Problem Set 7
Phy 211 - Fall 2002
Assigned: Friday, Oct. 25; Due: Thursday Oct. 31
DUE AT THE BEGINNING OF CLASS

Problem 1: More on Time Evolution for the Simple Harmonic Oscillator

In class we used Ehrenfest's Theorem to show that

$$\langle \hat{x}(t) \rangle = \langle \hat{x}(0) \rangle \cos(\omega t) + \frac{\langle \hat{p}(0) \rangle}{m\omega} \sin(\omega t) \,,$$

for any state of the simple harmonic oscillator. Rederive this result by showing

$$\langle \hat{x}(t) \rangle = \langle e^{i\hat{H}t/\hbar} \hat{x} e^{-i\hat{H}t/\hbar} \rangle \,,$$

then using the identity $e^A B e^{-A} = B + [A,B] + \frac{1}{2}[A,[A,B]] + ...$ and summing the series.

Problem 2: Uncertainty Principle for the Hydrogen Atom

Shankar, Problem 9.4.2 and 9.4.3

Problem 3: Uncertainty Relations for Angular Momentum

In Problem Set 4, we showed that the quantum mechanical operators for angular momentum obey the commutation relations:

$$[\hat{L}_x, \hat{L}_y] = i\hbar \hat{L}_z \,,$$
$$[\hat{L}_y, \hat{L}_z] = i\hbar \hat{L}_x \,,$$
$$[\hat{L}_z, \hat{L}_x] = i\hbar \hat{L}_y \,.$$

a) Use these commutation relations to derive uncertainty relations for the components of angular momentum.

b) The total angular momentum squared operator is defined by

$$\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2 \,.$$

Evaluate $[\hat{L}^2, \hat{L}_i]$, $i = x, y, z$. Of the observables $\hat{L}^2, \hat{L}_x, \hat{L}_y$ and $\hat{L}_z$, which can observed simulataneously?

Problem 4: More on Coherent States (Graduate Student Problem)

a) Show that

$$|z\rangle = \exp(-|z|^2/2 - z\hat{a}^\dagger)|0\rangle \,,$$

is a normalized coherent state, i.e. $\langle z|z\rangle = 1$. (Hint: the identity $e^A e^B = e^B e^A e^{[A,B]}$ is useful for this exercise.) What is $\langle z'|z\rangle$ for $z' \neq z$? Do coherent states $|z\rangle$ form an orthonormal basis?

b) Find the wave function, $\psi_z(x) \equiv \langle x|z\rangle$, for a coherent state. Show that $\langle \hat{x} \rangle, \Delta x$ obtained from this wavefunction agree with the answers calculated in class using annihilation-creation operators.