Assignment 15

1. Take the usual reference intensity for the db scale for sound, $I_0 = 10^{-12} \text{ W/m}^2$.
   a. What is the intensity of a sound of loudness 120 db?
   b. If this is the intensity of the sound at a rock concert at distance 90 m from the source, what is the average sound power output of the source?

2. Two waves of equal wavelength and frequency interfere. Separately the intensities of the two are $I_1$ and $I_2$.
   a. Show that the maximum and minimum possible intensities for the combine, wave are $I_{\text{max}} = (\sqrt{I_1} + \sqrt{I_2})^2$ and $I_{\text{min}} = (\sqrt{I_1} - \sqrt{I_2})^2$.
   b. Specify one possible phase difference between the waves in each of these two cases.

3. Questions about standing waves.
   a. Write wave functions for two waves which, when superposed, will produce a standing wave of amplitude 0.02 m, speed 10 m/s, and with nodes 0.1 m apart.
   b. For sound waves in a pipe, an open end is a pressure node, like a fixed end on a string. Explain why.
   c. If you pluck a guitar string exactly in the middle you get only the odd numbered harmonics. Explain.

4. Three successive harmonics of an organ pipe have frequencies 850, 1190 and 1530 Hz.
   a. Is the pipe open at both ends or closed at one end?
   b. What is the fundamental frequency?
   c. How long is the pipe? [Take the speed of sound to be 340 /s.]
5. The formula for the speed of sound in a gas is \( v = \sqrt{\frac{\gamma RT}{M}} \), where \( R \) is a universal constant, \( \gamma \) depends on the type of molecule in the gas but is essentially a constant, \( T \) is the Kelvin temperature, and \( M \) is the gram-molecular weight of the molecules.

a. Use this to explain why, if you fill your mouth cavity with a gas lighter than air (e.g., helium) your voice pitch rises.

b. Consider an organ pipe. As the temperature changes two things happen: the speed of sound changes, and the length of the pipe changes because of thermal expansion. Both increase if the temperature rises. Show that the effects of these on the pitch of the pipe tends to cancel.

c. For a woodwind instrument like the clarinet, where the pipe is made of wood, the thermal expansion is negligible. What is the effect on pitch as the temperature rises?

6. Some questions about the piano.

a. In the middle range of the piano the strings have the same mass per unit length and are at the same tension. The frequency of the note A below middle C is 220 Hz. The E above middle C has frequency 3/2 higher (330 Hz). What is the ratio of the lengths of the two strings?

b. The lowest note on the piano (also an A) has frequency 27.5 Hz. If its string were made of the same wire as that of A(220) and had the same tension, how long would it be? Take the length of A(220) to be 1 m.

c. In fact piano strings are no longer than about 2 m long. If the string for lowest note (A 27.5 Hz) is twice as long as that of A 220 Hz., what is the ratio of the mass per unit length of these strings?
7. More about the piano. So that music played in any musical key will sound equally good (or bad), the piano is tuned in “equal temperament”. This means that an octave (the range of pitches differing by a factor of 2) is divided into 12 parts such that each successive pitch (“half-step”) is \(2^{1/12}\) times higher than the previous one.

a. Refer to the two notes in 1(a) above. Assume that A has been tuned to 220 Hz. The note E is 7 half-steps higher. What is its frequency in equal temperament tuning?

b. Suppose these two notes are sounded at the same time. In addition to the fundamental, one hears (at least faintly) the higher harmonics of each note. Which harmonic of A has almost the same frequency as which harmonic of E?

c. What beat frequency can be heard between these two harmonics? (Counting these beats is how piano tuners formerly did their work.)

8. About the violin.

a. The four pitches of the open strings of the violin are G, D, A and E, in ascending order of frequency. Each string has frequency 3/2 that of the next lower one. If A has frequency 440 Hz, what are the frequencies of the other three strings?

b. The standard length of violin strings is 0.325 m and the normal tension for their use is 70 N. What should be the mass per unit length of the four strings?

c. As with all stringed instruments, the violin is “tuned” by adjusting the tension. Show that to change the frequency by 1% one must change the tension by about 2%.
9. More about the violin.

a. To produce frequencies other than those of the open strings, the player shortens the string by pressing a finger down over the string on the “fingerboard”, making that point in effect the far end of the string. Let the string length be \( L \). To what length should the player shorten the A string in order to produce the A one octave higher than that of the open string, i.e., twice the frequency?

b. How much should the player shorten the string to play the A two octaves above the open string, i.e., four times the frequency?

c. By placing a finger lightly on the string the player can make the whole string vibrate in a mode with a node where the finger is resting. What frequency is produced this way if the finger is resting at \( \frac{1}{4} \) of the length of the string? [See the drawing.]