Problem Set 2
Phy 211 - Fall 2003
Assigned: Thursday, Sept. 11; Due: Thursday, September 18
BEGINNING of class

Problem 1: Gaussian Integral

a) Evaluate the integral

$$I(a,b) = \int_{-\infty}^{\infty} dx \exp(-ax^2 + bx)\,.$$

(Hint: $I(a,b)$ can be related to $I(a,0)$ by completing the square. $I(a,0)$ is the Gaussian integral evaluated in class.)

b) Use this result to normalize the following wavefunction:

$$\psi(x) = N \exp\left(ikx - \frac{(x-x_0)^2}{4a^2}\right)\,.$$

c) What are $\langle x \rangle$ and $\langle p \rangle$ for this wavefunction?

Problem 2: Expectation Values

Consider a particle trapped in a one-dimensional box of length L whose wavefunction is given by

$$\psi(x) = A(L^2 - x^2); \qquad\qquad -L \le x \le L$$

a) Find A by normalizing the wavefunction.
b) Compute Δx and Δp for this state.

Problem 3: Dirac δ-function

a) Demonstrate

$$\begin{aligned} y\frac{d}{dy}\delta(y) &= -\delta(y)\,, \\ \delta(ay) &= \frac{1}{|a|}\delta(y)\,, \\ \delta(y^2 - a^2) &= \frac{1}{2|a|}[\delta(y-a) + \delta(y+a)]\,. \end{aligned}$$

Do this by showing that the convolution of the left and righthand side of these equations with an arbitrary smooth function, $f(y)$, is the same. For example,

$$\int dy\, f(y) y \frac{d}{dy}\delta(y) = -\int dy\, f(y)\delta(y)\,,$$

where the integral is over an interval that contains the point $y = 0$.

b) (Required for Graduate Students Only) Show that

$$\delta(y(x)) = \sum_i \frac{\delta(x_i)}{|y'(x_i)|},$$

where $y' = dy/dx$ and the x_i are zeroes of $y(x)$, i.e. $y(x_i) = 0$.

Problem 4: Waves on a String and Fourier Series

Consider a string stretched along the x-axis with its end points at $x = 0$ and $x = L$ fixed. The displacement of the string is described by a function $\psi(x, t)$ which obeys the following wave equation

$$\frac{d^2}{dx^2}\psi(x,t) = \frac{1}{v^2}\frac{d^2}{dt^2}\psi(x,t).$$

a) Expand $\psi(x, t)$ in a Fourier series:

$$\psi(x,t) = \sum_{n=-\infty}^{\infty} c_n(t)\sin(k_n x).$$

What are the possible allowed values of k_n?

b) From the wave equation for $\psi(x, t)$ derive an equation for the time dependence of $c_n(t)$. Find the general solution for $c_n(t)$ in terms of $c_n(0)$ and $dc_n/dt(0)$.

c) If at $t = 0$, $\psi(x, 0) = \sin(\pi x/L)$ and $d\psi(x, 0)/dt = 0$, what is $\psi(t)$?

Problem 5: Poisson Brackets (Required for Graduate Students Only)

Shankar, Exercise 2.7.2. This problem requires a reading of Section 2.7 which we did not cover in class.