Sample Problem with Recommended Format for Written Solution.

Problem:
A small block of mass $m$ is on a wedge as shown. The coefficient of static friction, $\mu_s$, is not sufficient to keep the block from sliding if the wedge is at rest. The wedge is being accelerated to the left at rate $a$.

a. What is the minimum value of $a$ such that the block does not slide down the wedge?

b. What is the maximum value of $a$ such that the block does not slide up the wedge?

Give answers in terms of $g$, $\mu_s$ and $\theta$.

Solution:

(a)

Newton’s 2nd law:

$F_x$: \[ N \sin \theta - f \cos \theta = ma \]

$F_y$: \[ N \cos \theta + f \sin \theta - mg = 0 \]

If block is on verge of sliding, \( f = \mu_s N \).

These equations give:

\[
\frac{a}{g} = \frac{\sin \theta - \mu_s \cos \theta}{\cos \theta + \mu_s \sin \theta}.
\]

(b) If the block is on the verge of sliding up, friction acts in the opposite direction, down the incline. The signs of the terms involving $f$ in the force component equations are reversed. This means that the signs in front of $\mu_s$ in the answer are reversed, so we obtain:

\[
\frac{a}{g} = \frac{\sin \theta + \mu_s \cos \theta}{\cos \theta - \mu_s \sin \theta}.
\]

[See the notes on the next page.]
Notes:

1. **Drawing.** The situation is already given in a drawing, so all that is needed is a drawing showing the forces acting on the block (the “free-body” diagram). Since (in part a) the block is about to slide down the plane, static friction must be up the plane to oppose the sliding. The diagram indicates the forces by their symbols, with arrows showing their directions.

2. **Coordinate axes.** It is generally best to choose coordinate axes so that one of them is along the direction of the *acceleration*, if that is known. Thus we choose the $x$-axis along $a$ (horizontal, to the left) and the $y$-axis vertical. This choice should be shown on the paper, with the axes clearly labeled.

3. **General law.** Here it is Newton’s 2\(^{\text{nd}}\) law. It is useful to write the name of the general law you are applying on the paper, to guide your thinking and to help the grader.

4. **Equations.** The components of $F = ma$ give two equations as shown. Since $a$ has only an $x$-component, the right side of the $y$-component equation is zero. Components of $N$ and $f$ must be carefully determined from the geometry. The component equations must be written down clearly and carefully. At this point there are two equations in three unknowns ($N$, $f$ and $a$).

5. **Static friction.** The maximum possible static friction force magnitude is $\mu_s N$. This gives the third equation. This equation must be written — or else use $\mu_s N$ in place of $f$ from the beginning, after explaining why. (Static friction is less than or equal to $\mu_s N$, so it must be explained why the maximum value is used.)

6. **Solution.** Solve the three equations for $a$, by eliminating $N$ and $f$.

7. **Repeat for part b.** The direction of $f$ is now reversed. Other forces remain the same. One can start over, rewriting the equations and solving the new ones. Or one can make the argument given, which is quicker. But the argument must be written down on the paper.

8. **Check answer.** The question asks for an acceleration, so the answer must have the dimensions of acceleration. (The solution shown solves for $a/g$ which must be dimensionless.) Checking dimensions is a good way to detect errors.

   Also look for limiting cases to see if they come out right. Here you can check $\theta = 0$ (which is like a block on the flat bed of an accelerating truck) and $\theta = 90^\circ$ (where the friction force must equal the weight to prevent sliding).

   The solution shown has all the necessary elements clearly displayed. You should not omit the few words that guide the argument, nor omit the drawings. Don’t leave the grader to guess your method from your equations.