

Final Exam

Phy 182 - Fall 2009

Saturday, Dec. 12, 3-5 pm

This exam contains 5 short answer questions and 3 problems. Useful equations are collected at the end of the exam. Please sign a pledge on your exam stating that you have abided by Duke's honor code.

Short Answer 1 (10 pts.)

Calculate the capacitance per unit length of two infinite concentric cylinders. Let the inner cylinder have radius a and outer cylinder have radius b .

Short Answer 2 (10 pts.)

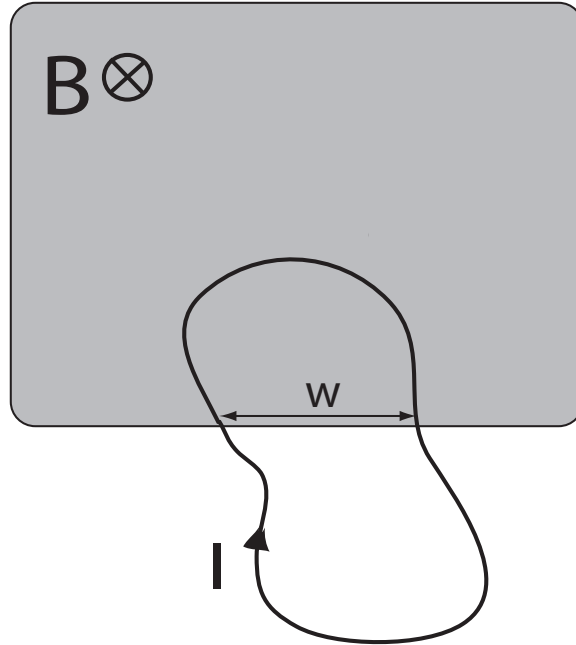
Consider a spherical conductor of radius R . The spherical conductor is held at a potential $V_0 = 0$ (relative to infinity). A charge q is placed a distance a from the center of the sphere.

a) Use the method of images to determine the force on the charge.

b) Now consider the case of $V_0 \neq 0$. This problem can also be solved using the method of images. Where do we place the additional image charge and what is its magnitude? What is the new force on the charge?

Short Answer 3 (10 pts.)

The gray area indicates a region of space with a uniform B pointing into the page. The loop is a wire carrying current I in the direction indicated by the arrow. The horizontal distance between the points on the loop of wire is w .



a) In this part of the problem, assume that the loop is NOT moving and the current is sustained by a battery (not shown). What is the total force on the wire, in terms of I , w and B ? Make sure you indicate direction as well as magnitude.

b) Now suppose we disconnect the battery and move the loop downwards with velocity v . If the total resistance in the loop of wire is R what is the current in the wire now? Make sure you indicate which way the current flows.

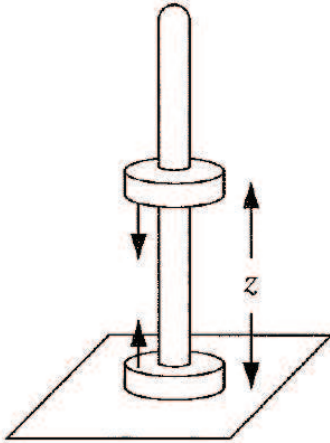
Short Answer 4 (10 pts.)

A sphere of radius R is filled with a linear dielectric material whose susceptibility is χ_e . A point charge q is embedded at the center of the sphere.

- a) Find the electric field, polarization and bound charge densities, σ_b and ρ_b .
- b) What is the total bound charge on the surface? Show that the total bound charge of the dielectric vanishes.

Short Answer 5 (10 pts.)

The figure shows a pair of toroidal magnets that can slide vertically along a pole. We will treat the magnets as dipoles with dipole moment $\vec{m}$ and mass m_d . If the dipole moments of both magnets are along the pole in the same direction, find the height z at which the top magnet floats above the bottom magnet which is sitting at the base of the pole.

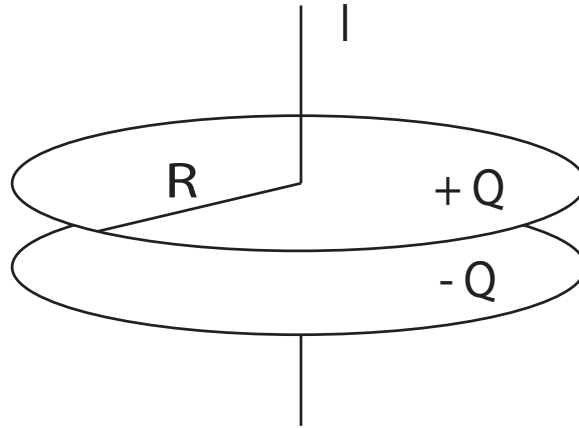


Problem 1 (20 pts.)

A sphere of homogeneous linear dielectric with permittivity ϵ is placed in a uniform electric field $\vec{E} = E_0 \hat{z}$. Find the electric field inside and outside the sphere.

Problem 2 (20 pts.)

The following figure depicts a capacitor being charged up by a current I . The radius of the capacitor plates is R , h is their separation and $h \ll R$. You may assume that the capacitor is being charged slowly enough that at any given time the charge Q is uniformly distributed over the plates' surface. You may also ignore fringing fields at the edges of the capacitor.



- Calculate the energy contained in the electric fields within the capacitor as a function of Q .
- Calculate the $\vec{B}$ field induced by the time-dependent $\vec{E}$ field within the capacitor.
- Evaluate the Poynting vector at the edges of the capacitor and show that the rate of energy flow into the capacitor is equal to the rate at which the energy you computed in part a) is changing with time.

Problem 3 (20 pts.)

a) What are the $\vec{E}$ and $\vec{B}$ fields of an infinite parallel plate capacitor moving with velocity v parallel to the plates, assuming that the plates carry electric charge per unit area of $\pm\sigma$ *in their own rest frame*.

b) Consider an infinite solenoid oriented along the x -axis. The solenoid has N turns per unit length each carrying current I . What is the $\vec{B}$ field measured by an observer moving with velocity v in the x -direction? What is the $\vec{B}$ field measured by an observer moving with velocity v in the z -direction? (Do NOT assume $v \ll c$).

c) Calculate the minimum energy a positron (e^+) must have in order for the reaction $e^+e^- \rightarrow \mu^+\mu^-$ to take place,

i) when the electron (e^-) is at rest and the positron has energy E ,

ii) when the electron and positron have the same energy E and opposite momentum.

Express your answer in units of the electron mass, m_e , and c . The mass of the positron is equal to the mass of the electron and the muon (μ^-) and antimuon (μ^+) have masses of $210 m_e$.

Formulae Sheet

Maxwell Equations in Vacuum

$$\begin{aligned}\vec{\nabla} \cdot \vec{E} &= \frac{\rho}{\epsilon_0} & \vec{\nabla} \cdot \vec{B} &= 0 \\ \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} &= 0 & \vec{\nabla} \times \vec{B} - \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} &= \mu_0 \vec{J}\end{aligned}$$

Maxwell Equations in Matter

$$\begin{aligned}\vec{\nabla} \cdot \vec{D} &= \rho_f & \vec{\nabla} \cdot \vec{B} &= 0 \\ \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} &= 0 & \vec{\nabla} \times \vec{H} - \frac{\partial \vec{D}}{\partial t} &= \vec{J}_f\end{aligned}$$

Displacement Field and $\vec{H}$

$$\vec{D} \equiv \epsilon_0 \vec{E} + \vec{P} \quad \vec{H} = \frac{1}{\mu_0} \vec{B} - \vec{M}$$

Linear Media

$$\vec{D} = \epsilon \vec{E} \quad \vec{H} = \frac{1}{\mu} \vec{B}$$

Lorentz Force Law

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Conductors

$$\vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B}) \quad V = I R \quad P = V I$$

Inductance

$$\Phi_1 = M_{12} \Phi_2 \quad \Phi = \oint d\vec{a} \cdot \vec{B} = L I$$

Electromagnetic Field Energy

$$U = \frac{1}{2} \int d^3x \left(\epsilon_0 \vec{E}^2 + \frac{1}{\mu_0} \vec{B}^2 \right)$$

Poynting Vector, Momentum

$$\vec{S} = \frac{1}{\mu_0} (\vec{E} \times \vec{B}) \quad \vec{P} = \int d^3x \epsilon_0 (\vec{E} \times \vec{B})$$

Solution to Laplace's Equation in Spherical Coordinates

$$V(r, \theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos \theta)$$

$$P_0(x) = 1 \quad P_1(x) = x \quad P_2(x) = \frac{3x^2 - 1}{2} \quad \int_0^1 dx P_l(x) P_m(x) = \frac{2}{2l+1} \delta_{lm}$$