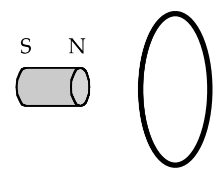
Physics 182

Assignment 4

1. A dipole (electric or magnetic) in a non-uniform field will in general experience a net force. The electric case was the subject of a problem on the midterm exam; here we examine the magnetic case. Let the dipole moment be **m**, and let the field be **B**. The potential energy of alignment is $U = -\mathbf{m}\cdot\mathbf{B}$. As in the electric case, one can show that if **m** is parallel to **B**, the force on **m** is toward the region where B is stronger, while if **m** is antiparallel to **B** the force is away from that region.

   Consider a small cylindrical bar magnet, which creates a field with lines emanating from its north pole and entering its south pole, much like the field of a short solenoid. This magnet is near a fixed circular loop of wire as shown.

   a. Suppose the magnet is moved toward the loop. Find the direction of the induced current in the loop, the direction of the resulting magnetic moment of the loop, and the direction of the force the magnet exerts on it. Explain your choices.

   b. Repeat if the magnet is moved away from the loop.

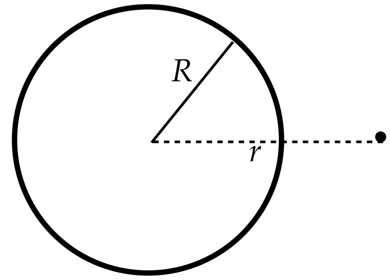
   c. Use these results to explain the following demonstration. A small cylindrical magnet is dropped down a long vertical copper tube. It is observed that the magnet quickly reaches a terminal speed and takes much longer to traverse the tube than does a non-magnetic object.

a. Looking toward the loop from behind the magnet, the field lines pass into the area bounded by the loop, and the flux is increasing. By Lenz's law the induced current is counter-clockwise to produce flux in the opposite direction. The magnetic moment of this current is toward the magnet, opposite to the field of the magnet. The force on the loop is thus away from the stronger field, i.e., a repulsion by the magnet.

b. Now the flux is decreasing, so the induced current is clockwise to produce more flux. The magnetic moment is away form the magnet, parallel to its field. The force is an attraction, pulling the loop toward the magnet.

c. Induced currents above the falling magnet attract it upwards, those below the magnet repel it upwards. The force increases with the speed, so it produces a terminal velocity.

2. Shown in cross section is a long solenoid that produces a B-field uniform within its windings and zero outside. The current in the windings are varied so that B is changing at rate dB/dt. Take the direction of the B-field to be into the page. We are interested in the fields at the point indicated.



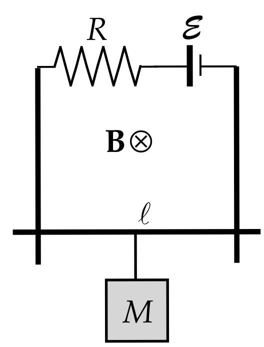
   a. Use the integral form of Faraday's law to find the E-field (magnitude and direction) at the point, if $r > R$. [Make use of the symmetry.]

   b. Repeat for $r < R$.

a. Choose a circle about the symmetry axis, passing through the point. By the symmetry the induced E-field has the same magnitude at all points on the circle, and is tangent to it, so $\oint \mathbf{E} \cdot d\mathbf{r} = 2\pi r E$. The flux through the circle is that through the cross-section, or $\Phi = B \cdot \pi R^2$. Faraday's law gives $2\pi r \cdot E = \pi R^2 \cdot dB/dt$, or $E = dB/dt \cdot R^2 / 2r$. The direction (by Lenz's law) is upward if $dB/dt > 0$, since an induced current would run counter-clockwise.

b. Now the flux through the circle is only $\Phi = \pi r^2 \cdot B$, so we find $E = dB/dt \cdot r/2$. Same direction.

3. Refer to the situation in a problem on the midterm exam, concerning a primitive motor. We will explore more fully the source of the energy that lifts the massive block.



   a. Find the emf $\mathcal{E}_0$ required to hold the block at rest.

   b. If the block moves upward at speed v, what is the induced back emf $\mathcal{E}'$ in the circuit?

   c. Taking this into account, what must be the emf supplied by the battery to keep the block moving at constant speed?

   d. The additional emf implies an additional amount of power $\mathcal{E}'I$ delivered by the battery. Show that it is exactly the power that lifts the block. [Recall that power input from a force is $P = \mathbf{F} \cdot \mathbf{v}$.]

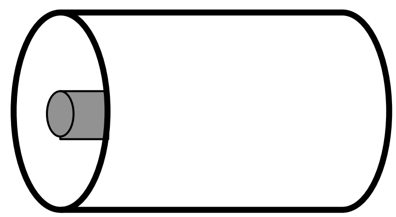
a. The force magnetic on the bar must equal the weight, so $I\ell B = mg$. But $I = \mathcal{E}_0 / R$, so $\mathcal{E}_0 = mgR / \ell B$.

b. The motional emf is $\mathcal{E}' = B\ell v$, opposite to the emf of the battery by Lenz's law.

c. The total emf must be the value of $\mathcal{E}_0$ in (a) because the total force on the bar is still zero, so the new emf of the battery must be $\mathcal{E}_0 + B\ell v$.

d. The power supplied by the extra emf is $B\ell v \cdot I = B\ell v \cdot (mg / B\ell) = mgv$, which is the power needed to lift the block at constant speed. [The magnetic forces can support the block but cannot do any work on it.]

4. Shown in perspective is a section of a coaxial cable. The inner wire has radius *a* and the outer thin cylindrical sheath has radius *b*. The section has length *d*. You are to calculate the self inductance of this section of the cable, by two methods.



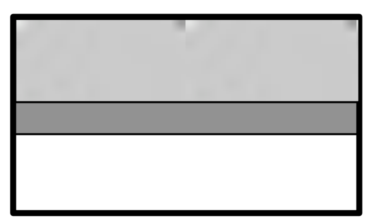
Let current *I* be carried to the right in the inner conductor and back to the left in the sheath.

a. What is the B-field in the region between the conductors? What is the magnetic energy density η_m in that region?

b. Integrate η_m over the volume of the region to find the total magnetic energy in this section of the cable. [For the volume element d^3r use a thin cylindrical volume of radius *r*, thickness *dr*, and length *d*.]

c. Use the formula for stored energy in an inductor to find *L*.

The second method uses the flux rule. Shown is the section of the cable in a view from the top.



d. Calculate the magnetic flux through the hatched rectangular area shown. [Consider the flux through a thin strip of length *d* and width *dr* parallel to the symmetry axis at distance *r* from that axis.]

e. Let the current be changing at rate dI / dt. Find the induced emf, and use it to find *L*.

a. The field in the gap (by Ampere's law) is $B = \mu_0 I / 2\pi r$. It is zero outside the cable, and we assume it is negligible inside the inner wire. The energy density is $\dfrac{\mu_0 I^2}{8\pi^2}\dfrac{1}{r^2}$.

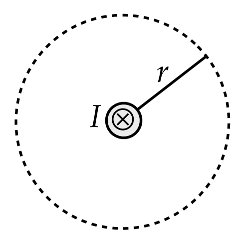
b. We have $U = \dfrac{\mu_0 I^2}{8\pi^2}\displaystyle\int_a^b \dfrac{2\pi r d \cdot dr}{r^2} = \dfrac{\mu_0 I^2 d}{4\pi}\ln(b/a)$.

c. Comparing to $U = \frac{1}{2}LI^2$ we find $L = \dfrac{\mu_0 d}{2\pi}\ln(b/a)$.

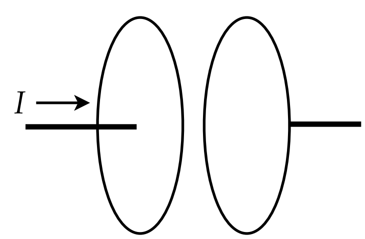
d. The flux is $\Phi = \displaystyle\int_a^b \dfrac{\mu_0 I d}{2\pi r}dr = \dfrac{\mu_0 d}{2\pi}\ln(b/a)\cdot I$.

e. From the flux rule we have $\mathcal{E} = -d\Phi/dt = -\dfrac{\mu_0 d}{2\pi}\ln(b/a)\cdot\dfrac{dI}{dt}$. Comparing to $\mathcal{E} = -L\dfrac{dI}{dt}$, we find the same result as in (c).

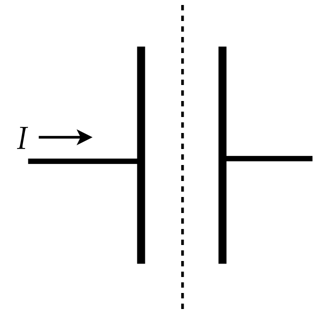
5. One uses Ampere's law to derive the B-field around a long straight wire as follows. Shown is the situation looking along the wire in the direction of the current *I*. By the symmetry the B-field lines must be circles around the wire as center, so we choose a circle of radius *r* for our path in the integral form of Ampere's law: $\oint \mathbf{B}\cdot d\mathbf{r} = \mu_0 I_{enc}$. The left side becomes $2\pi r \cdot B$, while the right side is simply $\mu_0 I$, so we get $B = \mu_0 I / 2\pi r$.



However, if the current is not steady there is a problem. Consider a capacitor with circular plates, shown in perspective view. The current carries charge to the plates. We ask for the B-field at a point in a plane passing through the gap between the plates, indicated by the dotted line in side view. The situation still has axial symmetry, so the field lines are circles



around the symmetry axis (the wire). We can repeat the analysis given above. But now the current passing through the surface bounded by the integration path is zero, because no current runs between the plates. We conclude that the B-field is zero in this plane. That is wrong. You are to show how including the displacement current in Ampere's law solves the problem.



a. Let the plates have area A and at the moment shown carry uniform charge Q. What is the E-field in the gap?

b. What is the displacement current density in the gap in terms of I? [Recall that $I = dQ / dt$.]

c. Show that for a point in the given plane but outside the gap (i.e., at a distance from the symmetry axis greater than the radius of the plates) the B-field is given by the formula derived above.

d. Find the B-field at a point in the plane but inside the gap.

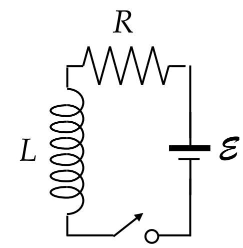
a. We have $E = \sigma / \varepsilon_0 = Q / A\varepsilon_0$.

b. The density is $\varepsilon_0 \cdot dE / dt = \varepsilon_0 \cdot (1 / A\varepsilon_0) \cdot dQ / dt = I / A$.

c. Using the usual geometry we have $2\pi r \cdot B = \mu_0 \int \varepsilon_0 \cdot dE / dt \cdot dA = \mu_0 I$, which leads to the formula above.

d. For r inside the gap the flux of the displacement current density is $\int \varepsilon_0 \cdot dE / dt \cdot dA = (I / A) \cdot \pi r^2$, so we find from Ampere's law $2\pi r \cdot B = \mu_0 I \cdot (\pi r^2 / A)$, or $B = \frac{1}{2} \mu_0 I \cdot r / A$.

6. When the switch is closed in the circuit shown, the current rises toward its steady final value I_m.



a. Write the circuit equation (Kirchhoff's loop rule) and arrange it as a differential equation for $I(t)$. Solve it with the initial condition $I(0)=0$.

b. Show that the power supplied by the battery ($\mathcal{E}I$) is equal to the power going to Joule heating in the resistor plus the power gong to build the magnetic field energy in the inductor.

a. We have $\mathcal{E} - L\dfrac{dI}{dt} - IR = 0$. This gives $\dfrac{dI}{dt} + \dfrac{R}{L}I = \dfrac{\mathcal{E}}{L}$. The solution is

$I(t) = \dfrac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$.

b. The power to the resistor is I^2R, and the power to the magnetic field is $LI \cdot dI/dt$. The total is $I \cdot \left[IR + L \cdot \dfrac{\mathcal{E}}{R} \cdot \dfrac{R}{L} e^{-Rt/L}\right]$. The quantity in [ ] is in fact $\mathcal{E}$, which proves the claim.