

Problem 1 [3 pts]

Take the diameter AB of the Earth's orbit as 3×10^8 km and consider a star S at a distance d , such that $SA = SB$ and the angle $ASB = 2$ arcseconds. Calculate d . This is the distance unit of one *parsec*. Derive the relation of a parsec to a light year.

Problem 2 [2 pts]

To get a feel for distance scales in the universe, what are the following distances in parsecs (pc). (You'll need to look up some of these distances yourselves, and some are given in the attachment of constants and conversion factors.)

- (a) Diameter of a proton (in SI units it is about $1 \text{ fm} = 10^{-15} \text{ m}$ (femtometer))
- (b) Radius of the Earth
- (c) Distance between the Earth and Sun
- (d) Size (diameter) of the Milky Way
- (e) Distance from the Milky Way to the next nearest galaxy
- (f) Distance from the Milky Way to the furthest known galaxy
(do a search to find out what this currently is)

Problem 3 [3 pts]

Given that the Sun takes about 2×10^8 years to make a circular orbit around the Galactic Centre, at a distance of 10 kpc, estimate the mass of the Galaxy contained within the solar orbit, assuming that it is spherical and ignoring any effects of mass lying outside the orbit. Assuming the Sun is a typical star and is at the outer edge of the galaxy, estimate the number of stars in our galaxy. (Note: the unit "kpc" is a *kilo-parsec*. See also the conversions attached to this problem set.)

Problem 4 [2 pts]

The lifetime of the particle called the pi meson (or pion), is $\tau_\pi = 2.5 \times 10^{-8} \text{ s}$ when the pion is at rest relative to the observer measuring its decay time. What is the lifetime measured by an observer at rest for pions travelling with a speed of $v = 0.999c$?

Problem 5 [4 pts]

Text problem 1.1. Explain both in the frame of the observer and the muon.

Problem 6 [5 pts]

Text problem 1.4

Problem 7 [5 pts]

Text problem 1.6

Problem 8 [4 pts]

(a) Write down the Lorentz transformations relating observations in a reference frame (t', x', y', z') that is moving with constant speed v relative to another reference frame (t, x, y, z) .

State clearly the assumptions you are making for the form in which you write these.

(b) Show by explicit computation that the proper time interval, $d\tau$, where;

$$c^2 d\tau^2 = c^2 dt^2 - (dx^2 + dy^2 + dz^2)$$

is invariant under Lorentz transformations. That is, $d\tau^2 = (d\tau')^2$.

Problem 9 [4 pts]

Relativity Handout: Problem 18

Problem 10 [4 pts]

In class we derived the relativistic form of the kinetic energy to be:

$$K = m_0 c^2 (\gamma - 1)$$

Show that this reduces to the familiar $K = \frac{1}{2} m_0 v^2$ for $v \ll c$ (the Newtonian limit).
(Hint: using a Taylor expansion will be useful.)

Problem 11 [4 pts]

Relativity Handout: Problem 44

Problem 12 [4 pts]

Text problem 1.9

Problem 13 [6 pts]

Text problem 1.10

Useful constants and conversions

Speed of light	$c = 2.99792458 \times 10^8 \text{ m/s}$
Planck's constant	$h = 6.626 \times 10^{-34} \text{ Js}$
	$\hbar = \frac{h}{2\pi}$
Gravitational constant	$G = 6.672 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$
Boltzmann constant	$k = 1.38 \times 10^{-23} \text{ J/K}$
Radius of Sun	$R_{\odot} = 6.96 \times 10^8 \text{ m}$
Mass of Sun	$M_{\odot} = 1.989 \times 10^{30} \text{ kg}$
Radius of Earth	$R_{\oplus} = 6.38 \times 10^6 \text{ m}$
Mass of Earth	$M_{\oplus} = 5.974 \times 10^{24} \text{ kg}$
Average Earth to Moon distance	$r_{EM} = 3.84 \times 10^8 \text{ m}$
Average Earth to Sun distance	$r_{ES} = 1.496 \times 10^{11} \text{ m} = 1 \text{ AU (Astronomical Unit)}$
Mass of electron	$m_e = 9.11 \times 10^{-31} \text{ kg} = 0.511\text{MeV}/c^2$
Mass of proton	$m_p = 1.67 \times 10^{-27} \text{ kg} = 938.279\text{MeV}/c^2$
Mass of neutron	$m_n = 1.68 \times 10^{-27} \text{ kg} = 939.573\text{MeV}/c^2$

$$1 \text{ J} = 10^7 \text{ erg} = 0.239 \text{ cal}$$

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

$$1 \text{ keV} = 10^3 \text{ eV}, 1 \text{ MeV} = 10^6 \text{ eV}, 1 \text{ GeV} = 10^9 \text{ eV}$$

$$1 \text{ light year (lyr)} = 9.4605 \times 10^{15} \text{ m}$$

$$1 \text{ Parsec (pc)} = 3.0856 \times 10^{16} \text{ m} = 3.26 \text{ lyr}$$