Assignment 4

Reading: Dynamics 2

Key concepts: Radial forces, effective gravity.

1. A block of mass $m$ slides at constant speed down an incline of angle $\theta$.
   a. What is the coefficient of kinetic friction $\mu_k$? \textit{Ans}: $\mu_k = \tan \theta$.
   b. The same block is sent up the incline, starting with speed $v_0$. How far up the incline will it slide before coming momentarily to rest? \textit{Ans}: 
      \[
      \frac{v_0^2}{2g \sin \theta + \mu_k \cos \theta}.
      \]
   c. Will it slide back down by itself? Justify your answer.

2. Questions about a car rounding a curve of radius 216 m. Use $g = 10 \text{ m/s}^2$ and ignore air effects.
   a. Suppose the curve is on level ground. How fast can a car go around it if $\mu_s$ between tires and road is 0.6? \textit{Ans}: 36 m/s.
   b. Suppose the curve is banked so a car at this speed does not need friction to round the curve. What is the banking angle $\theta$? \textit{Ans}: $\tan \theta = 0.6$.
   c. Now suppose there is rain on the banked curve, which reduces $\mu_s$ to 0.3. What is the minimum speed a car must maintain on the curve to avoid sliding down the bank? \textit{Ans}: 23.4 m/s.

3. A bucket is sliding to the right across a floor, and the level of the water in the bucket makes the angle shown with the horizontal.
   a. Is the bucket speeding up or slowing down?
   b. What is the magnitude of its acceleration, in terms of $g$ and $\theta$? [What angle does the water surface make with $\mathbf{g}_{\text{eff}}$?]
      \textit{Ans}: $a = g \tan \theta$.
   c. What is the friction coefficient $\mu_k$ between the bucket and the floor?
4. Three questions about what makes things move.
   a. Explain how you walk. That is, what force makes you move forward, and how do you bring it about?
   b. Explain how a car accelerates and brakes, in terms of what forces are acting and how the driver brings them about.
   c. In a game of tug of war two opposing teams pull on a rope stretched between them. By the 3rd law, they pull equally hard, in opposite directions, on the rope. How, then, does one team win?

5. Three questions about non-inertial frames.
   a. A woman who normally weighs 120 lbs. stands on a scale in an elevator that is accelerating upward at a rate $\frac{1}{4}g$. Draw the vectors for $g$, $a$, and $g_{eff}$, and find the reading of the scale. Ans: 150 lbs.
   b. Explain the spin cycle in a washing machine. Let the basket of the machine rotate about a vertical axis and explain, from the reference frame of the clothes in the basket, how the spinning removes much of the water.
   c. An object dropped from rest in a car moving horizontally and accelerating forward will fall along a line tilted toward the rear of the car. What line will a helium balloon released from rest at the floor of the car follow? Draw pictures for the two cases.

6*. A box is resting on the bed of a pickup truck that is accelerating to the right as shown. You are to draw vectors representing the forces on the box in the reference frame of the truck.
   a. Draw the vectors for the force of (effective) gravity and the force exerted on the box by the truck bed, in the case where static friction is sufficient to keep the box at rest relative to the truck bed.
   b. Draw the same two vectors for the case where the acceleration is too large to allow static friction to keep the box at rest relative to the truck bed.
   c. In the case in (b) draw the vector representing the net force on the box.
7. Three questions about driving.
   
   a. You are driving over the crest of a hill, so the road has the shape of an arc of a vertical circle of radius $R$. What is the maximum speed you can have and still maintain contact with the road?
   
   b. You are driving at speed $v$ when suddenly at distance $d$ in front of you a tree falls across the road. You have two options. You can turn, using the maximum static friction force for the turn, in a quarter circle that you hope has a radius less than $d$. Or you can go straight ahead, braking with the maximum static friction force, and hope to stop before hitting the tree. Which option is better, and why?
   
   c. Race car drivers don’t speed up during a turn, only after they have returned to straight parts of the track. Explain why.
   
8. Three questions about tension in cords.
   
   a. A real rope has mass and will break if the tension in it exceed a certain limit. Use these facts to explain why a real rope cannot be stretched into a horizontal line between two posts.
   
   b. A heavy picture is hung by two wires from a single support as shown. The wires will break if the tension in them exceeds the weight of the picture. What is the minimum angle the wires make with the horizontal? $\text{Ans: } 30^\circ$.
   
   c. Your car is stuck in snow, but luckily there is a tree nearby, and you have a stout rope. You attach the rope to the front of your car, then wrap it around the tree and pull it as tight as you can, in the situation shown. Then you exert as large a force as you can perpendicular to the rope at its midpoint. Explain why this works to pull your car out of the snow.
9. A ball of mass $m$ attached to a massless string of length $R$ is being whirled in a vertical circle as shown.

a. What is the minimum speed it needs at the top of the circle, if it does not leave the circle? \textit{Ans:} $\sqrt{Rg}$.

b. If it has that speed at the top, then the speed at the bottom will be $\sqrt{5Rg}$. [We will see how to arrive at this result soon.] What is the tension in the string at that point? \textit{Ans:} $6mg$.

c. Answer the same questions if the string is replaced by a massless rigid rod, in which case the speed at the bottom can be as small as $2\sqrt{Rg}$. 