Problem 1: Wigner 3-j symbol
Gottfried and Yan, Chapter 3, Problem 7.

Problem 2: Addition of Three Vectors
Gottfried and Yan, Chapter 3, Problem 8.

Problem 3: Rotation of a Vector
Gottfried and Yan, Chapter 4, Problem 2.

Problem 4: Rotation matrices for $j = 1$

a) Evaluate $d^{(1)}_{mm'}(\beta)$ by expanding $e^{-ijy/\beta}$ in a power series. Then check that this result agrees with the general formulae in Eqs. (144-145) of Gottfried and Yan.

b) Use the result of part a) to work out how the components of a vector operator:

\[ V_{\pm 1}^{(1)} = \mp \frac{V_x \pm iV_y}{\sqrt{2}}, \quad V_0^{(1)} = V_z \]

transform under a rotation about the $y$-axis by an angle $\beta$. Check that this result agrees with what you expect from the result of Problem 4.

Problem 5: Identities for Rotation Matrices

a) Evaluate

\[ \sum_{m=-j}^{j} m |d^{(j)}_{mm'}(\beta)|^2 \]

for arbitrary $j$. Check your result using the rotation matrices for $j = 1/2$ and $j = 1$.

b) Prove

\[ \sum_{m=-j}^{j} m^2 |d^{(j)}_{mm'}(\beta)|^2 = \frac{1}{2} j(j+1) \sin^2 \beta + m'^2 \frac{1}{2} (3 \cos^2 \beta - 1) . \]