**Reading:** Electrostatics 2.

**Key concepts:** Electrostatic potential, conductors as equipotentials, calculating $V$ by superposition and from $E$, torque on a dipole, electrostatic potential energy.

1. **Questions about the relations between $E$ and $V$ in electrostatics.**
   a. If $E$ is uniform and in the $x$-direction, i.e., $E = E \cdot \hat{i}$, where $E$ is a constant, derive the formula for $V(x)$, where $V(0) = V_0$. Ans: $V(x) = V_0 - E \cdot x$.
   
   b. If $V$ is constant in a region, what is $E$ in that region?
   
   c. Show from the definition of $V$ in terms of $E$ that if one moves in the direction of the E-field the potential decreases.
   
   d. As one moves along the surface of a conductor the potential does not change. Does this mean that $E$ is the same at all points on the surface?

2. **Shown are two situations involving point charges on the $x$-axis, separated by distance $\ell$.**
   Let the location of $+q$ be the origin and choose $V(\infty) = 0$. In each case:
   
   a. At what points on the finite $x$-axis is the total potential zero?
   
   b. Answer the same question for the total E-field of the charges.

3. **An electron (charge $-e$) is released from rest in the vicinity of other charges.**
   
   a. Does it move toward higher or lower potential?
   
   b. Does it move toward higher or lower potential energy?
   
   c. Does it move parallel or opposite to $E$?

4. **Shown are lines of constant potential, each line 5 V lower than the one to its left.**
   
   a. What is the direction of the E-field? How do you know?
   
   b. Is the magnitude of $E$ increasing to the right? To the left? Not changing?
5. Questions about field lines.
   a. Can lines of the E-field cross each other? Explain.
   b. Can lines of equal potential cross each other? Explain.

6. “Grounding” means connecting an object to the earth by a conductor. The earth is itself a very large conductor which can accept or supply significant amounts of charge without measurably changing its potential. One usually chooses that potential to be zero. “Shielding” means surrounding a region of space by a grounded conductor. Here are some examples:
   a. An uncharged conducting spherical shell surrounds a point charge as $q$ shown. Charge $q$ appears on the outer surface of the shell. (Why?) What are the E-field and potential outside the shell? Does it matter whether the point charge is at the center of the shell’s cavity?
   
   b. We ground the shell. (The icon represents grounding.) The charge on the outer surface of the shell all runs off to the earth. What is the E-field outside the shell now?
   
   c. We bring up another point charge $+Q$ and place it near but outside the shell. How does the grounded shell manage to remain at potential zero?
   
   d. Is the force between the grounded shell and the charge $+Q$ an attraction or a repulsion? Does this force depend in any way on $q$ (its magnitude, its sign, or its location within the shell)?

7. Questions about electrostatic potential energy.
   a. The relation $U = qV$ is often interpreted as follows: If a point charge $q$ is placed at a point where the potential due to other charges is $V$, the point charge $q$ acquires potential energy $U = qV$. Explain why this is wrong.
   
   b. If $V = 0$ at infinite distance, then a system of charges that has total energy (kinetic plus potential) greater than zero is unbound. Explain why.
   
   c. One can describe the total potential energy of a system of charges either as (1) the work done by an agent to bring the charges in from infinite distance and place them at rest at their final locations, or, (2) the negative of the work done by electrostatic forces while the system is being assembled. Explain why these give the same result.
8. Shown is a charged conductor in the shape of an ellipsoid.
   a. Compare the potential at the surface at the points shown.
   b. Compare the magnitude of $E$ at those points.

9. A pair of thin concentric conducting spherical shells, of radii $a$ and $b$, are arranged as shown. The inner shell carries charge $+q$ while the total charge on the outer shell is $+Q$. Give answers in terms of these quantities.
   a. Find the magnitude of the E-field $E(r)$ where $r$ is the distance from the center, in three regions: I, $r > b$; II, $a < r < b$; III, $r < a$.
      
      Ans: I: $E = k(Q + q) / r^2$; II: $E = kq / r^2$; III: $E = 0$.
   b. Write the formula for the potential in those three regions, taking $V(\infty) = 0$.
      
      Ans: I: $V = k(Q + q) / r$; II: $V = kQ / b + kq / r$; III: $V = kQ / b + kq / a$.
   c. Suppose small holes are drilled through the spheres along the dotted line in the figure, and an electron (charge $-e$) is released from rest in the hole at the top of the outer shell (distance $r = b$ from the center). What is the kinetic energy of the electron when it is at the center of the spheres?
      
      Ans: $K(r = 0) = keq \left[ \frac{1}{a} - \frac{1}{b} \right]$.

10. Shown is a coaxial cable, made of an inner cylindrical conducting wire of radius $a$ and an outer conducting sheath of inner radius $b$. The wire carries uniform positive charge per unit length $\lambda$. The sheath is grounded ($V = 0$). Also shown is a cross section, with a point (indicated by the dot) at distance $r$ from the symmetry axis.
   a. What is the charge per unit length on the inner surface of the sheath? What is the direction of $E$ at the indicated point? How do you know?
   b. Use Gauss’s law (using a cylindrical surface passing through the point) to find the magnitude of the E-field at the point indicated. 
      
      Ans: $E = 2k\lambda / r$.
   c. Find the potential on the wire’s surface. 
      
      Ans: $V(a) = 2k\lambda \cdot \ln(b / a)$.
11. Uncharged conducting spheres $A$ and $B$ have radii $R_A$ and $R_B$. They are far apart so any charge on one does not affect the potential on the other. Take $V(\infty) = 0$.
   
a. Charge $Q$ is placed on $A$. What is the potential at its surface?
   
b. A long wire is connected between the spheres. What is the potential on the surface of the two spheres? [How does the charge divide between them?]
   
c. Express the E-field magnitudes at the surfaces of the two spheres in terms of the potential you found in (b).
   
d. Suppose the potential at the surface of $B$ is 1000 V. Find the value of $R_B$ for which the E-field magnitude at the surface is $10^6$ V/m.

12. Two point charges $+q$ are fastened at the bottom corners of an equilateral triangle of side $a$ as shown.
   
   ![Diagram of a triangle with two charges at the bottom corners](image)
   
a. Let charge $+q$ be placed at the top corner. It is released from rest while the others remain fixed.
      
i. Describe its motion.
      
ii. When it is far from the others, what is its kinetic energy? [What are the initial and final potential energies?]
   
b. Now let the charge at the top be $-q$, and again it is released from rest.
      
i. Describe its motion qualitatively.
      
ii. What is its maximum kinetic energy? [At what point in the motion is the potential energy the smallest?]

13. Questions about accelerating a proton (charge = $+e$).
   
a. Using $K = \frac{1}{2}mv^2$, through what potential difference $V$ must a proton at rest must be accelerated to reach the speed of light, $c$. Ans: $V = mc^2 / 2e$.
   
b. The correct kinetic energy formula (from relativity) is $K = mc^2(\gamma - 1)$, where $\gamma = 1 / \sqrt{1 - v^2 / c^2}$ . Use the binomial approximation to show that if $v / c \ll 1$ this gives $K = \frac{1}{2}mv^2$ as an approximation.
   
c. The correct formula shows it would take an infinite amount of energy to accelerate a particle to speed $c$. Find the ratio $v / c$ for a proton accelerated through the potential difference you found in (a). Ans: $v / c = \sqrt{5/9}$.