Problem 1

a) Show that the fermionic current, \( j^a_\mu = \bar{\psi}t^a \gamma^\mu \psi \), is covariantly conserved:
\[
\mathcal{D}_\mu j^a_\mu = 0.
\]
Show this in two ways: 1) by using the equations of motion for \( \psi \), and 2) starting from the equation of motion for the field strength,
\[
\mathcal{D}_\mu F^a_{\mu\nu} = -g j^a_\nu.
\]

b) Verify the Bianchi identity:
\[
e^{\mu\nu\rho\sigma} \mathcal{D}_\nu F^a_{\rho\sigma} = 0.
\]

Problem 2

Consider the axial gauge condition, \( n^\mu A^a_\mu = 0 \), where \( n^\mu \) is an arbitrary four-vector. Add to the Lagrangian the gauge fixing term
\[
\mathcal{L} = -\frac{1}{2\xi} (n^\mu A^a_\mu)^2.
\]

a) Compute the gauge boson propagator. Note that \( \xi \) has mass dimension \(-2\) unlike the case of the Lorentz gauge. In axial gauge in order to obtain a propagator that behaves as \( \sim k^{-2} \) for large \( k \) (where \( k \) is the virtual four-momentum in the propagator), one has to take the limit \( \xi \to 0 \). Show that this has the effect of enforcing \( n^\mu A^a_\mu = 0 \) by checking that \( n^\mu \Pi_{\mu\nu}(k) = 0 \), where \( \Pi_{\mu\nu}(k) \) is the numerator of the gauge boson propagator.

b) Show that ghosts decouple in this gauge.

Problem 3

Peskin & Schroeder, Problem 15.5.