Supplementary Problems for Topics I

1. A small block of mass $m$ is on a wedge as shown. The wedge is being accelerated to the left at rate $a$.

   a. If there is no friction between the block and wedge, for what value of $a$ will the system move with the block at rest relative to the wedge?

   Now suppose there is friction, but the coefficient of static friction, $\mu_s$, is not sufficient to keep the block from sliding if the wedge is at rest.

   b. What is the minimum value of $a$ such that the block does not slide down the wedge?

   c. What is the maximum value of $a$ such that the block does not slide up the wedge?

   Give answers in terms of $g$, $\mu_s$ and $\theta$.

   a. Choose the $x$-axis to be in the direction of the acceleration, i.e., to the left. The only forces on the block are gravity and the normal force from the wedge. We have

   $\begin{align*}
   F_x & : N \sin \theta = ma \\
   F_y & : N \cos \theta - mg = 0
   \end{align*}$

   Solving we find $a = g \tan \theta$.

   b. In this case there is maximum static friction up the incline, so we have

   $\begin{align*}
   F_x & : N \sin \theta - f_s \cos \theta = ma \\
   F_y & : N \cos \theta + f_s \sin \theta - mg = 0
   \end{align*}$

   where $f_s = \mu_s N$. Solving we find

   $a = g \frac{\sin \theta - \mu_s \cos \theta}{\cos \theta + \mu_s \sin \theta}$.

   c. Here the friction force is down the plane. Reversing the signs in front of the friction force is the same as reversing the signs in front of $\mu_s$, so we find

   $a = g \frac{\sin \theta + \mu_s \cos \theta}{\cos \theta - \mu_s \sin \theta}$.
2. A ball of mass \( m \) is attached to a massless string of length \( L \), which is attached to a ceiling as shown. The ball is pulled back until the string makes angle \( \theta \) with the vertical, and it is released from rest. At the bottom of the swing, the string encounters a peg at distance \( d \) below the ceiling. The upper part of the string stops moving, while the ball now rotates about the peg in a smaller circle than before.

a. Suppose the ball makes a complete circle about the peg. What is the minimum speed it must have at the top of that circle?

b. At what minimum value of \( \theta \) must the ball be released initially so that it makes a complete circle about the peg? [Take gravitational potential energy to be zero at the bottom of the swing.]

c. Clearly \( \theta \) cannot be greater than 90°. What is the smallest value of \( d \) for which the ball can make a complete circle about the peg?

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a. The radius of the circle is \( L - d \). Since at the top of the circle the tension acts downward, the minimum downward acceleration occurs when \( T = 0 \), or when \( mg = mv^2 / (L - d) \). Thus \( v_{\text{min}}^2 = g(L - d) \).

b. Conservation of energy. Take \( U = 0 \) at the bottom of the swing. Then the initial potential energy is \( mgL(1 - \cos \theta) \) and the potential energy at the top of the circle is \( 2mg(L - d) \). Thus

\[
mgL(1 - \cos \theta) = \frac{1}{2}mv_{\text{min}}^2 + 2mg(L - d) = \frac{5}{2}mg(L - d), \quad \text{so} \quad \cos \theta = \frac{5d - 3L}{2L}.
\]

c. Set \( \cos \theta = 0 \) to find \( d = 3L / 5 \).