PHY-105: Nuclear Reactions in Stars (continued)

Recall from last lecture that the nuclear energy generation rate for the PP reactions (that main reaction chains that convert hydrogen to helium in stars similar to the Sun – see previous handout) could be written as a power law thus:

$$\epsilon_{PP} \approx \epsilon'_{0,PP} \rho X^2 \left(\frac{T}{10^6 \mathrm{K}} \right)^4$$

for temperatures near (within 50% of roughly) 1.5×10^7K, where
$\epsilon'_{0,PP} = 1.07 \times 10^{-12} \mathrm{Jm^3 kg^{-2} s^{-1}}$
Also, the CNO gave (around the same temperature):

$$\epsilon_{CNO} \approx \epsilon'_{0,CNO} \rho X X_{CNO} \left(\frac{T}{10^6 \mathrm{K}} \right)^{19.9}$$

where $\epsilon'_{0,CNO} = 8.24 \times 10^{-31} \mathrm{Jm^3 kg^{-2} s^{-1}}$, and where X is the mass fraction of H and X_{CNO} is the mass fraction of oxygen/carbon/nitrogen (an average of the three).
We noted the much larger temperature dependence for the CNO cycle compared to the PP chains. This means that lower mass stars, with cooler core temperatures, generate most of their nuclear energy via the PP chains, whereas in more massive stars (somewhat more massive than the Sun) with higher core temperatures the CNO cycle becomes increasingly important. As you can see from the results of these different nuclear reactions, this would play an important role for the elemental composition of stellar cores.

Also recall what happens to T and ρ as the mean molecular weight increases as a result of hydrogen burning.

The triple alpha process

We can see from the equation that we derived for T_{QM} (the typical temperature required for two nuclei to overcome the Coulomb barrier and interact via the strong nuclear force and thus be able to partake in a nuclear reaction, after accounting for quantum mechanical effects – see class notes), that the ${}^{4}_{2}$He produced from hydrogen burning can only begin to "burn" itself when the temperature rises to about 64 times that required for hydrogen burning (**verify this!**).

When the temperature does become sufficent for helium nuclei to undergo "fission" (as result of the density and temperature increasing due to the fractional collapse of the star as the hydrogen runs out), the main reactions that occur are:

$$\boxed{\begin{array}{lcl} {}^{4}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} & \leftrightarrow & {}^{8}_{4}\mathrm{Be} \\ {}^{8}_{4}\mathrm{Be} + {}^{4}_{2}\mathrm{He} & \rightarrow & {}^{12}_{6}\mathrm{C} + \gamma \end{array}}$$

This is the so-called **triple alpha process** (because it involves three alpha particles, or helium nuclei).

The rate of energy generated by this process as a power law in T, centered around 10^8K is:

$$\epsilon_{3\alpha} \approx \epsilon'_{0,3\alpha}\rho^2 X_{He}^3 \left(\frac{T}{10^8\mathrm{K}}\right)^{41}$$

where X_{He} is the mass fraction of helium nuclei in the stellar core where the reaction occurs.

Note the extremely strong temperature dependence.

Question: if the temperature increases by 10% what does the energy generation rate increase by ?

The result of the triple alpha process is energy generated (in the form of photons) and carbon-12 nuclei. When the amount of ${}^{12}_{6}$C present becomes significant other reactions start to compete with the triple alpha process:

$$\begin{array}{lcl} {}^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He} & \rightarrow & {}^{16}_{8}\mathrm{O} + \gamma \\ {}^{16}_{8}\mathrm{O} + {}^{4}_{2}\mathrm{He} & \rightarrow & {}^{20}_{10}\mathrm{Ne} + \gamma \end{array}$$

However, these can only happen when the temperature becomes large enough to overcome the much larger Coulomb barrier compared to that of two helium nuclei interacting.

If the star is massive enough though, this will happen, and if temperatures of about 6×10^8K can be reached then carbon begin to burn via reactions such as:

$$\begin{array}{rcl} {}^{12}_{6}\mathrm{C} \ + \ {}^{12}_{6}\mathrm{C} & \rightarrow & {}^{16}_{8}\mathrm{O} + 2 \ {}^{4}_{2}\mathrm{He} \\ {}^{12}_{6}\mathrm{C} \ + \ {}^{12}_{6}\mathrm{C} & \rightarrow & {}^{20}_{10}\mathrm{Ne} + \ {}^{4}_{2}\mathrm{He} \\ {}^{12}_{6}\mathrm{C} \ + \ {}^{12}_{6}\mathrm{C} & \rightarrow & {}^{24}_{12}\mathrm{Mg} + \gamma \end{array}$$

plus several other possibilities.

If the temperature can reach around 10^9K then oxygen can subsequently burn via:

$$\begin{array}{rcl} {}^{16}_{8}\mathrm{O} \ + \ {}^{16}_{8}\mathrm{O} & \rightarrow & {}^{28}_{14}\mathrm{Si} + \ {}^{4}_{2}\mathrm{He} \\ {}^{16}_{8}\mathrm{O} \ + \ {}^{16}_{8}\mathrm{O} & \rightarrow & {}^{32}_{16}\mathrm{S} + \gamma \end{array}$$

amongst other possible reactions.

However, this can't keep going on forever even if the star is massive enough such that temperature can keep rising due the gravitational collapse at each successive stage of nuclear burning. **Recall** that for nuclei around a mass number of A ~ 56 (for example ${}^{56}_{26}$Fe and some isotopes of Ni), energy can no longer be released by nuclear fission reactions, so that when/if fission reactions occur with these heavy nuclei energy is actually *absorbed*. This has dire consequences for the star which can no longer produce energy to withstand the gravitational collapse, and so.....its lights out, literally!

Of course, the star can only reach this stage **if** the temperature is high enough for the Coulomb barrier to be overcome. If the star is not massive to reach this stage then nuclear burning, and therefore gravitational collapse, will occur earlier. For example, if star was so light (and these exist, but they have to be less than about half the mass of the Sun), that the temperature for helium burning could never be reached, then the star would collapse after hydrogen burning leaving a cold dense mass of helium nuclei. If it stops after helium burning, a cold core of mostly carbon and oxygen (and a bit of nitrogen) is left – this is the end result of a *white dwarf* ("cold" here is a relative term – its still very hot, in fact "white" hot, hence its name, but not hot enough for the heavier nuclei of oxygen and carbon to "burn"). We'll look at this possibility, and what happens to much more massive stars in detail later.

But before that, I want to briefly look at how energy is transported in stars. We've seen how it is produced, but somehow the energy produced in the core has to get to the surface of the star.

But first, as an interesting aside, a very famous mystery in astrophysics that was recently (10 years or so ago) solved was the *solar neutrino problem.* It is worth spending a bit of time on this.....

The solar neutrino problem

From the PP reactions, that dominate energy production in the Sun, we see that neutrinos, ν_e, are produced, and we can determine fairly precisely their rate of production from these reactions.

Neutrinos have no charge, so they can't interact electromagnetically, and they don't experience the Strong nuclear force (which acts on the property of "colour" which only quarks possess). They can *only* experience the Weak nuclear force, which, because it is several orders of magnitude weaker than the others, means that neutrinos can travel through the Sun (and Earth) without interacting at all (or rather with only a small probability of interacting – to stop 50% of a beam of neutrinos requires they travel through about a light-year of lead!). However, they can be, and are, detected on Earth in various experiments (because they still do have a tiny probability of interacting with some detector material to give a signal that is then measured, so a very small fraction of those that pass through a detector do in fact "stop").

In one of the first solar neutrino detection experiments (in the early 1970's by Raymond Davis and collaborators), a huge detector containing about 100,000 gallons of C_2Cl_4 (perchloroethylene) was located about 1 mile underground, in an old gold-mine in Homestake, SD.

As solar neutrinos travelled through the Earth and the huge tank of C_2Cl_4 one isotope of the chlorine present, ${}^{37}_{17}\mathrm{Cl}$, can interact with ν's of sufficient energy via the following interaction:

$$ {}^{37}_{17}\mathrm{Cl} + \nu_e \quad \rightarrow \quad {}^{37}_{18}\mathrm{Ar} + e^- $$

where, ${}^{37}_{18}\mathrm{Ar}$ is a radioactive isotope of Argon with a half-life of $\tau \sim 35$ days.

Every few months the accumulated ${}^{37}_{18}\mathrm{Ar}$ was purged from the tank and the number of Argon atoms counted (from their radioactive decays). The theory of nuclear reactions in the Sun and neutrino interactions with the detector predicted a rate of about 1.5 Argon atoms produced per day, however, only about 0.5 per day were observed, about a third of the prediction.

This was called the **Solar neutrino problem** and was confirmed by various other experiments in the following years.

When an observation doesn't agree with theory, there is no shortage of physicists espousing a solution. In this case the most promising were:

- Something was wrong with the predictions of nuclear reaction rates and/or energy transportation rates in the Sun. A popular theory was the existence of *Weakly Interacting Massive Particles* (or WIMPS, also a leading candidate for Dark Matter in the Universe – more on that later). It was though that WIMPS, because they are massive, are attracted to the Sun's centre where they could be responsible for transporting some of the energy near the centre to other regions, thereby relieving the neutrinos of some of their role in doing this.

- Neutrinos are **not** massless (a shocking possibility at the time). If ν's have a very small mass and $m(\nu_e) < m(\nu_\mu) < m(\nu_\tau)$ then they can experience so-called neutrino oscillations from one type to another:

$$\nu_e \leftrightarrow \nu_\mu \qquad \nu_e \leftrightarrow \nu_\tau \qquad \nu_\mu \leftrightarrow \nu_\tau$$

  The rate of these oscillations depends on the the Δm's (mass differences) between the different ν species.

Subsequent experiments have conclusively shown these oscillations do in fact take place, with the consequence that neutrinos have mass. (This details of this consequence is outside the scope of the course, but a qualitative way to think about it is that if neutrinos were massless then they would travel with the speed of light and hence their "clocks" will be stopped according to our frame of reference. If their clocks are stopped then they can't do *anything* we can observe. Because they *are* observed to do something, that is, oscillate into another species which you can think of as an event in Special Relativity, then their clocks can't be stopped, they must be travelling with a speed less than c, and therefore they must have mass.)

The result for the Davis experiment, meant that some of the ν_e's emitted by the Sun will oscillate to ν_μ and ν_τ along their journeys to Earth. The Davis experiment could only detect ν_e's due to the reaction given above, so the fact that the measured rate was low meant some had oscillated to ν_μ's and ν_τ's which couldn't be detected (in fact about 2/3 of them had).
Some experiments are now designed to detect ν_μ's and ν_τ's, and the rates observed agree remarkably well theoretical predictions.

Energy Transport

The energy produced in a stellar core must somehow be transported to the surface. Recall that we thus far have differential equations relating Pressure, Mass, and Luminiosity to the radius, r;

$$\frac{\mathrm{dP}}{\mathrm{dr}} = -\frac{\mathrm{GM_r}\rho}{\mathrm{r}^2} \qquad \text{(hydrostatic equilibrium)}$$

$$\frac{\mathrm{dM_r}}{\mathrm{dr}} = 4\pi\mathrm{r}^2\rho \qquad \text{(mass conservation)}$$

$$\frac{\mathrm{dL_r}}{\mathrm{dr}} = 4\pi\mathrm{r}^2\rho\epsilon \qquad \text{(energy generation)}$$

but do not yet have an equation relating the temperature to r. Such an equation will depend on how energy is transported and distributed in the star.

There are three main energy transport mechanisms at play in stellar interiors. These are same mechanism as for thermal transfer you should be familiar with from thermodynamics, though their details are somewhat different in the environments of stellar interiors. They are:

- **Radiation:** the energy produced in the nuclear reactions is carried to the surface in the form of photons, which are being absorbed and re-emitted as they interact with matter.
- **Convection:** similarly to the more familiar convection in air, hot stellar mass regions carrying energy in the form of kinetic energy of the particles tend to move outward, while cooler regions tend to move inward. This is a very complicated mechanism to accurately describe in stellar models.
- **Conduction:** energy is transported through the collisions between particles. This is generally an insignificant mechanism of stellar energy transport.

Radiation is the most important so we'll look at that in more detail.

Radiation

The rate at which energy flows through the stellar material via the mechanism of radiation is determined by the *opacity* of the material, κ. You can qualitatively think of opacity as the resistence of the stellar material to the passage of radiation.

The *Radiation Transport Equation* relates the pressure due to radiation, P_{rad}, at a radius, r, to the outward radiative flux, F_{rad}:

$$\frac{dP_{rad}}{dr} = -\frac{\bar{\kappa}\rho}{c}F_{rad}$$

Note that dP_{rad}/dr is the *radiation* pressure gradient, and that the stellar material density, ρ, is a function of r as usual. Also note that κ has units of m^2kg^{-1}.

Recall the form of the Steffen-Boltzmann law; $P_{rad} = \frac{1}{3}aT^4$.

$$\Rightarrow \quad \frac{dP_{rad}}{dr} = \frac{1}{3}a(4T^3)\frac{dT}{dr}$$

$$\Rightarrow \quad \frac{dT}{dr} = -\frac{3}{4ac}\frac{\bar{\kappa}\rho}{T^3}F_{rad} = -\frac{3\kappa L_r\rho}{16\pi acr^2T^3}$$

(verify this)
Note that this equation implies that as the flux or opacity increases, the temperature gradient becomes steeper, in this case more negative.

So now we have our long sought after equation relating T to r (which we need because it will tell us what can and can't happen in a given region of the star, most importantly what nuclear reactions can occur). But we still need to calculate the opacity, κ.

To understand how to calculate κ we must first understand the ways a photon can interact with matter. It can lose energy in 4 main ways, as we'll discuss, but the rates of each interaction depend on the photon's frequency, ν, making a detailed calculation of κ difficult. Here, I'll just qualitatively discuss what goes into the calculation, then assuming values for κ, what they actually mean.

Next, we'll look at the possible photon interactions (you've already seen two of them).

Photon Interactions

(1) **Bound-bound absorption**: A photon is absorbed by a bound electron in an atom moving it to a higher bound energy level. If the electron was initially in the ground state with an energy E_1, and finally in the first excited state with energy E_2, then since the photon was completely absorbed we must have:

$$E_\gamma = h\nu_\gamma = E_2 - E_1$$

These processes are what produce the observed spectral (absorption) lines of stars. Typically, temperatures are such that $h\nu_\gamma$ is much larger than $E_2 - E_1$ so this doesn't play a significant role in energy transport.

(2) **Bound-free absorption**: A photon is absorbed by a bound electron such that the electron escapes (atom ionized):

$$E_\gamma = h\nu_\gamma = -E_1$$

Again, generally not important in typical stars, and depends on how many bound electrons exist.

(3) **Free-free absorption**: A free electron of energy E_i absorbs a photon resulting in an increase of the electron's energy to E_f, where:

$$E_\gamma = h\nu_\gamma = E_f - E_i$$

(4) **Scattering**: (see HW#3 problem 1)
This process (Compton scattering) does not lead to the absorption of a photon but does slow down the rate at which energy escapes from a star. We have:

$$E_{\gamma f} - E_{\gamma i} = \frac{hc}{\lambda_f} - \frac{hc}{\lambda_i}$$

where;

$$\lambda_f - \lambda_i = \frac{h}{m_e c}(1 - \cos\theta)$$

Note that the reverse of the absorption processes can (and do) also occur, which is how photons propagate through the stellar material.

As mentioned, calculation of opacity, κ, difficult because all these processes for all the electrons in the stellar material (both bound and free) must be considered. But in general we find: κ is low at both very high and low temperatures. At high temperatures most photons have high energy ($\sim kT$), and are less easily absorbed than lower energy photons (and most atoms are ionized so only the 3rd and 4th processes above contribute). At low temperatures most atoms are not ionized and so there are fewer free electrons available to scatter radiation or take part in free-free absorption, and most photons don't have enough energy to ionize atoms.

Opacity has a maximum value at intermediate temperatures where bound-free and free-free absorption are important.

For example: In the Sun;

$$\begin{aligned} \rho_c &\sim 10^5\,\mathrm{kgm^{-3}} \\ \kappa &\sim 0.1\,\mathrm{m^2kg^{-1}} \\ \Rightarrow \quad \kappa\rho_c &\sim 10^4\,\mathrm{m^{-1}} \end{aligned}$$

Note that the units of $\kappa\rho_c$ are $\mathrm{m^{-1}}$, implying the inverse of this quantity has units of length – this is called the "mean free path" of the photon (which comes from the definition of κ: larger κ implies larger "resistance" to photon's propagation which implies shorter paths between interactions (mean free paths)).

So, a typical photon travelling from the centre of the Sun is absorbed or scattered after it has travelled about 10^{-4} m.

Further out when $\rho \sim 10^3\,\mathrm{kgm^{-3}}$, we find $\kappa \sim 10\,\mathrm{m^2kg^{-1}}$ so that the mean free path of photons (radiation) is again about 10^{-4} m.

Also note that in the Sun,

$$\mathrm{T_c} \sim 1.5 \times 10^7\,\mathrm{K} \quad \Rightarrow \quad \frac{\mathrm{dT}}{\mathrm{dr}} \sim 2 \times 10^{-2}\,\mathrm{Km^{-1}}$$

Therefore, $\Delta\mathrm{T}$ between when a typical photon is emitted or absorbed is $\sim (2 \times 10^{-2}\mathrm{Km^{-1}})(10^{-4}\mathrm{m}) \sim 2 \times 10^{-6}\,\mathrm{K}$. That is, there is a very small temperature difference between typical interaction lengths of the photons, which is why thermal equilibrium is a very good approximation allowing us to use the Black-body (Planck) distribution for photon energies, $B_\nu(T)$.