Exam II

Solutions

In questions or problems not requiring numerical answers, express the answers in terms of the symbols given, and standard constants such as $g$. If numbers are required, use $g = 10 \text{ m/s}^2$.

Part A: Multiple choice questions. Check the best answer. Each question carries a value of 4 points.

1. A correct and general statement of the law of conservation of angular momentum of a system is:
   - [ ] If the net external torque about any point is zero, the total angular momentum about all points is conserved.
   - [ ] The product of the moment of inertia and angular velocity of a body is conserved.
   - [ ] If the net external force on a system is zero, the total angular momentum is conserved.
   - [X] If a component of the net external torque about a point is zero, that component of the total angular momentum about that point is conserved.

2. You move a heavy cylindrical oil drum of mass $M$ up an incline at constant speed, by applying a force with your hands at the top of the drum as shown. The drum rolls up the incline without slipping. Which of the following is NOT true?
   - [ ] As the drum moves a distance $d$ on the incline you do work $Mgd \sin \theta$.
   - [ ] The static friction force is up the incline, with magnitude $\frac{1}{2}Mg \sin \theta$.
   - [ ] The applied force has magnitude $\frac{1}{2}Mg \sin \theta$.
   - [X] Half of the work done on the drum is done by static friction.
     [Static friction never does work.]

3. Satellite A’s orbit around the earth is a circle of radius $2R$; satellite B’s orbit is an ellipse with perigee distance $R$ and apogee distance $3R$. They have equal mass.
   - [ ] The total energy of A is greater than that of B. [Same]
   - [ ] The period of B is greater that that of A. [Same]
   - [ ] The largest speed of B is the same as the speed of A. [Larger]
   - [X] When B is also at distance $2R$ from the center of the earth its speed is the same as that of A.
4. An SUV rounds a curve on a level road, moving at the instant shown into the page. Its CM is at height $h$ above the road, and the distance between the wheels shown is $w$. The question being posed is this: If its speed increases beyond the maximum safe value, will the SUV slide outward on the curve or roll over? Which of these is WRONG?

- The answer does not depend on the total mass of the SUV and its contents.
- The torque (about the CM) tending to cause rollover comes from friction between the tires and the road.
- The total normal force on the inner (closer to the center of the curve) wheels is the same as on the outer wheels. [Larger on outer wheels]
- The larger the ratio $h/w$, the higher the chance of rollover.

Part B: Check True or False. Each question carries a value of 3 points.

1. Despite what one sometimes reads, it is impossible for the CM of a high jumper who clears the bar actually to pass under the bar.
   - True
   - False [It’s done all the time]

2. As the disk shown rolls freely (without slipping) up the incline, slowing down, the static friction force is directed up the incline.
   - True [Friction torque must be counter-clockwise]
   - False

3. When the sun and moon are on opposite sides of the earth, the daily high tides are much smaller than when both bodies are on the same side of the earth.
   - True
   - False [Same. These are spring tides.]
Part C: Problems. Work problems in the space provided, indicating your method clearly. A correct final answer supported by no argument, or a fallacious one, will receive little or no credit. The problems carry the point values shown.

1. Romeo and Juliet sit at opposite ends of a boat 4 m long, at rest on the water, while Romeo sings a serenade. To show her appreciation, Juliet moves to where Romeo sits to give him a kiss. The boat’s mass is 80 kg, Juliet’s is 50 kg and Romeo’s 70 kg. Assume that the water offers no resistance to motion of the boat. It takes Juliet 6 s to reach Romeo. All answers are numerical.
   a. Assume she moves at constant speed \( v \) relative to the water, while Romeo and the boat move the other way with constant speed \( v' \). What is \( v' / v \)?
   b. Find \( v \) and \( v' \) from the fact that she travels the length of the boat to reach Romeo. [Use your answer to (a).]
   c. How far has the boat moved relative to the water when she reaches him?

[15 points]

a. Total momentum remains zero, so \( m_{\text{Juliet}} \cdot v = (m_{\text{Romeo}} + m_{\text{Boat}}) \cdot v' \). This gives \( v' / v = 50 / 150 = 1 / 3 \).

b. Her speed relative to the boat is \( v_{\text{rel}} = v + v' \). Since \( v_{\text{rel}} \cdot 6 = 4 \) we have \( v + v' = 2 / 3 \). Using (a) we find \( v = 1 / 2 \) m/s and \( v' = 1 / 6 \) m/s,

c. The boat moves distance \( v' \cdot 6 = 1 \) m.
2. Shown is a car on a level road, with the total normal forces from the road on the front and rear wheels indicated. Also indicated are the horizontal distances of the wheels from the CM of the car. The CM is at height \( h \) above the road. The values of these distances are: \( x_1 = 1 \) m, \( x_2 = 2 \) m, \( h = 1 \) m. The car has mass \( M \).

a. If the car is at rest (or moving with constant velocity), find \( N_1 \) and \( N_2 \) in terms of \( M \) and \( g \).

b. Suppose the car accelerates to the left at rate \( a = 0.1g \). Find \( N_1 \) and \( N_2 \). [What force produces the acceleration, and where does it act?]

c. Repeat if the car is accelerating to the right (braking) at the same rate.

d. Wheels are not rigidly attached to a car, but are mounted on springs to allow some vertical motion and smooth out the ride. Explain why if you apply the brakes forcefully the front of the car dips and the rear rises.

[20 points]

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\begin{align*}
\text{CM} & \quad x_1 \quad x_2 \\
N_1 & \quad N_2
\end{align*}
\]

\[
\begin{align*}
a. \quad \text{Vertical forces: } N_1 + N_2 &= Mg. \text{ Torques about CM: } 2 \cdot N_1 = 2 \cdot N_2. \text{ Solving we find } N_1 = \frac{2}{3}Mg, \quad N_2 = \frac{1}{3}Mg. \\
b. \quad \text{Since friction is the only horizontal force, } f = Ma = 0.1Mg. \text{ The vertical forces remain the same, but friction adds a new torque, so we have } f \cdot 1 + N_1 \cdot 1 - N_2 \cdot 2 = 0, \text{ or } N_1 - 2N_2 + 0.1Mg = 0. \text{ Combining this with the equation from the vertical forces we find } N_1 = (1.9/3)Mg \quad \text{and } N_2 = (1.1/3)Mg. \\
c. \quad \text{Now the friction force (and torque) is in the opposite direction, so we have } N_1 - 2N_2 - 0.1Mg = 0, \text{ which gives } N_1 = 0.7Mg \text{ and } N_2 = 0.3Mg. \\
d. \quad \text{As (c) shows, applying the brakes creates a counter-clockwise torque that increases } N_1 \text{ and reduces } N_2. \text{ Thus the front springs are compressed more and the rear springs less than when the car is not accelerating.}
\end{align*}
\]
3. A uniform rod, pivoted at its center, has mass $M$, length $\ell$, and moment of inertia about the pivot $I_0 = \frac{1}{12}M\ell^2$. It is balanced horizontally at rest as shown. A ball of putty of mass $M/3$ is dropped from height $\ell/2$ above the right end of the rod; in the collision it sticks to the rod.

a. What is the angular speed $\omega$ of the system immediately after the collision? [What is conserved in the collision?]

b. What is the total energy after the collision, taking potential energy to be zero at the level of the pivot?

c. What is the angular speed when the ball is at the bottom of its swing?

d. To what height $h$ (above the level of the pivot) will the ball rise during the swings of the system?

[Give all answers in terms of $M$, $\ell$ and $g$.]

[20 points]

| a. | Angular momentum about the pivot is conserved. The initial value is $L_i = (M/3) \cdot v_0 \cdot (\ell/2)$, where $v_0 = \sqrt{g\ell}$ is the speed with which the ball strikes the rod. After the collision the value is $L_f = I_{tot} \cdot \omega$, where $I_{tot} = \frac{1}{12}M\ell^2 + (M/3) \cdot (\ell/2)^2 = \frac{1}{6}M\ell^2$. Using $L_i = L_f$ we find $\omega = \sqrt{g/\ell}$. |
| b. | After the collision $K = \frac{1}{2}I_{tot}\omega^2 = \frac{1}{12}Mg\ell$. This is the total energy, since $U = 0$. |
| c. | The rod’s potential energy never changes, so we need consider only the ball. We have $\Delta U = -(M/3) \cdot g \cdot (\ell/2) = -\frac{1}{6}Mg\ell$, so the final kinetic energy is $K_f = \frac{1}{12}Mg\ell + \frac{1}{6}Mg\ell = \frac{1}{4}Mg\ell$. Set this equal to $\frac{1}{2}I_{tot}\omega_f^2$ to find $\omega_f = \sqrt{3g/\ell}$. |
| d. | When the system is momentarily at rest, $(M/3) \cdot gh = \frac{1}{12}Mg\ell$, or $h = \ell/4$. |
4. A spacecraft of mass \( m \) is projected upward from the earth’s surface with initial speed \( v_0 = \sqrt{Rg} \), where \( R \) is the radius of the earth and \( g \) is the gravitational field at its surface. Give all answers in terms of \( m \), \( g \) and \( R \).

a. Taking potential energy to be zero if the spacecraft is infinitely far from earth, what is the total energy? [What is \( GM \), the product of the gravitation constant and the earth’s mass, in terms of \( g \) and \( R \)？]

b. When the spacecraft is at distance \( r \) from the center of the earth, what is its speed, in terms of \( r \) and the given quantities?

c. To what maximum value of \( r \) will it go?

d. What must the initial speed \( v_0 \) be if the spacecraft is to escape to infinity?

[20 points]

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\text{a. The gravitational force on a mass } m \text{ at the surface of the earth is }\ F = G \frac{Mm}{R^2}. \text{ This is also equal to } mg, \text{ so we have } g = GM/R^2. \text{ The potential energy when } m \text{ is at the surface of the earth is } U = -G \frac{Mm}{R} = -mgR, \text{ so the total energy is } E = \frac{1}{2}mv_0^2 + U = -\frac{1}{2}mgR. \\
\text{b. By conservation of energy } \frac{1}{2}mv_0^2 + U(r) = -\frac{1}{2}mgR, \text{ where } U(r) = -G \frac{Mm}{r^2} = -mg \frac{R^2}{r}, \text{ so we find } v(r)^2 = gR \left( \frac{2R}{r} - 1 \right). \\
\text{c. Set } v(r) = 0 \text{ to find } r_{\text{max}} = 2R. \\
\text{d. The total energy must be zero (or greater), so } \frac{1}{2}mv_0^2 - mgR = 0, \text{ or } v_0 = \sqrt{2gR}.]
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Median = 48.0
SD = 14.4

Median = 60.5
SD = 14.2