Assignment 6

Reading: Energy 2

Key Concepts: Potential energy, total energy, conservation of mechanical energy, equilibrium and turning points.

1. Some statements about the law of conservation of mechanical energy. Comment on their validity and give your reasons.
   a. The law says that the sum of the kinetic and potential energies of a system remains constant.
   b. The law says that if the net external force is zero then the sum of the kinetic and potential energies of a system remains constant.
   c. The law says that if only conservative forces do work then the sum of the kinetic and potential energies of a system remains constant.
   d. The law says that if all the external forces are conservative then the sum of the kinetic and potential energies of a system remains constant.

2. Some short questions.
   a. Prove by direct calculation that the work done by a constant force on a particle is independent of the path followed by the particle.
   b. The static friction force exerted on an object by a stationary surface does no work on the object. Why not?
   c. When you start your car moving from rest there is an obvious increase in kinetic energy. But the external force (static friction) that causes the acceleration does no work. Does this contradict conservation of energy? Explain.
   d. The kinetic energy of a moving body is a property of its motion, regardless of what is happening to any other things it might be interacting with. Explain the difference between that and the work done on the body. What about potential energy?
3. A block slides on the inside of a vertical frictionless circular track, moving clockwise. The block has mass \( m \) and the track’s radius is \( R \). At point A the block has the minimum necessary speed. Give answers in terms of \( m \), \( R \) and \( g \). Find:

a. Its speeds at A, B and C. \( \text{Ans:} \, \sqrt{Rg}, \sqrt{3Rg}, \sqrt{5Rg} \).

b. The normal force from the track at C. \( \text{Ans:} \, 6mg \).

c. Its acceleration (magnitude and direction) at B. \( \text{Ans:} \, \sqrt{10} \cdot g \), below left horizontal by \( \tan^{-1}(1/3) \).

4*. Consider the block and track in the previous question but now let the block move on the outside of the track.

a. As the block moves past the top, what is the maximum speed it can have and stay on the track? \( \text{Ans:} \, \sqrt{Rg} \).

b. Suppose when it is as shown and moving clockwise it has speed \( v \). What is the normal force exerted by the track at that instant, in terms of \( v \), \( R \), \( g \) and \( \theta \)? [Draw a free body diagram with axes radial and tangent to the track.] \( \text{Ans:} \, mg \cos \theta - \frac{mv^2}{R} \).

c. Let the block start at the top with a non-zero but very small speed and slide clockwise. At what value of \( \theta \) will be on the verge of leaving the track? [Find the speed in terms of \( \theta \).] \( \text{Ans:} \, \cos \theta = \frac{2}{3} \).

5*. Two questions about potential energy functions.

a. A non-ideal spring exerts a force obeying the formula \( F(x) = -4x + 4x^3 \). Find the potential energy function and plot it for \(-2 \leq x \leq 2\). \( \text{Ans:} \, 2x^2 - x^4 \).

b. The potential energy function of a force is \( U(x) = -2e^{-x/2}/x \). Find the formula for the force and plot it for \( 0.1 \leq x \leq 2 \). \( \text{Ans:} \, -e^{-x/2}\left(\frac{1}{x} + \frac{2}{x^2}\right) \).
6. A 4 kg block starts from rest on a frictionless track at height 2 m above the bottom. It slides 2 m across a rough horizontal floor with $\mu_k = 0.2$, then slides on a frictionless floor until it reaches a spring of stiffness 200 N/m. Use $g = 10 \text{ m/s}^2$.

a. What is the kinetic energy of the block as it reaches the rough floor the first time? Ans: 80 J.

b. What is its kinetic energy as it leaves the rough floor the first time?
   Ans: 64 J.

c. How far does it compress the spring before coming momentarily to rest?
   Ans: 0.8 m.

d. How high up the track does it rise when it is moving back to the left the first time? Ans: 1.2 m.

e. Where, measured from the left edge of the rough floor, does the block finally come to rest? Ans: 2 m.

7. Two identical blocks of mass $m$ are connected by a string and arranged as shown. The table is frictionless. The hanging block is attached to the spring, which is uncompressed when the system is released from rest. Let the coordinate $y$ be measured downward from the initial position of the hanging block, and let its gravitational potential energy be zero at that level.

a. Through what distance $y_1$ does the hanging block drop before its acceleration becomes zero? [The string is taut until then.] Ans: $mg / k$.

b. At that instant, what is its speed $v_1$? Ans: $\sqrt{\frac{m}{2k}} \cdot g$.

c. After that the string becomes slack because the hanging block is slowing down while the one on the table is not. What value $y_2$ does the hanging block reach before coming momentarily to rest? [Assume the block on the table continues at constant speed.] Ans: $\frac{mg}{k} \cdot \left(1 + \frac{1}{\sqrt{2}}\right)$. 
8. In the same situation as the previous question, let the coefficient of kinetic friction between the block and the table be $\mu_k$, and let it be large enough so the string always remains taut. Now what is the distance $y_2$ that the hanging block falls before the system comes momentarily to rest? [What is the change in total mechanical energy?] Ans: $\frac{2mg}{k} \cdot (1 - \mu_k)$.

9. In our later discussion of satellite orbits we will deal with a potential energy function which has the form

$$U(r) = \frac{A}{r^2} - \frac{B}{r}$$

where $A$ and $B$ are positive constants and $r > 0$.

a. Let the total energy be $E$. The turning points are the roots of the equation $U = E$. Any quadratic equation can be written as $(r - r_1)(r - r_2) = 0$, where $r_{1,2}$ are the roots. Solve for $r_1 + r_2$ and $r_1 r_2$ in terms of the constants. Ans: $r_1 + r_2 = -\frac{B}{E}$, $r_1 r_2 = -\frac{A}{E}$.

b. Take $A = 1$, $B = 3$ and $E = -2$. Plot $U$ and $E$ and show the turning points.