Midterm 1
Phy 182 - Fall 2009
Monday Sep. 28, in class exam.

This exam contains 3 short answer questions and 3 problems. Useful equations are collected at the end of the exam. Please sign a pledge on your exam stating that you have abided by Duke’s honor code.

Short Answer 1 (5 pts.)

What is the electric field for
a) an infinite line charge with charge density \( \lambda \) per unit length, and
b) an infinite uniformly charged plane with charge density \( \sigma \) per unit area?

For each case draw a diagram indicating the charge and the field, and clearly identify the coordinates you are using.
Short Answer 2 (5 pts.)

For an electric field of the form $\vec{E} = k \hat{r}/r$, what is the charge density?
Short Answer 3 (5 pts.)

In the figure below, $\vec{E}_1$ is the electric field just above the surface of a dielectric with permittivity $\epsilon$. This field is at an angle of $45^\circ$ with respect to the normal to the dielectric surface. The region above the dielectric has the permittivity of free space, $\epsilon_0$. $\vec{E}_2$ is the electric field just below the surface of the dielectric. The electric field $\vec{E}_2$ is at an angle $\theta$ with respect to the normal. What is $\theta$ in terms of the permittivity $\epsilon$?
Problem 1 (15 pts.)

The figure depicts a cross sectional view of two infinite conducting metallic half-cylinders of radius $R$ separated by infinitesimal distance. The upper half-cylinder is held at potential $+V_0$ and the bottom half-cylinder is held at potential $-V_0$. Find the electrostatic potential everywhere in space. You can express your answer in the form of an infinite series.
Problem 2 (15 pts.)

Consider two co-centric metallic spheres of radius $a$ and $b$, with $a < b$. Charge $+Q$ is on the inner sphere and $-Q$ is on the outer sphere.

a) What is the capacitance of this system?

b) Verify the formulae for the energy stored in a capacitor by calculating the total energy in the electric field.
Problem 3 (15 pts.)

An ideal dipole with dipole moment $p \hat{z}$ sits at the center of a sphere of radius $R$ that is filled with a linear dielectric whose permittivity is $\epsilon$.

Find the electrostatic potential inside and outside the sphere.
Useful Formulae

Gauss’ Law

\[ \nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad \int d\vec{a} \cdot \vec{E} = \frac{Q_{\text{enc}}}{\epsilon_0} \]

Displacement Field

\[ \vec{D} = \epsilon_0 \vec{E} + \vec{P} \]

Linear Dielectrics

\[ \vec{D} = \epsilon \vec{E} \quad \frac{\epsilon}{\epsilon_0} = \epsilon_r = 1 + \chi_e \]

Capacitors

\[ Q = CV \quad W = \frac{1}{2} CV^2 \]

Solution to Laplace’s Equation in Spherical Coordinates

\[ V(r, \theta) = \sum_{l=0}^{\infty} \left( A_l r^l + B_l \frac{r^{l+1}}{r^{l+1}} \right) P_l(\cos \theta) \]

Multipole Expansion

\[ V(\vec{r}) = \frac{1}{4\pi \epsilon_0} \left( \frac{Q}{r} + \frac{\vec{p} \cdot \vec{r}}{r^3} + \ldots \right) \]

Vector Calculus (more on next page)

\[ \hat{r} = \frac{\vec{r}}{r} \quad \nabla r = \hat{r} \quad \nabla_i \vec{r}_j = \delta_{ij} \quad \nabla \cdot \frac{\vec{r}}{r^2} = 4\pi \delta^3(\vec{r}) \]

Fourier Transform

\[ \frac{1}{\pi} \int_{-\pi}^{\pi} d\theta \sin(n \theta) \sin(m \theta) = \delta_{nm} \]