MidTerm 1
Phy 211 - Fall 2002
Thursday, October 3

Problem 1

Consider a quantum mechanical system which is described by a two dimensional Hilbert space spanned by basis vectors denoted $|1\rangle$ and $|2\rangle$. Let us introduce an operator $\hat{\Omega}$ whose matrix elements in this basis are:

$$\langle 1|\hat{\Omega}|1\rangle = \langle 2|\hat{\Omega}|2\rangle = a$$
$$\langle 1|\hat{\Omega}|2\rangle = \langle 2|\hat{\Omega}|1\rangle = b$$

a) Find the eigenvectors and eigenvalues of $\hat{\Omega}$.
b) Suppose the system is in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}|1\rangle + \frac{i}{\sqrt{2}}|2\rangle\,.$$

What is the probability that when $\hat{\Omega}$ is measured the result is a? b? a+b? a-b?
c) Compute $\langle\hat{\Omega}\rangle$, $\Delta\hat{\Omega}$ for this state.

Problem 2

Consider the state vectors

$$|\tilde{x}\rangle = U(\hat{x})|x\rangle = e^{ig(\hat{x})}|x\rangle\,,$$

where $g(x)$ is a real function of x.

a) Show that $U(\hat{x})$ is a unitary operator.
b) Evaluate the matrix elements:

$$\langle\tilde{x}'|\hat{x}|\tilde{x}\rangle \qquad \langle\tilde{x}'|\hat{p}|\tilde{x}\rangle\,.$$

Problem 3

Consider the operator

$$T(a) = \exp\left(\frac{i\hat{p}a}{\hbar}\right)\,,$$

where a is a real number.

a) Is $T(a)$ Hermitian? Unitary?
b) Show $T(a)\,\hat{x}\,T^{\dagger}(a) = \hat{x} + a$
c) **Graduate Student Problem:** Evaluate $\langle x|T(\hat{a})|f\rangle$ for arbitrary state vector $|f\rangle$.

Problem 4

Consider a particle moving in 2 dimensions (x-y), described by the following Lagrangian

$$\mathcal{L} = \frac{\mu}{2}(\dot{x}^2 + \dot{y}^2) - B\,\dot{x}\,y\,.$$

a) Find the canonical momenta, p_x and p_y. Find the Hamiltonian.

b) **Graduate Student Problem:** Now consider p_x and p_y in the limit $\mu \to 0$. One of the momenta should vanish in this limit. Determine the **quantum mechanical** commutation relations for the other momentum and its canonical conjugate coordinate. Is it possible to measure x and y simultaneously in this limit? If not, what kind of uncertainty relation do you expect x and y to obey?