Problem 1 [6 pts]

Let’s go back again to look at problem 9 from assignment #3.

Explain, being as quantitative as you can: (more so than I am here!)

(a) Will the distribution predicted in part (c) be in fact the distribution that actually occurs? Why or why not?

The distributions predicted will not be that for hydrogen atoms as a whole because at high enough temperatures (i.e., 10,000 K) most H will be ionized, thus leaving very few electrons in any energy level to participate in energy level transitions (here you should give an indication of how many for a given temperature). However, for the small number of H atoms that remain neutral, this will be their expected distribution.

(b) Given the result from part (b) why aren’t the Paschen series lines (those to/from the n=3 state) considered in our treatment of the hydrogen lines we expect to observe?

We found that the Balmer series lines are strongest at T around 10,000 K, a result of when enough H is neutral (Saha) and some of this neutral H is in the n=2 state (Boltzmann). Even though only a small fraction is in the n=2 state it is more than the neutral H that remains at much higher temperatures. (Make sure you understand this interplay between the Saha and Boltzmann equations in predicting the relative strength of lines.) At around 10,000 K the Paschen series lines will only be significant if their are significant numbers of electrons in the n = 3 state. For neutral H at 10,000 K we have:

$$\frac{N_3}{N_2} = .... = \frac{9}{4}e^{-2.2} \Rightarrow N_s \sim \frac{1}{4}N_2$$

So the Paschen lines will exist but at a strength about a quarter that of the Balmer lines. Note that this ratio will drop rapidly as T decreases.

(c) In order to calculate \(N_2/N_{\text{total}}\) needed for estimates of the Balmer series strength, we made the assumption that \(N_I = N_1 + N_2\) in order to give us ratios we could easily calculate from the Boltzmann and Saha equations. Justify this assumption.

Again, at 10,000 K, we have:

$$\frac{N_2}{N_1} = 2e^{(-3.40eV+13.6eV)/0.8625eV} = 1.5 \times 10^{-5}$$

$$\Rightarrow N_I = N_1 + N_2 + N_3 + ... = N_1 + (1.5 \times 10^{-5}N_1 + (0.4 \times 10^{-5}N_1 + ....$$

So for our rather rough approximations we only use the first two terms. (Note that even though the second term is much less than the first, we need to keep it in order to get the required ratio \(N_2/N_{\text{tot}}\) in terms of \(N_2/N_1\) which is what the Boltzmann equations gives us.)
Problem 2 [5 pts]
In class we used the approximation that nearly all the H-I atoms are in the ground state so that we could estimate the partition function;

\[ Z_1 \approx g_1 = 2(1)^2 = 2 \]

Verify that this statement is correct for a temperature of 10,000 K by evaluating the first three terms in the partition function.

At 10,000 K we have for the first three terms:

\[ Z = g_1 + g_2 e^{(E_1-E_2)/kT} + g_3 e^{(E_1-E_3)/kT} \]

where \( kT = 0.8617 \text{ eV} \), \( E_1 - E_2 = -10.2 \text{ eV} \), \( E_1 - E_3 = -12.1 \text{ eV} \), \( g_1 = 2 \), \( g_2 = 8 \), and \( g_3 = 18 \).

Therefore:

\[ Z = 2 + 5.8 \times 10^{-5} + 1.4 \times 10^{-5} \]

thus justifying neglecting the terms beyond the first.

Problem 3 [5 pts]
Explain clearly why the calcium-II absorption lines in the Sun are about a factor of 400 stronger than the H-I Balmer lines when there is only one calcium atom for every 500,000 hydrogen atoms in the Sun’s surface layer.

See class notes where we went through this in detail. Here I just expect you to repeat that argument and fill in some of the calculations I didn’t have time to do in lecture.

Problem 4 [6 pts]
Text, Problem 2.7

Given \( \chi_i = 0.754 \text{ eV} \), \( n_e = 10^{17} \text{ cm}^{-3} = 10^{23} \text{ m}^{-3} \). When \( H^- \) ionizes it gives neutral \( H \), so in the Saha equation we can take, \( N_I \) = number of \( H^- \) atoms, and \( N_2 \) = number of \( H \) atoms (singly ionized \( H^- \)). Since \( \chi_i >> kT \) (we’ll see from the final result that the temperature will be in a range where this is a very good assumption) the Boltzmann factors in \( Z_I \) and \( Z_{II} \) are much smaller than 1, so that \( Z_{II} = 2 \) and \( Z_I = 1 \) (only one possibility for 2 electrons in the ground state).

\[
\frac{N_{II}}{N_I} = \frac{4}{10^{23}} (\frac{2\pi(9.11 \times 10^{-31} \text{ kg})(1.38 \times 10^{-23} \text{ J/K})T}{(6.626 \times 10^{-34} \text{ Js})^2})^{3/2} e^{-\frac{0.754 \text{ eV}}{(8.62 \times 10^{-5} \text{ eV/K})T}} = (9.66 \times 10^{-2})T^{3/2} e^{-\frac{8750 \text{ K}}{T}}
\]

For \( N_{II} = N_I \):

\[
\Rightarrow T^{3/2} e^{-\frac{8750 \text{ K}}{T}} = 10.4 \Rightarrow Te^{-\frac{5830 \text{ K}}{T}} = 4.75
\]

You can solve this numerically to get:

\[ T \approx 1100 \text{ K} \]

That is, \( H^- \) ions only play a role for very cool stars.
Problem 5 [3 pts]
Sirius, the brightest appearing star in the night sky, has an apparent bolometric magnitude of: \( m_{bol} = -1.55 \).
The distance to Sirius is 2.6 pc. Determine the absolute bolometric magnitude of Sirius and compare it with that of the Sun. What is the ratio of Sirius’ intrinsic luminosity to that of the Sun?

From \( m - M = 5 \log \left( \frac{d}{10 \text{ pc}} \right) \) we get:

\[
M = -1.55 - 5 \log(0.26) = 1.38 \text{ for Sirius}
\]

For the Sun, \( M = 4.76 \), so that Sirius is a much brighter star intrinsically than the Sun. We can also see this through the ratio of their luminosities. We have:

\[
M_{\text{sirius}} = M_{\text{sun}} = 2.5 \log \left( \frac{L_{\text{sirius}}}{L_{\text{sun}}} \right) \Rightarrow 2.5 \log \left( \frac{L_{\text{sirius}}}{L_{\text{sun}}} \right) = 3.38 \Rightarrow \frac{L_{\text{sirius}}}{L_{\text{sun}}} = 22.5
\]

Problem 6 [5 pts]
Assuming that 10 eV could be released by every atom in the Sun through chemical reactions, estimate how long the Sun could shine at its current rate through chemical processes alone. For simplicity assume that the Sun is composed entirely of hydrogen. Is it possible that the Sun’s energy is entirely based on chemical reactions? (You will need to look up the mass and luminosity of the Sun and the mass of a hydrogen atom – given in class and on assignment 1.)

Assuming Sun is 100% hydrogen, and \( M_\odot = 2 \times 10^{30} \text{ kg} \) (mass of Sun), \( m_H = 1.67 \times 10^{-27} \text{ kg} \) (mass of hydrogen atom), \( L_\odot \approx 4 \times 10^{26} \text{ J/s} = 2.5 \times 10^{45} \text{ eV/s} \) (intrinsic luminosity of the Sun), and let \( \epsilon \) be the energy released by every atom through chemical reactions (10 eV).

Then, the total energy that could be released by chemical reactions is:

\[
E_{\text{chem}} = \epsilon \frac{M_\odot}{m_H} = (10 \text{ eV}) \frac{2 \times 10^{30} \text{ kg}}{1.67 \times 10^{-27} \text{ kg}} = 1.2 \times 10^{58} \text{ eV}
\]

With \( L_\odot = 2.5 \times 10^{45} \text{ eV/s} \), we can estimate the time these chemical reactions could power the Sun at this luminosity:

\[
t_{\text{chem}} = \frac{E_{\text{chem}}}{L_\odot} = \frac{1.2 \times 10^{58} \text{ eV}}{2.5 \times 10^{45} \text{ eV/s}} = 4.8 \times 10^{12} \text{ s}
\]

This is about 150,000 years, MUCH shorter than the lifetime of the Sun, so we wouldn’t be around if this was the primary energy source!