PHY-105 Assignment #2 Due before 8:00pm Monday, 14 February 2011

Problem 1 [4 pts]
In its rest frame, quasar Q2203+29 produces a hydrogen emission line of wavelength 121.6 nm. Astronomers on Earth measure a wavelength of 656.8 nm for this line. Determine the redshift parameter (z) and the speed of recession for this quasar.

Problem 2 [3 pts]
Quasar 3C 446 is extremely variable: its luminosity at optical wavelengths has been observed to change by a factor of 40 in as little as 10 days. Using the redshift parameter $z = 1.404$ measured for 3C 446, determine the time for the luminosity variation as measured in the quasar's rest frame.

Problem 3 [6 pts]
Using the aid of space-time diagrams:

(a) Show that if two events are space-like separated, there is a Lorentz frame in which they are simultaneous. Does a Lorentz frame exist in which they are simultaneous if the two events are time-like separated ? Why or why not ?

(b) Show that if two events are time-like separated, there is a Lorentz frame in which they occur at the same point (same spacial coordinate values). Is this also true if the two events are space-like separated ? Why or why not ?

(c) If in a Lorentz frame two events A and B occur at times t_A and t_B respectively where $t_B > t_A$ (that is, B occurs after A), does a Lorentz frame exist in which A occurs after B ? If this is possible, what must the relationship between A and B in the first Lorentz frame be (space-like, time-like, or null, separated) ?

Problem 4 [6 pts]
Show that the total relativistic energy of a particle travelling with speed v, $E = \gamma m_0 c^2$, can also be expressed as $E^2 = p^2c^2 + m_0^2c^4$, where m_0 is the particle's rest mass, and p is the relativistic momentum, $p = \gamma m_0 v$.

Problem 5 [6 pts]
(a) A neutral pion can decay to two photons thus: $\pi^0 \rightarrow \gamma + \gamma$.
The π^0 has a rest mass of $135\,\text{MeV}/c^2$. If a π^0 at rest in a particular frame of reference decays to two photons, use the conservation of relativistic energy and momentum to determine the energy of both photons. What are the wavelengths and frequencies of these photons ?

(b) A rarer decay mode for the π^0 is to an electron and a positron: $\pi^0 \rightarrow e^+ + e^-$. Given that the mass of the electron (and positron) is $0.511\,\text{MeV}/c^2$, determine the total energy, kinetic energy, and speed of the electron and positron.

Problem 6 [8 pts]
Consider the collision of a photon with wavelength λ_i with an electron of mass m_e at rest. After the collision the scattered photon makes an angle θ with respect to the initial photon direction, and has wavelength λ_f. Write down the conservation of relativistic energy and momentum equations and using these derive the Compton scattering equation:

$$\Delta\lambda \;=\; \lambda_f - \lambda_i \;=\; \frac{h}{m_e c}(1 - \cos\theta)$$

Problem 7 [4 pts]
Calculate the energies (in eV) and wavelengths (in nm) of the longest wavelength photons emitted by a downward transitioning electron in each of the Lyman, Balmer, and Paschen series of the hydrogen atom. In what regions of the electromagnetic spectrum are these ?

Problem 8 [3 pts]
Show that, at room temperature, the thermal energy $kT \approx 1/40\,\mathrm{eV}$. At what temperature is kT equal to 1 eV ? to 13.6 eV ? (the latter is the temperature at which the thermal energy is sufficient to ionize hydrogen.)

Problem 9 [3 pts]
The cosmos is pervaded by a 3 K radiation field, which is regarded as the "echo" of the Big Bang. This radiation field is called the Cosmic Microwave Background. Calculate the energy and wavelength of this radiation. In what region of the electromagnetic spectrum is this ?

Problem 10 [7 pts]
Consider a gas of neutral hydrogen atoms.

(a) At what temperature will equal numbers of atoms have electrons in the ground state and in the second excited state ($n = 3$) ?

(b) At a temperature of 85,400 K, when an equal number (N) of atoms are in the ground state and in the first excited state, how many atoms are in the second state ($n = 3$) ? Express your answer in terms of N.

(c) As the temperature $T \to \infty$, how will the electrons in the hydrogen atom be distributed, according to the Boltzmann equation ? That is, what will be the relative numbers of electrons in the $n = 1, 2, 3, \ldots$ orbitals ?