Problem Set 3

PHY 760 - Fall 2014

Assigned: Wednesday, Sep. 17 Due: Friday, Sep. 26

Reading: RHB, Chs. 13.2

Problems: RHB Ch.13: 21, 22, 24, 26, 28

Additional Problem:

A damped harmonic oscillator is governed by the equation

$$\frac{d^2}{dt^2}x(t) + 2\gamma\frac{d}{dt}x(t) + \omega^2 x(t) = F(t)\,, \tag{1}$$

where γ is a damping constant, ω is the angular frequency of the oscillator in the absence of damping, and $F(t)$ is a driving force. Suppose that this driving force vanishes for $t < 0$ and is nonvanishing for $t > 0$. Furthermore, suppose the oscillator is at rest at $t = 0$ when the driving force turns on, i.e., $x(0) = dx(0)/dt = 0$. In this problem you will find $x(t)$ for $t > 0$. For parts a)-c) you can assume that the two roots of the equation $s^2 + 2\gamma s + \omega^2 = 0$ are different.

a) Solve Eq. (1) using Laplace transforms by first solving for $\bar{x}(\omega)$ in terms of the Laplace transform of $F(t)$, then using the convolution theorem to obtain $x(t)$ as an integral involving $F(t)$.

b) Your result can be written in the form

$$x(t) = \int_0^\infty dt' G(t,t')F(t')\,.$$

(You may need to insert a step function into the integrand to put it in this form.) Deduce the Green's function $G(t,t')$ from your answer to part a).

c) Solve the differential equation

$$\left(\frac{d^2}{dt^2} + 2\gamma\frac{d}{dt} + \omega^2 x(t)\right) G(t,t') = \delta(t-t'),$$

using the Laplace transform and verify it is the Green's function you found in part b). To get the correct $G(t,t')$ you must impose the same boundary conditions imposed on $x(t)$, i.e., $G(0,t') = dG(0,t')/dt = 0$.

d) If the roots of the equation $s^2 + 2\gamma s + \omega^2 = 0$ are the same, what are $x(t)$ and $G(t,t')$?

Please write down how many hours you spent on this problem set.