PHY-105: Final Stages of Stellar Evolution

First lets summarize what we covered last lecture (March 23). For typical stars, once the H in the core depletes ϵ_{PP} must cease. But now, although T_c isn't yet hot enough to initiate He burning, it has increased such that H burning continues to generate energy in a shell (or envelope) around the small, predominantly He core. In this core T_c is almost constant ($L_r \approx 0 \;\; \Rightarrow \;\; \frac{dT}{dr} \approx 0$). This means that for the core to support the material above it, the required $\frac{dP}{dr}$ has to be the result of a continuous increase in ρ.

As the H burning shell continues to consume H, the isothermal He core grows in mass, until it becomes too great and can no longer support the material above it. The maximum fraction of a star's mass that can exist in an isothermal core and still support the outer layers us given by the Schonberg-Chandrasekhar limit:

$$\left(\frac{M_{ic}}{M}\right)_{SC} \approx 0.37 \left(\frac{\mu_e}{\mu_{ic}}\right)^2 .$$

Refer to the class notes to make sure you understand what each term means. For a star like the Sun, we saw that this fraction is about 8%.

What consequently happens to the star depends critically on mass, but we'll look at typical cases. When the SC limit is reached, core begins to collapse causing evolution to occur on a much faster timescale. The core collapse releases gravitational potential energy, about half of which goes into heating up the core. When helium burning can occur further collapse is prevented by the energy released (radiation pressure) from He burning. (We note here that this isn't entirely true, but we'll keep to this somewhat simplistic picture. In reality, when the density gets very large, there is a component of the pressure provided by electron degeneracy pressure – see previous handout.) As each successive fuel supply is exhausted in the core, these regions contract/collapse and heat up until the next series of nuclear reactions can occur. If the star is massive enough this can occur all the way up to the neighbourhood of Fe.

For stellar masses of $M <\sim 8 - 10M.$, nuclear burning stops when the Carbon-Oxygen core is reached, and typically the surrounding mass shells have been expelled by the release of energy from H/He burning. The hot central object that is revealed is called a *white dwarf*; a C-O core maintaining hydrostatic equilibrium via electron degeneracy pressure. The maximum mass that a degenerate electron gas can support is $\sim 1.4M.$ – this is called the *Chandrasekhar mass limit*, and is the maximum mass of a white dwarf (note that is not the mass of the original star which has had most of its mass from the outer shells blown off).

For $M >\sim 10M.$ star's core can get hot enough to burn C, O, and then Si, etc., up to Fe. As we saw these later stages take successively less time (Si burning on the order of days !). When the central regions of stars composed of nuclei in the neighbourhood of Fe, no further energy releasing reactions can occur – but T still very high and some nuclei will have sufficient kinetic energy to participate in endothermic nuclear reactions.

At the extremely high temperature's now present in the core, γ's have enough energy to destroy heavy nuclei: called *photodisintegration*. Refer to class notes for details. Photodisintegration is highly endothermic → core collapses, pressure from electron degeneracy only thing supporting the star. With significant protons now in the core, these electrons can now be captured via:

$$p^+ + e^- \;\; \rightarrow \;\; n + \nu_e$$

resulting in a huge escape of energy by neutrinos (called a "neutrino burst"). Core subsequently collapses extremely rapidly.

If the **initial** mass of the star was $<\sim 25M.$, the remnant in the inner core will stabilize and become a "neutron star", supported by "neutron degeneracy pressure".

If $M > 25M.$, even neutron degeneracy pressure cannot support star, and the final collapse will be complete producing a "Black Hole". In both these cases, their formation is accompanied by the enormous production of neutrinos in the neutrino burst (total energy released in a matter of seconds ~ 100 times more than the Sun will produce in its entire main sequence lifetime !!). As the core collapses it produces shock waves which blow of the outer layers producing a large γ release – this is what we call a "Type II supernova".

See text for more details. Also refer to text for the discussion of the most likely mechanism behind a "Type I supernova". In particular read Chapter 4, sections 4.1, 4.2, 4.3, and understand the different types and characteristics of supernovae.

The exact mechanisms behind types I and II supernovae and somewhat uncertain still, but we think we understand what they are most likely die to. For Type I the mechanism is an "exploding white dwarf" (see section 4.3).

For type II, as we have seen, the mechanism is the catastrophic collapse of an iron core. Photodisintegration $\rightarrow \nu$ burst, which preceeds maximum light and can be used to "trigger" on supernovae. Remnant is either a neutron star or black hole – we'll look at both of these in more detail next lecture(s). With photodisintegration, core collapses extremely rapidly, dissociating itself from the outer layers. However, when neutrons become degenerate this collapse suddely stops, causing shock waves (or "bounce") to move outwards, initiating the type II supernova explosion.

Type II supernova occur about once every 50 years in the Milky Way, but observations are rare: most well-known being; Crab Supernova (SN 1054), which gives us the Crab Nebula which is still expanding at $\sim 1500km/s$ with $L \sim 10^5 L.$ 950 years later(!); SN 1987A which was observed quite recently (in 1987 as the name implies).

As we've seen type II supernovae require $p^+ + e^- \quad \rightarrow \quad n + \nu_e$. We can estimate the density at which this reaction is possible, to give us some insight into the state of stellar matter in the cores that are just about to supernova. The next 2 pages go into some detail of this estimate.

M. Kruse, Spring 2009

PHY-105: Estimate of density of star's core for required for electron capture reaction

We have seen that if a star's Main-Sequence mass is greater than about $10M_\odot$ the end result will be a Neutron Star or a Black Hole. If the star is massive enough, such that the photon energies become sufficient for photodisintegration (see class notes), we end up with a hot mix of protons (among other things) that can capture electrons via:

$$p^+ \;+\; e^- \;\;\rightarrow\;\; n \;+\; \nu_e$$

resulting in a huge release of energy via neutrinos.
I'll also note here, that this electron capture reaction can occur additionally for protons in heavier nuclei, converting them to neutrons. In fact, this is important in the formation of a neutron star (we'll look at this process in more detail later).
But first, lets get an idea of the conditions in the core of this star, assuming it is now composed mostly of protons and electrons (that will be relativistic and degenerate). We can get a rough estimate of the density required for the electron capture reaction above to occur, assuming for simplicity that the neutrino carries away a negligible amount of energy (not true, but fine for an order of magnitude estimate). Then, from relativistic energy conservation for this reaction we have:

$$\gamma m_e c^2 \;+\; m_p c^2 \;\;=\;\; m_n c^2$$

where, as usual, $\gamma = 1/\sqrt{1 - v^2/c^2}$ where v is the average electron speed, and we are assuming the proton's and neutron's speed is negligible.
Then, we get (verify),

$$v^2 \;\;=\;\; \left[1 - \left(\frac{m_e}{m_n - m_p}\right)^2\right] c^2$$

We now want to relate this to density. If ρ is the density of matter in the core which is composed of nuclei ${}^A_Z X$ (on average), and the mass and volume of the core is M and V respectively, then $\rho = M/V$, where $M = A m_H N_X$ where N_X is the total number of nuclei in the core. If n_X is the number density (N_X/V) of nuclei, then we have:

$$\rho \;\;=\;\; A m_H n_X \;\;=\;\; A m_H \frac{n_e}{Z}$$

where n_e is the number density of electrons (recall there will be about Z electrons per nucleus).
So now we need an estimate for n_e. Now, assume these electrons are as closely packed as allowable by the Pauli Exclusion Principle (a reasonable assumption since its only the electron degeneracy pressure that's providing any support to the core). Then, if the average electron momentum is p, the closest they can be separated is given by the Heisenberg Uncertainty Principle: $\Delta p \Delta x \sim \hbar$.
For this rough estimate we can take $p \sim \Delta p$ (if they're very closely packed then Δp can be large and be a reasonable estimate for p).

So if we have electrons separated by Δx then each electron occupies an average volume of Δx^3, giving:

$$p \approx m_e v \approx \hbar n_e^{1/3}$$

$$\Rightarrow \quad \rho \approx \frac{A}{Z} m_H \frac{(m_e v)^3}{\hbar^3}$$

$$\Rightarrow \quad \rho \approx \frac{A}{Z} m_H \left(\frac{m_e c}{\hbar}\right)^3 \left[1 - \left(\frac{m_e}{m_n - m_p}\right)^2\right]^{3/2}$$

Now, we are assuming mostly only protons and electrons present, so that $A/Z \sim 1$, and with:

$$m_p = m_H = 938.279 \, MeV/c^2 = 1.67 \times 10^{-27} \, kg$$

$$m_n = 939.573 \, MeV/c^2$$

$$m_e = 0.511 \, MeV/c^2 = 9.11 \times 10^{-31} \, kg$$

$$\hbar = 1.055 \times 10^{-34} \, Js \,, \qquad c = 3.00 \times 10^8 \, ms^{-1}$$

we calculate:

$$\rho \approx 2.3 \times 10^{10} \, kg \, m^{-3}$$

A more detailed calculation gives a value about a factor of two smaller than this, so we're not too far off in this rough calculation.

Some comparisons:

- 1 cm^3 (about a teaspoon) would weigh about 100,000 kg
- Density of water is $\rho_{water} = 1.0 \times 10^3 \, kgm^{-3}$
- Density of lead is $\rho_{Pb} = 1.1 \times 10^4 \, kgm^{-3}$
- For the Sun the average density is, $\bar{\rho_\odot} = 1.4 \times 10^3 \, kgm^{-3}$, and the density in the Sun's core is, $\rho_{\odot c} = 1.6 \times 10^5 \, kgm^{-3}$.

So this is rather exotic material in the cores of large-mass stars at the end of their lives. And this is nothing compared to the densities of Neutron Stars, which we'll look at soon!