Supplementary Problems for Topics IV

1. One mole of an ideal gas is compressed at constant temperature $T$ from volume $V$ to volume $2V/3$.
   a. Express the heat released in this process in terms of the given quantities and $R$.
   All the heat released by the gas is absorbed by a block of ice at 0° C. Some of the ice melts; the melted part has mass $m$.
   b. Express the latent heat of fusion $L$ for ice in terms of the given quantities.

2. An engine uses one mole of an ideal monatomic gas ($c_v = 1.5R$) in the following cycle:
   1. Starting at temperature $T_0$ and pressure $P_0$, the pressure is doubled at constant volume.
   2. In a step represented on the P-V diagram by a straight line, the volume is doubled and the pressure cut in half.
   3. In another straight line step, the system is returned to its original state.
   a. Make a careful plot of the cycle on a P-V diagram.
   b. Find the heat in $Q$, the work out $W$ and the change in internal energy for each step. Give all answers in terms of $R$ and $T_0$.
   c. Find the total work done per cycle by the engine, the total heat taken in (in those steps where heat is actually taken in), and the efficiency.
   d. Find the Carnot efficiency for an engine running between the two extreme temperatures in this cycle.

3. Repeat Prob. 2, except that the second step is this:
   2. Keeping the temperature fixed, the volume is doubled.

4. Repeat Prob. 2, except that the second step is this:
   2. In an adiabatic step, the pressure is cut in half.
5. An ice cube of mass 40 g at its melting point (0° C) is placed in a container with 300 g of water at 10° C, and the system is isolated from the surroundings. Use the following data: latent heat of fusion of ice \( (L) = 3 \times 10^5 \text{ J/kg}; \) specific heat of water \( (c) = 4 \times 10^3 \text{ J/kg-K}. \)

a. What is the final equilibrium temperature? [Assume all the ice melts.]

b. Find the change in entropy of the system. [Remember to use temperatures in K.]

c. Use the approximation \( \ln(1-x) \approx -x \) (for small values of \( x \)) to show that the total entropy of the system increases.

6. A cyclic engine has 20\% efficiency, and does 100 J of work per cycle.

a. What is the total heat taken in per cycle?

b. What is the total heat expelled per cycle?

c. If this engine runs on a reversible Carnot cycle, and heat is taken in at temperature 400 K, at what temperature is heat expelled?

7. A heat pump is a refrigerator for which the hot and cold reservoirs can be interchanged. In summer, when it functions as an air conditioner, it takes heat \( Q_C \) per cycle from the interior of the house at temperature \( T_C \) and expels heat \( Q_H \) to the outdoors at temperature \( T_H \). In winter, it takes heat \( Q_C \) per cycle from outdoors at temperature \( T_C \) and expels heat \( Q_H \) to the interior at temperature \( T_H \). Assume the device runs on a Carnot cycle and consumes energy \( W \) per cycle.

a. For summer operation, one wants the ratio \( Q_C / W \) to be as large as possible. Express this in terms of the temperatures. Evaluate it for indoor temperature 22° C and outdoor temperature 32° C.

b. For winter operation, the corresponding quantity is \( Q_H / W \). Express this in terms of the temperatures. Evaluate it for indoor temperature 19° C and outdoor temperature 4° C.
8. An air-conditioner is used to maintain temperature \( T_C \) inside the house, while heat flows into the house by conduction through the walls, windows, etc. Call the area of the exterior of the house \( A \) and call its average thermal conductivity to the interior \( K \). The outdoor temperature is \( T_H \), and the air-conditioner works on a reversible Carnot cycle, consuming energy at a rate \( dW / dt \).

a. To keep the temperature constant, the air-conditioner must take heat out of the house at rate \( dQ_C / dt \). Express \( dW / dt \) in terms of this quantity and the temperatures.

b. Express \( dW / dt \) in terms of \( K \) and the temperatures. [What is the net rate of heat flow into the house?]

c. Find the rate of increase in total entropy \( dS / dt \), in terms of \( K \) and the temperatures.