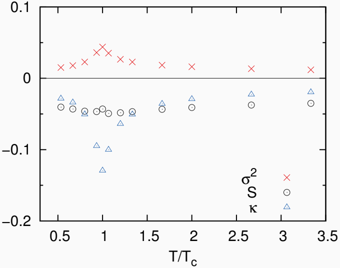
What about the skewness?



$\tilde{\lambda}_3 = 10,\ \tilde{\lambda}_4 = 1,\ \tilde{\lambda}_6 = 0$



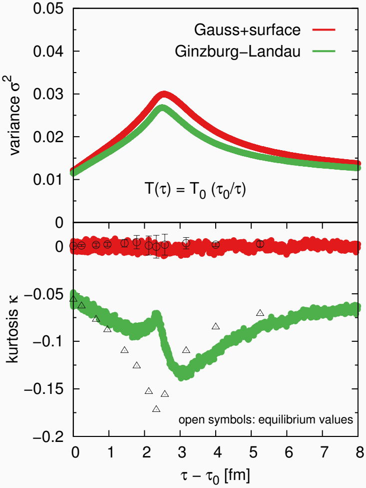
$\tilde{\lambda}_3 = 10,\ \tilde{\lambda}_4 = 10,\ \tilde{\lambda}_6 = 0$



$\tilde{\lambda}_3 = 5,\ \tilde{\lambda}_4 = 4,\ \tilde{\lambda}_6 = 0$

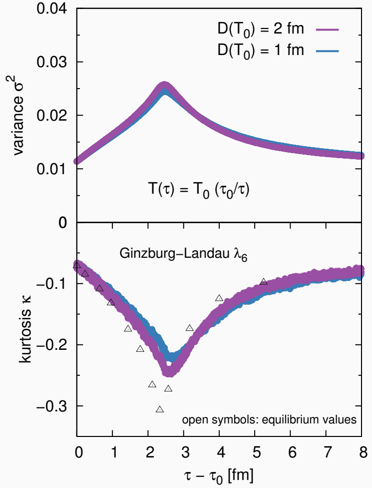
- Delicate interplay between the parameters when $T \to T_c$, where for $\tilde{\lambda}_6 = 0$ all parameters $\to 0$.

"Spurious" dynamical effects?



- "Hörnchen" at T_c when all couplings $\to$ 0...
- spurious? due to instable potential?
- goes away for finite λ_{2n} at T_c

Dynamics with $D = 2$



- for larger D (shorter relaxation time):
  - $\rightarrow$ the maximum of variance becomes larger
  - $\rightarrow$ minimum of kurtosis becomes smaller
  - $\rightarrow$ extrema are closer to T_c