Final Exam

Solutions

In questions or problems not requiring numerical answers, express the answers in terms of the symbols for the quantities given, and standard constants such as g. In numerical questions or problems, use g = 10 m/s².

Part A: Choose the best answer; 4 points each.

1. The acceleration of a particle has constant magnitude a. Which of the following is wrong?

☛A. If its direction is also constant, the particle moves in a straight line. [Projectile motion.]

B. If the direction of the acceleration is perpendicular to the direction of the velocity, the speed does not change.

C. If the particle is moving in a circle of radius R at constant speed v, then $a = v^2/R$.

D. The total force on the particle has constant magnitude.

2. A system is subject to zero net external force and zero net external torque about the z-axis. The only mechanical quantities that are certainly conserved are:

A. All components of the linear momentum

B. The z-components of angular momentum and linear momentum.

☛C. All components of linear momentum and the z-component of angular momentum.

D. The z-component of angular momentum..

3. Three objects with the same mass roll without slipping from the same height down an incline: (1) a hoop of radius R; (2) a sphere of radius R; (3) a sphere of radius $2R$. When they reach the bottom the speeds are in the order:

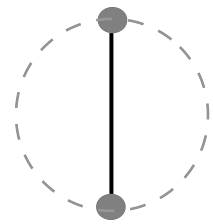
A. (1) = (2) = (3).

B. (1) < (2) < (3).

C. (1) > (2) = (3).

☛D. (1) < (2) = (3). [Hoop has more of its KE in rotation, less in CM motion.]

4. Two ice skaters, each of mass m, are holding the ends of a massless rope of length $2R$ and skating around in circles, each with speed v, with the center of the rope motionless. They move their hands along the rope until they are only distance R apart. Which of the following is wrong?



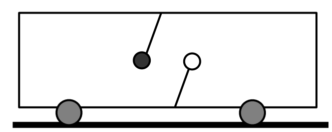
A. The center of the rope remains fixed.

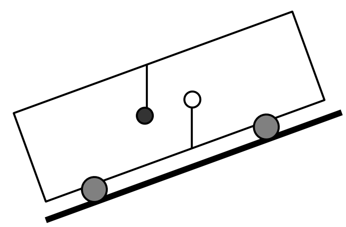
B. Angular momentum of the system is conserved.

☛C. Their new kinetic energy is twice the original kinetic energy. [4 times.]

D. Their new speeds are $2v$ each.

5. A railroad car has a ball of known mass m suspended from the ceiling, and a helium balloon fastened by a string to the floor. As seen from inside the car, the situation is as shown in the top figure. There are two possibilities for the angles the strings make with the ceiling: (1) the car is on a level track but accelerating to the right; (2) the car is at rest on an incline, as shown in the bottom figure. To determine, from inside the car, which is correct, you can:





A. Cut the string to the ball and seen how it falls.

B. Cut the string to the balloon and see how it rises.

C. Measure the tension in the string attached to the ball and compare it to the ball's weight as measured by a scale in the car. [These will be the same.]

☛D. Measure the tension in the string attached to the ball and compare it to mg.

6. A satellite is in a circular orbit around the earth. It is desired to give it enough energy to escape earth's gravity. Take potential energy to be zero at infinite separation.

☛A. The new total energy must be at least zero.

B. The speed must be at least doubled. [KE must be doubled, not speed.]

C. The satellite must be sent directly away from the earth.

D. None of the above is true.

7. A tenor at a party sings the sustained high C (about 1050 Hz) at the end of the aria *Di quella pira*. A crystal wine glass on a table cracks. The best explanation is:

   A. The glass was too full of wine and the pressure cracked the glass.

   B. The person drinking from the glass hated Verdi's operas.

   ☛ C. The natural frequency of vibration of the empty glass was about 1050 Hz.

   D. As with many tenors, the note was sung a bit sharp.

8. The longest pipes on an organ are usually about 5 m long, and are open at both ends. Take the speed of sound to be 340 m/s. Which of these is wrong?

   A. The frequency of the fundamental is about 34 Hz.

   B. The same fundamental frequency can be produced by pipes half as long if one end is closed.

   C. If one end is closed, only odd numbered harmonics are produced.

   ☛D. One of the above is not true.

Part B: Choose T or F, depending on whether the statement is true or false; 3 points each.

1. The direction of a conservative force is toward lower potential energy. **T**

2. Only torques due to internal forces can change the angular momentum about the CM. **F** [External.]

3. When you unscrew the cap from a bottle, your thumb and fingers exert opposite forces but parallel torques. **T**

4. In the orbiting space station, hot air does not rise as it does on earth. **T** [No buoyant force because $g_{eff} = 0$.]

5. If your distance from the source of a sound is doubled, the loudness decreases by about 3 db. **F** [About 6 db.]

6. The hammers that set the strings of the piano into vibration hit the string 1/7 of the length from one end; this prevents the 7th harmonic from being produced. **T**

Part C: Problems. Indicate your method clearly. An answer supported by no argument, or a fallacious one, may receive little or no credit. Each part of a problem carries the value of 5 points.

1-1. A fireworks projectile of mass 2 kg is launched into the air with initial velocity components $(v_0)_x = 10$ m/s and $(v_0)_y = 50$ m/s.

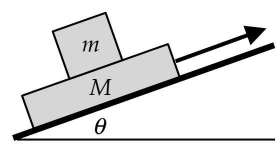
a. How long does it take for it to reach the highest point in its flight?

b. When it is at that highest point an explosion breaks the projectile into two pieces of mass 1 kg. One piece is brought to rest by the explosion and drops to earth. How far from the point of launch does the other piece land? [What is its velocity just after the explosion? Draw a picture.]

a. At the top $v_y = 0$, so $(v_0)_y - gt = 0$, or $t = 50/10 = 5$ s.

b. After the explosion, the moving piece has $v_x = 20$ m/s and $v_y = 0$. It falls for 5 s before landing, so it moves 100 m after the explosion, plus 50 m before the explosion, or a total of 150 m from the launch site. [The other piece lands at the same time 50 m from the launch site; the CM lands at 100 m.]

1-2. A box of mass m sits on a sled of mass M which is being pulled up a frictionless incline as shown. The static friction coefficient between the sled and the box is μ_s.



c. Find the maximum acceleration the system can have if the crate is not to slip backwards on the sled. [Draw a free-body diagram for the box.]

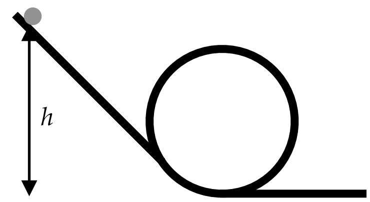
d. For that maximum acceleration, what is the tension in the rope pulling the sled? [Treat the sled and box as a single system.]

c. The maximum friction force (up the incline) is $f_s = \mu_s N = \mu_s mg\cos\theta$. The total force is $f_s - mg\sin\theta = mg(\mu_s\cos\theta - \sin\theta) = ma$, so $a = g(\mu_s\cos\theta - \sin\theta)$.

d. Fiction is now internal, so the total external force is $T - (M+m)g\sin\theta = (M+m)a$, or $T = (M+m)g(\mu_s\cos\theta - \sin\theta) + (M+m)g\sin\theta = \mu_s(M+m)g\cos\theta$.

Or, consider the sled alone: $T - Mg\sin\theta - f_s = Ma = Mg(\mu_s\cos\theta - \sin\theta)$, leading to the same result.

2-1. A small ball of mass m, radius r, and moment of inertia $I = \frac{2}{5}mr^2$, rolls down the track shown, and then rolls around the inside of the circular part of the track, which has radius $R >> r$.



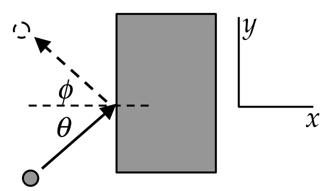
a. If it barely makes it across the top of the circle, what is its kinetic energy at that point, in terms of m, R and g?

b. What is the minimum height h for it to make it across the top of the circle?

a. The normal force from the track is zero, so the ball's CM speed must obey $mg = mv^2 / R$, or $v^2 = Rg$. So the CM kinetic energy is $\frac{1}{2}mv^2 = \frac{1}{2}mRg$. The rolling condition gives $\omega = v / r$, so its rotational kinetic energy is $\frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}mr^2\right)(v/r)^2 = \frac{1}{5}mv^2 = \frac{1}{5}mRg$. The total kinetic energy is $\frac{7}{10}mRg$.

b. Conservation of energy: $mgh = \frac{7}{10}mgR + 2mgR = 2.7mgR$, or $h = 2.7R$.

2-2. A ball of mass m collides elastically with the surface of a block of mass M. The ball's initial momentum $\mathbf{p}$ and final momentum $\mathbf{p'}$ make angles θ and ϕ with the normal to the block's surface as shown. The surface is frictionless so the block and ball can exert no force on each other in the y-direction. The final momentum $\mathbf{P}$ of the block thus has no y-component.



c. Use $K = p^2 / 2m$ for kinetic energy, and write the equations that result from the conservation laws in terms of the variables given.

d. Show that if $M \to \infty$, then $p' = p$, $\phi = \theta$, and $P = 2p\cos\theta$.

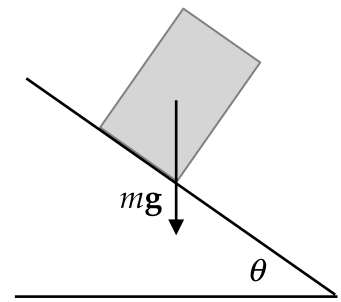
c. Conservation of P_x: $p\cos\theta = -p'\cos\phi + P$. Conservation of P_y: $p\sin\theta = p'\sin\phi$.

Conservation of K: $\dfrac{p^2}{2m} = \dfrac{p'^2}{2m} + \dfrac{P^2}{2M}$.

d. Since P is not infinite, conservation of K shows that as $M \to \infty$, $p' \to p$. Then conservation of P_y shows that $\phi \to \theta$, and conservation of P_x gives $P \to 2p\cos\theta$.

[A very heavy block, or a wall, receives considerable momentum from the collision but very little energy.]

3. A tall crate is on an incline. The angle of the incline is such that the line along the direction of the crate's weight passes through its lower right corner as shown.



   a. If it at rest because of static friction, show that it does not tip over (barely).

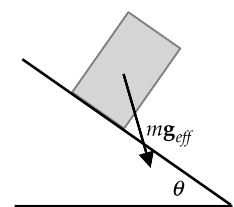
   b. If it is sliding but kinetic friction is slowing it down, show that it tips over.

   c. If it is sliding down with increasing speed, show that it does not tip over.

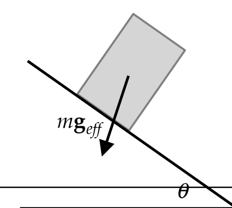
   [Consider torques about the lower right corner. Draw diagrams showing the direction of $\mathbf{g}_{eff}$.]

a. None of the forces give torques about the lower right corner because they all have zero moment arm. (The normal force acts effectively at that corner, and friction and gravity act alongs lines through it.)

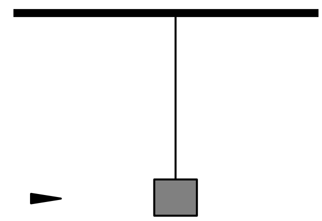
b. The acceleration is up the incline, so $\mathbf{g}_{eff} = \mathbf{g} - \mathbf{a}$ is tilted to the right of $\mathbf{g}$ so its line of action passes to the right of the lower right corner as shown, giving a cw torque. There is no force that gives a ccw torque, so the crate rotates cw.



c. Now the acceleration is down the incline, so $\mathbf{g}_{eff} = \mathbf{g} - \mathbf{a}$ is tilted to the left of vertical, and its line of action pass to the left of the lower right corner as shown. This gives a ccw torque, easily balanced by the cw torque of the normal force.



4-1. A wood block of mass M is suspended by a massless string of length ℓ as shown. A bullet of mass m moving horizontally at speed v strikes the block and sticks in it. The block-bullet system then swings, reaching a maximum height h above its original level before swinging back down. We wish to determine v from a measurement of h.



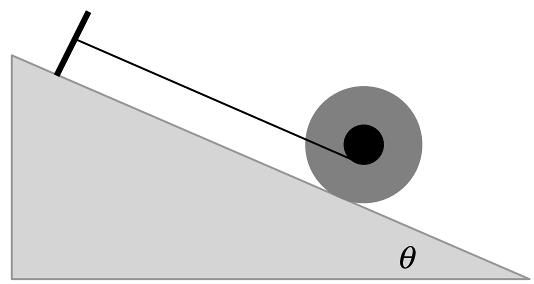
a. What is the speed v' of the block-bullet system just after the bullet sticks in the block? [What is conserved in the collision?]

b. Find v in terms of h and the masses? [What is conserved in the subsequent motion?]

a. Horizontal momentum (or angular momentum about the suspension point) is conserved. We have $mv = (M+m)v'$, so $v' = (m/M+m)v$.

b. Conservation of energy: $\frac{1}{2}(M+m)v'^2 = (M+m)gh$, so $v^2 = 2gh\left(\frac{M+m}{m}\right)^2$.

4-2. A wheel of mass m is held at rest on an incline as shown. The wheel has radius R and the axle with the string wrapped around it has radius $R/2$. There is enough friction so the wheel does not move.



c. What direction is the static friction force? How do you know?

d. What is the tension T in the string?

c. The tension gives a cw torque about the CM. Gravity and the normal force give no torque because they have no moment arms. So friction must give a cow torque, and thus must be directed down the incline.

d. Torques about CM: $T\cdot(R/2) = f_s \cdot R$, so $f_s = T/2$. Forces up the incline: $T - f_s - mg\sin\theta = 0$, so $T = 2mg\sin\theta$. [To hold it at rest with less tension, wrap the string around the axle the other way, so friction acts up the incline.]

5-1. Questions about using Kepler's 3rd law.

a. An astronomical unit (AU) is the distance from the sun to the earth. So for earth's orbit, $a = 1$ AU and the period T is 1 yr. The period of Halley's comet is about 75 yr. What is its semi-major axis a, in AU?

b. The comet's perihelion distance (closest approach) is about 0.6 AU. What is is aphelion distance (furthest recession)?

c. The radius of moon's orbit around earth is about 4×10^8 m and its period is about 28 days. What is the radius of a satellite orbit around the earth that has a period of 1 day?

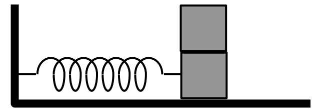
[Set up the calculations so all you have to do is use a calculator to get the answers. If you have time and a good calculator, do the numbers.]

a. For earth, using AU for distance and yr for time, $a^3 / T^2 = 1$. So for the comet $a^3 / (75)^2 = 1$, or $a = (75)^{2/3} = 17.8$ AU.

b. Call the aphelion distance x. Then $x + 0.6 = 2a = 35.6$ AU, or $x = 35$ AU.

c. For the moon (using m for distance and days for time) $a^3 / T^2 = (4 \times 10^8)^3 / (28)^2 = 8.16 \times 10^{22}$. So for the satellite $a^3 / 1^2 = 8.16 \times 10^{22}$, or $a = 4.34 \times 10^7$ m. [The actual number is 4.22×10^7 m. This kind of orbit is used for communications satellites, because it stays above a particular spot on earth.]

6-1. Shown are two identical blocks of mass m, with one on top of the other and the lower block attached to a spring of stiffness k. The lower block slides on a frictionless surface. The coefficient of static friction between the blocks is μ_s.



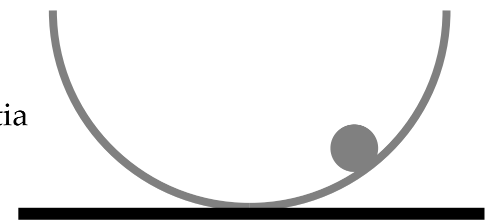
a. Assuming the top block does not slip on the bottom one, what is the angular frequency ω of the oscillation?

b. What is the largest value of the oscillation amplitude A for which the top block will not slip?

a. We have $\omega = \sqrt{k/2m}$.

b. The maximum acceleration is $a_{\max} = \omega^2 A = (k/2m)A$. This must be provided by friction. Use the maximum friction force $f_s = \mu_s mg$. Then $\mu_s mg = ma_{\max}$, or $a_{\max} = \mu_s g$. So $A = 2\mu_s mg/k$.

6-2. A small disk rolls back and forth across the bottom of a circular track of radius R. The disk has radius $r << R$, mass m, and moment of inertia $\frac{1}{2}mr^2$. We specify its CM location by distance s measured along the arc of the track, with $s = 0$ at the bottom. In the same approximation used for a pendulum, the gravitational potential energy is $U(s) = \frac{1}{2}m\left(\frac{g}{R}\right)s^2$. You are to find the angular frequency ω of the oscillation.



c. Write the kinetic energy of the disk in terms of its CM speed v.

d. Let the disk start from rest at $s = A$. Find its maximum CM speed $v_{\max}$. Then use the SHM relation between $v_{\max}$ and A to find ω.

c. The CM kinetic energy is $\frac{1}{2}mv^2$ and the rotation kinetic energy is $\frac{1}{2}I\omega^2 = \frac{1}{2}\cdot\frac{1}{2}mr^2\cdot(v/r)^2 = \frac{1}{4}mv^2$, so $K = \frac{3}{4}mv^2$.

d. Conservation of energy. Maximum speed is when $s = 0$, so we have $\frac{1}{2}\frac{mg}{R}A^2 = \frac{3}{4}mv_{\max}{}^2$. This gives $v_{\max}{}^2 = \frac{2}{3}(g/R)A^2$. Using $v_{\max} = \omega A$ we find $\omega^2 = 2g/3R$. [As with the pendulum, it is independent of the mass.]

7-1. Consider three wave functions describing waves in a certain medium:

$$y_1 = A\cos(20\pi x - 2000\pi t)$$
$$y_2 = A\cos(20.1\pi x - 2010\pi t)$$
$$y_3 = A\cos(20\pi x + 2000\pi t)$$

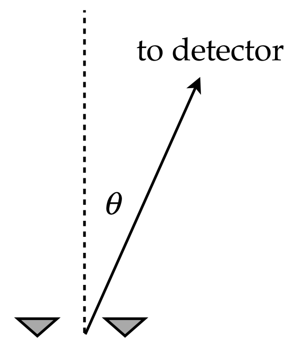
a. Which pair will combine to give a standing wave, and how far apart are the nodes?

b. Which pair will combine to give beats, and what is the beat frequency?

a. y_1 and y_3 give a standing wave. the wavelength is $\lambda = 2\pi / k = 0.1$ m, so the nodes are $\lambda / 2 = 0.05$ m apart.

b. y_1 and y_2 give beats. The two frequencies are $f_1 = \omega_1 / 2\pi = 1000$ Hz and $f_2 = \omega_2 / 2\pi = 1005$ Hz. The beat frequency is 5 beats/s.

7-2. The speakers shown emit sound of Intensity I_0 each, and of wavelength λ, in phase with each other. They are separated by horizontal distance $7\lambda / 2$.



c. What intensity is detected at $\theta = 0$? At $\theta = \pi / 2$?

d. At how many other angles between 0 and $\pi / 2$ is the intensity the same as at $\theta = 0$?

a. At $\theta = 0$ the path difference is zero, so $\delta = 0$ and $I = 4I_0$. At $\theta = \pi / 2$ the path difference is $7\lambda / 2$, so $\delta = (2\pi / \lambda) \cdot 7\lambda / 2 = 7\pi$, and $I = 0$.

b. The intensity is $4I_0$ when $\delta = 2\pi, 4\pi, 6\pi$ so there are three such angles.

8-1. A bicyclist rides at speed 1% of the speed of sound toward a large vertical wall. He is blowing a horn of frequency 400 Hz.

a. What is the frequency of the sound received by the wall?

b. How many beats per second does he hear in the combination of the sound he hears from the reflection and the sound directly from the horn?

[Use the binomial approximation.]

a. The wall is a stationary receiver, so we have

$$f_1 = 400 \cdot \frac{v}{v - 0.1v} = 400 \cdot \frac{1}{1 - .01} \approx 400 \cdot (1 + .01) = 404 \text{ Hz.}$$

b The wall reflects the frequency it receives. The bike rider, moving toward the wall, hears $f_2 = f_1 \dfrac{v + 0.1v}{v} = 404 \cdot (1.01) \approx 408$ Hz. So he hears $408 - 400 = 8$ beats/s.

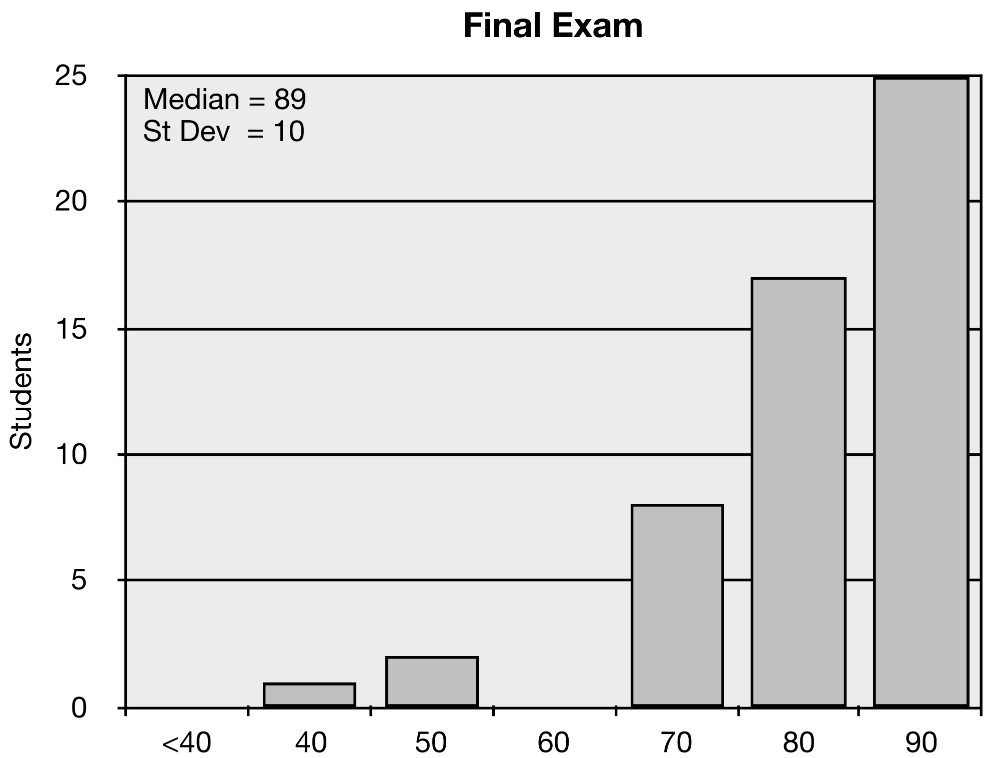
8-2. The string on a piano that produces frequency 440 Hz is 1.5 m long. The lowest note on the piano has frequency 55 Hz. The strings have the same tension. The mass per unit length of the string is μ_0 for 440 Hz and μ_1 for 55 Hz.

c. If $\mu_1 = \mu_0$, how long would the string for the lowest note be?

d. The string for the lowest note is only 2 m long. What is the ratio μ_1 / μ_0? [Find the ratio of the wave speeds.]

c. Since $f = v / 2L$, the string would be 8 times as long, or 12 m. [Much too long for any room.]

d. Using $f = v / 2L$, or $v = 2Lf$, we have $v(440) / v(55) = (1.5/2) \cdot (440/55) = 6$. Using $v^2 = T / \mu$ we have $\mu_1 / \mu_0 = 36$. {Take a look inside a piano.]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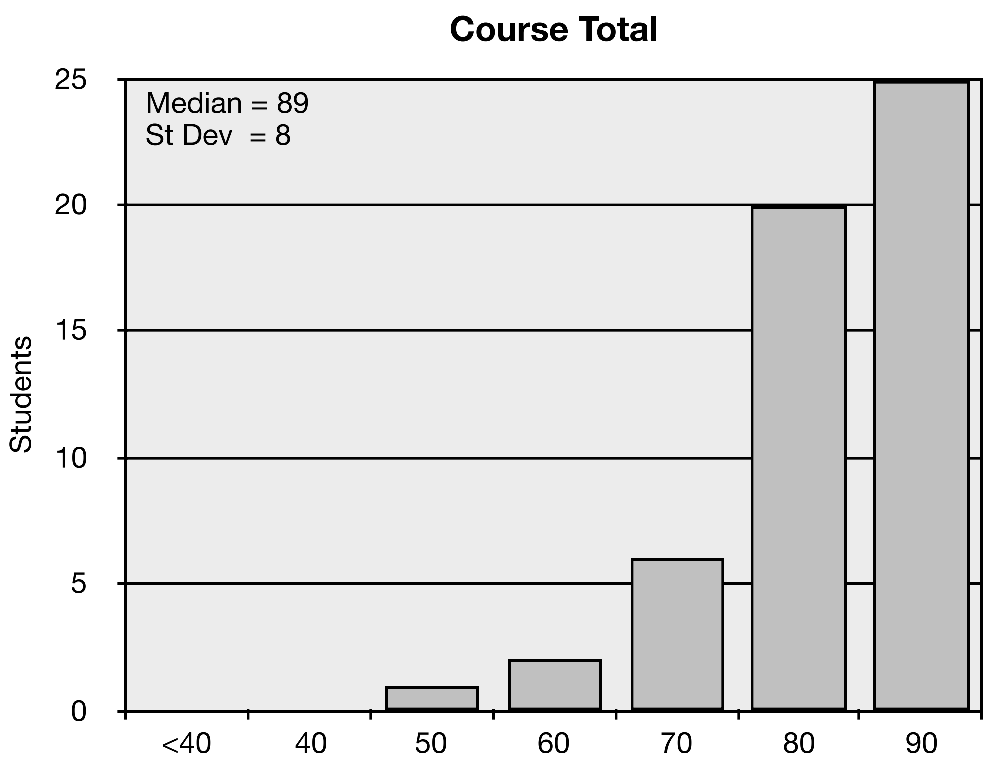
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