Problem Set 9
PHY 465 - Spring 2014
Assigned: Thursday, Apr. 20

Reading: Shankar 19.1-5

Problem 1: Scattering in the Born Approximation

Shankar 19.3.2

Problems 2-3: The Partial Wave Expansion

Shankar 19.5.4, 19.5.5

Problem 4:

Consider the scattering of an electron off of a cubic atomic crystal. Approximate the potential the electron feels as a δ-function at the location of each atom in the lattice, so the potential is

$$V(\vec{x}\,) = V_0 \sum_{\vec{n}} \delta(\vec{x} - \vec{n}\,a)\,.$$

Here a is the lattice spacing and $\vec{n} = (n_x, n_y, n_z)$ where n_i are integers between $-\infty$ and ∞. (We are treating the crystal as infinitely large.) Calculate the scattering amplitude in the Born approximation, then use the identity

$$\sum_{n=-\infty}^{\infty} e^{inx} = 2\pi \sum_{m=-\infty}^{\infty} \delta(x - 2\pi m)\,,$$

(for integers n and m) to show that scattering only occurs at angles determined by the Bragg diffraction law:

$$\sin\theta = \frac{\lambda}{2a}\sqrt{m_x^2 + m_y^2 + m_z^2}\,.$$

Problem 5:

In section 19.3 of the textbook, the Born approximation is used to calculate the differential cross section for an electron scattering from the Coulomb field of a nucleus of charge Z, assuming the nucleus is point-like. Generalize this result to an electron scattering off a nucleus which is not point-like but has an extended charge distribution, $\rho(\vec{x}\,)$. The electrostatic potential in this case is given by:

$$V(\vec{x}\,) = Ze \int d^3y \frac{\rho(\vec{y}\,)}{|\vec{x} - \vec{y}\,|}$$

Here the total charge of the nucleus is Ze so the density $\rho(\vec{y}\,)$ is normalized as $\int d^3y \rho(\vec{y}\,) = 1$.

a) Show that

$$\frac{d\sigma}{d\Omega} = \left.\frac{d\sigma}{d\Omega}\right|_{\text{Coulomb}} \times F(q)$$

where $d\sigma/d\Omega|_{\text{Coulomb}}$ is the differential cross section in the point-like limit. The function $F(q)$ is called the **form factor**; give the form factor as an integral over the function $\rho(\vec{x}\,)$.

b) What $F(q)$ in the limit $q \to 0$? Explain your result on physical grounds.

c) What is $F(q)$ for a uniform spherical charge distribution of radius R?