

Midterm 2

Phy 182 - Fall 2009

Monday Nov. 9, in class exam.

This exam contains 3 short answer questions and 3 problems. Useful equations are collected at the end of the exam. Please sign a pledge on your exam stating that you have abided by Duke's honor code.

Short Answer 1 (5 pts.)

a) What is the magnetic field created by an infinite uniform current density $\vec{K} = K_0 \hat{y}$ confined to the $y - z$ plane?

b) Now suppose the region $x > 0$ is filled with a linear medium with permeability μ_1 and the region $x < 0$ is filled with a linear medium whose permeability is μ_2 . What are $\vec{H}$ and $\vec{B}$ in this case? Check that your answer satisfies the appropriate boundary conditions at $x = 0$.

Short Answer 2 (5 pts.)

A hollow cylinder of length L and radius R has uniform charge density σ and is rotating at an angular frequency ω . The center of the cylinder is at $z = 0$. The charge is on the sides of the cylinder but not the top or the bottom. What is the magnetic field

- a) at a point on the positive z -axis at a distance $r \gg L, R$?, and
- b) at a point in the $z = 0$ plane at a distance $r \gg L, R$?

Make sure you indicate the magnitude and direction of the field in each case. Your answer need only be accurate to lowest order in L/r and R/r .

Short Answer 3 (5 pts.)

A particle of mass m and charge e is executing circular motion in the x-y plane in the presence of a uniform magnetic field $\vec{B} = B_0 \hat{z}$ field in the z-direction. How long does it take to execute one orbit?

Problem 1 (15 pts.)

Calculate the energy per unit length of an infinite solenoid with n turns per unit length carrying current I , by

- a) calculating the energy per unit length of the magnetic fields, and
- b) calculating the self-inductance per unit length of the solenoid and using the formulae for the work required to establish steady currents.

Problem 2 (15 pts.)

A coaxial cable consists of a straight cylindrical wire of radius a carrying current $\vec{I}(t) = I_0 \cos(\omega t) \hat{z}$ (where the z -axis is defined by the wire) and a coaxial cylinder of radius b which carries the opposite current.

a) Calculate the magnetic field in the region $a < r < b$. Make sure you indicate the direction in which the magnetic field is pointing.

b) Calculate the induced electric field due to the oscillating $\vec{B}$ field in the same region. Make you indicate the direction in which the electric field is pointing.

c) Calculate the displacement current density, J_d , and the total displacement current, I_d , in a length l .

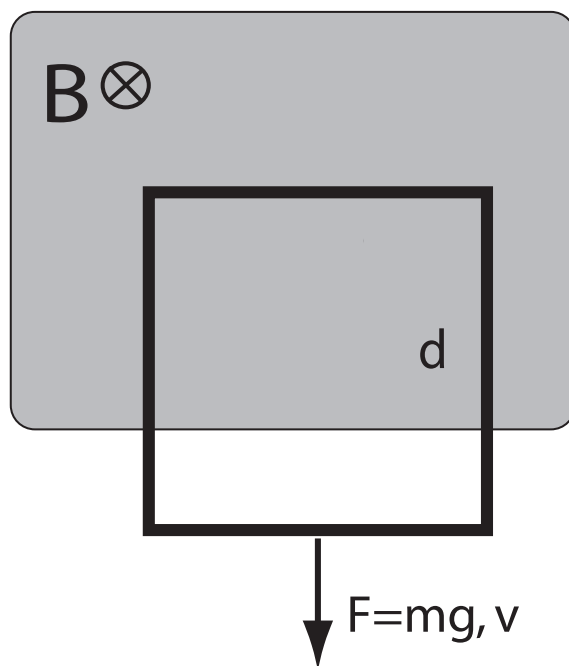
Problem 3 (15 pts.)

The figure on the next page depicts a square loop of wire with sides length d . The wire has cross sectional area A and conductivity σ and mass density ρ . The shaded area in the diagram depicts a region with magnetic field pointing into the page. The wire is held vertically so gravity is acting to pull the wire out of the region containing magnetic field.

a) If the wire is falling with velocity v , what is the emf induced in the wire? What is the magnitude of the current induced by the emf? Indicate the direction of current flow by drawing on the figure.

b) What is the net magnetic force acting on the loop? Indicate its direction by drawing on the figure.

c) What is the terminal velocity of the loop? Assume that the loop stays in the region of magnetic field long enough to attain terminal velocity. (Hint: when expressed in terms of σ and ρ the result should be independent of d and A .)



Formulae Sheet

Maxwell Equations in Vacuum

$$\begin{aligned}\vec{\nabla} \cdot \vec{E} &= \frac{\rho}{\epsilon_0} & \vec{\nabla} \cdot \vec{B} &= 0 \\ \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} &= 0 & \vec{\nabla} \times \vec{B} - \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} &= \mu_0 \vec{J}\end{aligned}$$

Maxwell Equations in Matter

$$\begin{aligned}\vec{\nabla} \cdot \vec{D} &= \rho_f & \vec{\nabla} \cdot \vec{B} &= 0 \\ \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} &= 0 & \vec{\nabla} \times \vec{H} - \frac{\partial \vec{D}}{\partial t} &= \vec{J}_f\end{aligned}$$

Displacement Field and $\vec{H}$

$$\vec{D} \equiv \epsilon_0 \vec{E} + \vec{P} \quad \vec{H} = \frac{1}{\mu_0} \vec{B} - \vec{M}$$

Linear Media

$$\vec{D} = \epsilon \vec{E} \quad \vec{H} = \frac{1}{\mu} \vec{B}$$

Lorentz Force Law

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Field of Magnetic Dipoles

$$\vec{A}_{\text{dip}}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2} \quad \vec{B}_{\text{dip}}(\vec{R}) = \frac{\mu_0}{4\pi} \frac{3\hat{r}(\vec{m} \cdot \hat{r}) - \vec{m}}{r^3}$$

Conductors

$$\vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B}) \quad V = I R \quad P = V I$$

Inductance

$$\Phi_1 = M_{12} I_2 \quad \Phi = \oint d\vec{a} \cdot \vec{B} = L I$$

Electromagnetic Field Energy

$$U = \frac{1}{2} \int d^3x \left(\epsilon_0 \vec{E}^2 + \frac{1}{\mu_0} \vec{B}^2 \right)$$

Energy for Capacitors, Steady Currents

$$W = \frac{1}{2} C V^2 = \frac{1}{2} \frac{Q^2}{C} \quad W = \frac{1}{2} L I^2$$