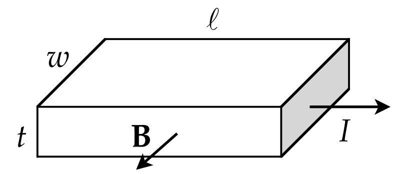
Physics 182

Assignment 3

1. A rectangular slab conductor carries uniform current I as shown. It is in a uniform B-field directed across the slab as shown. We are interested in the effect of the B-field on the microscopic particles whose flow constitutes the current.



a. Assume those particles carry positive charge. Which direction will they be deflected by the B-field?

By the Lorentz force $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$, if the moving charges are positive and **v** is to the right, the deflection is downward.

b. The deflection causes an accumulation of positive charge on one surface of the conductor and negative charge on the other, creating an E-field.. What direction is that field in this case?

Since positive charge accumulates on the bottom, **E** is upward.

c. The force on the moving charges due to the E-field balances that due to the B-field when equilibrium is established. What magnitude must the E-field have if the charges are moving with speed v?

We must have $qE = qvB$, or $E = vB$.

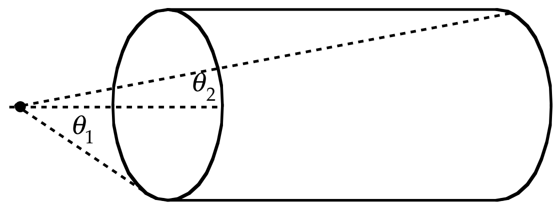
d. What will be the potential difference between the top and bottom of the slab? Which surface is at higher potential?

Assuming the E-field is uniform, $\Delta V = vBt$, with higher potential at the bottom.

e. How would things change if the charge carriers were negative, and moving in the direction opposite to I?

With negative charge, **v** would be to the left, deflecting the charges down again. This would mean the E-field is downward, and the higher potential would be at the bottom. {This is in fact what experiment (the Hall effect) shows, because the moving charges are electrons. The potential difference is very small because the speed v is very small.]

2. The B-field on the axis of a single circular loop of wire carrying a current is worked out in G. That formula can be used to find the field along the axis of a solenoid, which is a closely wound coil of circular cross-section as shown.



a. Let the coil be wrapped with n turns per unit length. Then the amount of current in a strip around the coil of width dz will be $dI = I \cdot n \cdot dz$. Integrate over z from one end of the coil to the other to find the field in terms of the two angles shown.

The strip makes a contribution $dB = \frac{\mu_0 nI}{2} \frac{R^2 dz}{(R^2 + z^2)^{3/2}}$. To integrate this we change the variable to θ, where $\tan\theta = R / z$. Then $R / \sqrt{R^2 + z^2} = \sin\theta$ and $dz = -(R / \sin^2\theta) d\theta$. The integral becomes $B = -\frac{\mu_0 nI}{2} \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta = \frac{\mu_0 nI}{2} (\cos\theta_2 - \cos\theta_1)$.

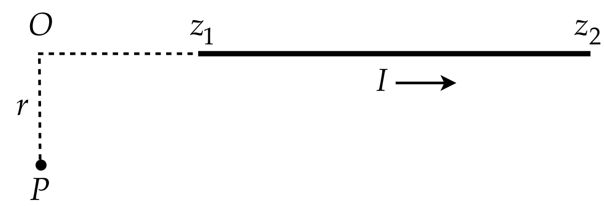
b. Show that if the solenoid is infinite (so the field point has to be inside it) the magnitude of the field is $\mu_0 nI$.

In that case $\theta_2 = 0$ and $\theta_1 = \pi$, so we get $B = \mu_0 nI$.

3. Use Ampere's law in integral form to find the field inside a toroidal coil. (This is worked out in G using the Biot-Savart law). Show that the field outside the coil is zero.

Use circular path inside the coil. By the symmetry, **B** is tangent to this circle and has the same magnitude at all points. So $\oint \mathbf{B} \cdot d\mathbf{r} = B \oint dr = 2\pi r \cdot B$. The linked current is NI, where N is the number of turns. Ampere's law gives $B = \frac{\mu_0 NI}{2\pi r}$. If the path is outside the coil, the current linked is zero, so the field is zero.

4. Find the vector potential **A** at point P for a finite straight section of wire carrying current I as shown. The ends of the wire segment are at z_1 and z_2. Use cylindrical coordinates (r, ϕ, z) and make the usual replacement $\mathbf{j}(r')d^3r' \to I\,d\mathbf{l}$ for situations involving wires.



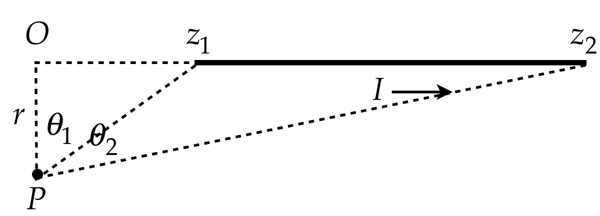
We have $A_z(r) = \frac{\mu_0 I}{4\pi}\int_{z_1}^{z_2} \frac{dz}{\sqrt{z^2+r^2}} = \frac{\mu_0 I}{4\pi}\ln\left(z+\sqrt{z^2+r^2}\right)\Big|_{z_1}^{z_2} = \frac{\mu_0 I}{4\pi}\ln\frac{z_2+\sqrt{z_2^2+r^2}}{z_1+\sqrt{z_1^2+r^2}}$.

5. Take your answer to the above question and show that it leads to the B-field derived from the Biot-Savart law in Example 5 in Chap 5 of G. [Use the fact that the B-field lines are circles around the z-axis, so **B** has only a ϕ-component. Look up the formulas for the curl in cylindrical coordinates in G.]

The only component of the curl that is non-zero is $B_\phi = -\frac{\partial A_z}{\partial r}$. Let $f(z,r) = z+\sqrt{z^2+r^2}$.

Then $\frac{\partial \ln(f)}{\partial r} = \frac{1}{f}\frac{\partial f}{\partial r}$. Using the angles shown below we find after some manipulation

that $\frac{1}{f(z_1,r)}\frac{\partial f(z_1,r)}{\partial r} = \frac{1-\sin\theta_1}{r}$, and similarly for z_2. Thus $B_\phi(r) = \frac{\mu_0}{4\pi}\frac{I}{r}(\sin\theta_2 - \sin\theta_1)$.



6. Since there is no magnetic equivalent of a point charge, the most elementary source of a magnetic field is a dipole. As is shown in Sec 5.4 of G, the vector potential of such a dipole, denoted by **m**, is given by

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\mathbf{r}}{r^3}.$$

Show that the B-field in this case is

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi r^5}[3(\mathbf{m}\cdot\mathbf{r})\mathbf{r} - \mathbf{m}r^2].$$

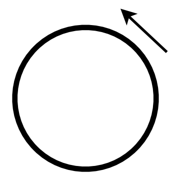
This is just a matter of carrying out the derivatives. Use the identity for $\nabla\times(\mathbf{A}\times\mathbf{B})$ given on the inside cover of G, and the fact that **m** is a constant.

7. A loop of wire carrying a current I creates a magnetic moment given in general by

$$\mathbf{m} = \tfrac{1}{2} I \oint \mathbf{r}' \times d\mathbf{l} \, .$$

As is shown in G, if the loop lies in a plane, then $\tfrac{1}{2}\oint \mathbf{r}' \times d\mathbf{l} = \mathbf{a}$, where $\mathbf{a}$ is a vector perpendicular to the plane of the loop, with magnitude equal to the loop's area. (The direction is given by a right hand rule: see G.)

a. Find the magnetic moment (magnitude and direction) of a circular loop of wire, of radius R, carrying current I counter-clockwise as seen from above.



Here $\mathbf{r}'$ is perpendicular to $d\mathbf{l}$, and the magnitude of $\mathbf{r}' \times d\mathbf{l}$ is $R^2\, d\theta$. The integral over the angle gives 2π and we have $\mathbf{m} = \pi R^2 I\, \mathbf{a}$.

b. Find the magnetic moment of a ring of total charge Q, uniformly distributed, of radius R, which rotates about its symmetry axis at angular speed ω. [It is like the current in (a).]

This is the same as (a), but we must find the current. The total charge Q passes a point in time equal to the period of the rotation, so $I = Q / T = Q \cdot \omega / 2\pi$. The magnetic moment is $\mathbf{m} = \tfrac{1}{2} R^2 Q\omega\, \mathbf{a}$.

c. Suppose the ring in (b) has mass m. What is the ratio of its magnetic moment to its angular momentum as it rotates? [This is called the *gyromagnetic ratio*.]

The angular momentum has magnitude $L = mR^2\omega$, so the gyromagnetic ratio is $Q / 2m$.