Problem Set 4
PHY 760 - Fall 2013
Assigned: Friday, Sep. 26, Due: Friday, Oct. 3

Reading: RHB, Ch. 4

Problems: Ch. 4: 4.6, 4.8, 4.10, 4.14, 4.16, 4.18, 4.20

Additional Problem 1:

a) Evaluate the function, $B(m,n)$, which is defined by the integral

$$B(m,n) = \int_0^1 dx\, x^{m-1}(1-x)^{n-1}\,,$$

in terms of gamma functions. (Hint: Start with the product $\Gamma(m)\,\Gamma(n)$ and use the integral representation of the gamma function.)

b) Obtain another integral for $B(m,m)$ by substituting $x = u^2$ into Eq. (1). Then combine this with the result for $B(m,m)$ in part a) and to derive the following formulae:

$$\Gamma(2z) = \frac{2^{2z-1}}{\sqrt{\pi}}\Gamma(z)\Gamma(z+1/2)\,.$$

Additional Problem 2:

By applying Laplace's method to the expression

$$n! = \int_0^\infty dt\, e^{-t}\, t^n$$

to obtain Stirling's approximation for the factorial. Then apply the **Euler-Maclaurin** summation formula

$$\begin{aligned}\int_1^n dx f(x) &= \frac{1}{2}f(0) + f(1) + f(2) + ... + f(n-1) + \frac{1}{2}f(n) \\ &\quad + \sum_{k=1}^{m} \frac{B_k}{k!}\left(f^{(k-1)}(n) - f^{(k-1)}(1)\right) + R_{m,n}\end{aligned}$$

to the function $f(x) = \log x$ to obtain the corrections to the asymptotic formula. In this formula, n is an integer, $f^{(k-1)}(x)$ is the $(k-1)$th derivative of $f(x)$, and $R_{m,n}$ is an error that is $O(1/n^m)$. The first line is the trapezoid rule for numerically approximating the integral. The second line is an asymptotic series for the corrections to this approximation.

Please write down how many hours you spent on this problem set.