Quiz 5

Solutions

Check the best answer.

1. A student sits on a stool that can rotate freely about a vertical axis. She holds a bicycle wheel with its axle vertical, spinning rapidly with its angular momentum \( L \) vertically upward. Nothing is moving except the wheel. She flips the axle 180° so it is vertical again but with the wheel spinning the other way. The angular momentum about the vertical axis of everything in the system except the wheel is:

- [ ] Zero.
- [ ] \( L \).
- [ ] \(-L\).
- [ ] \(2L\). [The total angular momentum must add to \( L\)]

Check T or F depending on whether the statement is true or false.

2. If a body has only two forces acting on it, of equal magnitude and opposite direction but not along the same line, linear momentum is conserved but angular momentum is not conserved about any point.

- [ ] T
- [ ] F

[The total external torque is not zero about any point.]
3. Two children are sitting at opposite ends of a thin plank of length \(d\) and mass \(M\), which is mounted on a frictionless vertical pivot at its center so it can rotate in a horizontal circle, as shown from above. The plank’s moment of inertia about the pivot is \(\frac{1}{12}Md^2\) and each child has mass \(\frac{1}{3}M\). Initially the children and the plank rotate about the pivot with angular speed \(\omega_0\). Treat the children as point masses, and give answers in terms of \(M\), \(d\) and \(\omega_0\).

a. What is the kinetic energy of the system?

b. As the system rotates, the children pull themselves along the plank until each is halfway to the pivot. What is the new angular speed?

c. How much work did they do in pulling themselves along the plank?

\begin{align*}
a. \quad \text{The total moment of inertia initially is } I_{tot} = \frac{1}{12}Md^2 + 2 \cdot \frac{1}{3}M(d/2)^2 = \frac{1}{4}Md^2. \quad \text{The kinetic energy is } K = \frac{1}{2}I_{tot}\omega_0^2 = \frac{1}{8}Md^2\omega_0^2. \\
b. \quad \text{The new moment of inertia is } I'_{tot} = \frac{1}{12}Md^2 + 2 \cdot \frac{1}{3}M(d/4)^2 = \frac{1}{8}Md^2. \quad \text{Conservation of angular momentum gives } I_{tot}\omega_0 = I'_{tot}\omega, \text{ or } \omega = 2\omega_0. \\
c. \quad \text{The new kinetic energy is } K' = \frac{1}{2}I'_{tot}\omega^2 = \frac{1}{4}Md^2\omega_0^2, \text{ so the children do work in the amount } \frac{1}{8}Md^2\omega_0^2.
\end{align*}