Problem 1 [4 pts]
In class we showed that the classical temperature required for 2 protons to overcome the Coulomb barrier and collide so that the strong nuclear force is experienced is much greater than the temperature in the Sun’s central region.

(a) Derive this conclusion (basically go through the derivation in class and substitute in the appropriate values).

(b) One possibility that might lead one not to conclude that this necessarily forbids nuclear reactions, is that the velocities used were averages, whereas in fact some particles will have much larger values as one can see from the Maxwell-Boltzmann distribution. If some nuclei have velocities 10 times the root-mean-square (rms) value for the Maxwell-Boltzmann distribution will these nuclei be able to overcome the Coulomb barrier for the temperature of the centre of the Sun (use $T_C = 2 \times 10^7$ K for the Sun)?

Using the Maxwell-Boltzmann distribution calculate, for $T_C$ of the Sun, the ratio of protons having velocities 10 times the rms value to those moving at the rms velocity.

From this what do you conclude concerning any possibility that classical particles can overcome the Coulomb barrier?

Problem 2 [3 pts]
In deriving the temperatures required for particles to overcome the Coulomb Barrier we are interested in the relative motion between the two interacting particles. Consider 2 particles of masses $m_1$ and $m_2$, respectively, which in a given coordinate system have positions and velocities given by $\vec{r}_1, \vec{r}_2$ and $\vec{v}_1, \vec{v}_2$. In this coordinate system the total kinetic energy will be:

$$K = \frac{1}{2} m_1 |\vec{v}_1|^2 + \frac{1}{2} m_2 |\vec{v}_2|^2$$

If $\vec{v}$ is the relative velocity between the two particles ($\vec{v} = d\vec{r}/dt$ where $\vec{r} = \vec{r}_2 - \vec{r}_1$) show that total kinetic energy can be written as:

$$K = \frac{1}{2} \mu v^2 \text{ where } \mu = \frac{m_1 m_2}{m_1 + m_2}, \, v = |\vec{v}|$$

($\mu$ is often called the reduced mass.)

(Hint: choose the origin of the coordinate system to be at the centre of mass of the two particles and write $\vec{r}_1$ and $\vec{r}_2$ in terms of $\vec{r}$.)

Problem 3 [3 pts]
Describe (qualitatively) how the wave nature of particles in quantum mechanics is required to explain how nuclear reactions can take place in the core of the Sun.
Problem 4 [5 pts]

Consider the three nuclear reactions in the PP I reaction chain. For each reaction calculate the energy released (in MeV). Assume the mass of the neutrino’s are zero. (Note that this energy is released in the form of kinetic energy of the final state particles. Note also that any energy carried away by the neutrinos is lost to the star.)

Compare the total energy released from the three PP I reactions to that calculated from simply using the binding energy of $^4_2$He, and quantitatively reconcile any differences. (Hint: the results you get should differ by roughly 10%.)

(Use: $m(^1_1\text{H}) = m_p = 1.6726231 \times 10^{-27}\text{kg}$, $m_n = 1.674929 \times 10^{-27}\text{kg}$, $m(^2_1\text{H}) = 2.0141\text{u}$, $m(^3_2\text{He}) = 3.0160\text{u}$, $m(^4_2\text{He}) = 4.00151\text{u}$ where $1\text{u} = 1.6605402 \times 10^{-27}\text{kg} = 931.49432\text{MeV}/c^2$, $m_e = 9.11 \times 10^{-31}\text{kg} = 0.511\text{MeV}/c^2$)

It will probably be easiest to begin by expressing all relevant masses in units of MeV/c$^2$.

Problem 5 [3 pts]

As the amount of helium nuclei (produced from the PP reaction chains) increases in a star’s interior, and its temperature rises, the so-called *triple alpha process* can occur and produce energy in the star’s interior. Calculate the total energy released (in MeV) from the 2 reactions that constitute this reaction chain. What can you conclude from this concerning what the major source of energy is over the lifetime of a star.

($m(^8_4\text{Be}) = 8.005305\text{u}$, $m(^{12}_{6}\text{C}) = 12\text{u}$ (by definition))

Problem 6 [2 pts]

For temperatures around $10^8\text{K}$, which is necessary for helium nuclei to overcome the Coulomb barrier and participate in “helium burning” via the triple alpha process discussed in problem 5, by what factor does the energy generation rate from the triple alpha process increase if the temperature rises by 5%? and, by 20%?

Problem 7 [4 pts]

Consider the reaction:

$$^{56}_{26}\text{Fe} + ^{32}_{16}\text{S} \rightarrow ^{88}_{42}\text{Mo}$$

where, $m(^{56}_{26}\text{Fe}) = 55.93494\text{u}$, $m(^{32}_{16}\text{S}) = 31.97207\text{u}$, and $m(^{88}_{42}\text{Mo}) = 87.921953\text{u}$ (obtained from: http://en.wikipedia.org/wiki/Table_of_nuclides)

(a) Approximately how many times greater must the temperature be, compared to that required for hydrogen burning, in order for the Fe and S nuclei to overcome their Coulomb barrier and for the reaction to be able to occur?

(b) Calculate the energy *released* in this reaction (recall that if this turns out to be negative then energy is required to be absorbed in order for the reaction to take place). Discuss the result and its implications on stellar evolution.
Problem 8 [3 pts]
A white dwarf is a very dense star, with its ions and electrons packed extremely close together (we’ll look at white dwarfs more closely in later lectures). Each electron may be considered to be located within a region of size $\Delta x \approx 1.5 \times 10^{-12}$ m. Use Heisenberg’s uncertainty principle to estimate the minimum speed of the electrons. Will the effects of Special Relativity be important for these stars?

Problem 9 [3 pts]
For the approximate conditions of the interior of our present-day Sun, calculate the ratio of the energy generation rate for the PP chain to the energy generation rate for the CNO cycle. Use for the Sun’s interior: $T_c = 1.6 \times 10^7$K, $\rho_c = 1.6 \times 10^5$kg m$^{-3}$, a mass fraction for hydrogen of 0.35, and a mass fraction for oxygen + carbon + nitrogen of 0.015. (Check the lecture notes for the relevant energy generation formulae.)
If a different star had 10 times $T_c$ of the Sun, 3 times $\rho_c$, and the same mass fractions, how would this ratio change?